The optimal taxation of air travel under monopolistic dynamic pricing

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Abstract

This article re-examines the question of how to optimally tax air travel within the model from Gallego and van Ryzin (1994), in which a monopolistic airline chooses its dynamic pricing policy to sell tickets to randomly arriving consumers over a finite time horizon until the plane departs. In general, the profit maximizing policy differs from the welfare maximizing policy. However, for a certain class of demand functions that includes constant elasticity and exponential demand functions, a simple policy instrument, namely a tax on vacant seats is sufficient to perfectly align profit maximization incentives with welfare maximization. Calibrating the model to predict a load factor of 80% (the current global average), the welfare maximizing tax on vacant seats leads to load factors of 97% for the constant elasticity demand function and 98% for the exponential demand function. These results suggest that club mechanisms for financing global public good institutions via aviation taxes will create stronger participation incentives if they do not constrain countries to use passenger taxes but instead allow them to use emissions taxes and even taxes on vacant seats.

1 Introduction

In 2018, 18.8% of available seat kilometers on passenger flights remained vacant in the world. If the vacant seats were taxed, then airlines would adjust their dynamic pricing policies so as to fill more seats. Using a second instrument such as a tax/subsidy on occupied seats, one could achieve that the available

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seat kilometers would be unchanged but the planes would be filled with more passengers.[1] Would such a reform increase welfare?

This article studies this question in the monopolistic dynamic pricing model introduced by Gallego and van Ryzin (1994). In the model, an airline sells tickets for a single flight over a finite time horizon to consumers that arrive according to a Poisson point process. At each point in time the airline posts a price, given how much time and how many vacant seats remain. When a consumer appears, her valuation for the flight is randomly drawn from a time-invariant distribution D, independently of the other consumers. If her valuation exceeds the price posted by the airline, she purchases the ticket. Otherwise, she does not purchase it and disappears.

An increase in the tax on vacant seats causes the profit maximizing airline to lower its Markovian pricing policy everywhere (shown in proposition 7): In other words: For each remaining time and remaining number of vacant seats, the airline will set a lower price. For welfare, this has two effects: On the one hand, it will lead to additional passengers being taken on board, which yields welfare gains, as long as ticket prices are strictly positive. On the other hand, if it turns out that many consumers with high valuations appear later, some of them will end up not taking the flight. This constitutes foregone welfare.

In proposition 5 I show that for certain thick-tailed distributions of valuations the second effect dominates the first. However, for the distributions of valuations that are most commonly used in the literature, which correspond to constant elasticity and exponential demand functions, the first effect always dominates the second. In fact, for a class of demand functions that includes constant elasticity and exponential demand functions, I show with proposition 3 that a tax on vacant seats with a certain strictly positive rate achieves that the incentives of the profit maximizing airline will be perfectly aligned with the objective of social welfare maximization. In particular, under this optimal tax on vacant seats the profit-maximizing airline will choose the welfare maximizing pricing policy.

These results rationalize the UK's attempt in 2010 to replace its passenger duty with a per-plane tax. The UK government seems to have been convinced that a per-plane-tax could increase social welfare by increasing load factors (see Dresner (2010)). However, the UK abandoned the reform, recognizing that only distance-based air passenger taxes like their passenger duty are unambiguously legal under international law (see Larsson et al. (2019)). Despite these legal problems, the EU included international aviation in its emissions trading scheme in 2012. However, other countries strongly opposed this measure which they considered to be illegal under international law. In the face a looming risk of

[1] To see that this claim is plausible, consider the following algorithm: Suppose the status quo load factor (i.e. proportion of occupied seats) is Z and we aim to raise the load factor to Z^* whilst leaving the passenger kilometers unchanged. Start by introducing a small tax on vacant seats. This will increase the average load factor. It might also change the number of passenger kilometers. Now adjust the tax/subsidy on all seats so as to achieve that the passenger kilometers is like under the status quo. This adjustment might also affect the average load factor. However, presumably the load factor will still be higher than under the status quo.

retaliatory measures, the EU exempted international flights to and from non-EU states from November 2012 onward.

By default, the EU will again include international flights in its emissions trading scheme from 2024 onward (Larsson et al 2019). This could again lead to tensions as in 2012. To reduce the risk of opposition by other countries, the EU countries could use distance-based air passenger taxes[2], since these are legal under international law (Larsson et al 2019). However, an alternative approach could be for the EU to initiate a club mechanism that would require participating countries to tax aviation emission (or to put in place emissions trading schemes), whilst also requiring them to allocate a certain fraction of the tax revenue to Global Public Good Institutions (GPGIs) (see Stern (2019) for a comparative analysis of this and two alternative such proposals). The larger the proportion of the tax revenue that would be raised for GPGIs, the more beneficial the mechanisms would be for those who do not participate. Thus if a sufficiently large proportion of the tax revenue was raised for GPGIs, then other countries would be unlikely to oppose the mechanism.

The results obtained in the current study weigh in favor of the use of emissions taxes instead of distance based air passenger duties for such a club mechanism. In fact, replacing a distance based air passenger duty with an emissions tax resembles the removal of a subsidy on vacant seats. Proposition 3 suggests that this would, under the assumptions identified in the proposition, increase social welfare for the countries to whose flights the measure is applied[3]. In fact, by further adding a tax on vacant seats such as a tax that is both proportional to emissions and to the proportion of seat kilometers left vacant, further welfare gains would be realized according to proposition 3. Thus a club mechanism allowing countries to use emissions taxes and even such taxes on vacant seats would increase the incentives for countries to participate.

The paper is organized as follows. Section 2 reviews the literature that this paper builds on. Section 3 defines the model and reproduces a standard heuristic derivation of the Bellman equation. Section 4 derives the main result about how a tax on vacant seats can for a certain family of demand functions perfectly align profit maximization incentives with welfare maximization. Section 5 gives the results from the numerical calibration. Section 6 explains a result that highlights that complementary regulations are potentially required to ensure that the introduction of a tax on vacant seats robustly increases social welfare. Section 7 discusses the implications of the results for the optimal design of international agreements. Section 8 concludes. Most proofs are relegated to the appendix.

[2]Taxes at the European level require unanimity, whilst for emissions trading scheme a simple majority suffices. Thus it would be more likely that the enthusiastic EU countries would individually set these taxes instead of hoping for consensus at the EU level.

[3]Proposition 3 shows that an appropriately chosen positive tax on vacant seats aligns profit maximization incentives with welfare maximization. I conjecture that social welfare is a single-peaked function in the tax on vacant seats.

2 Related Literature

Keen and Strand (2007,2012,2013). use Walrasian equilibrium analysis to study taxes on aviation fuel, sales taxes on tickets and trip taxes (passenger charges that depend on distance but not the ticket price). The current paper aims to complement the analysis by explicitly taking into account that the choice of taxes will affect load factors. In fact, the task of finding the optimal tax system for aviation can roughly be decomposed into two dimensions: Firstly, using taxes to influence the set of flights that are supplied and how they are supplied. Secondly, using taxes to influence for a given set of flights how the tickets are sold and in particular how many of them are sold. The Walrasian equilibrium analysis as done by Keen and Strand (2007,2012,2013). can inform the first dimension. It is on the second dimension that I focus the attention in this paper.

For this, I use the model from Gallego and van Ryzin (1994), where a monopolistic airline sells tickets over a finite time horizon to consumers that arrive following a Poisson process. The airline chooses an entire Markovian pricing policy, specifying the price to set conditional on the number of remaining seats and time until departure. Taxes can influence the entire Markovian pricing policy. McAfee and te Velde (2006) apply this model to do welfare analysis for the case of an exponential distribution of consumer valuations. They do so by comparing the Bellman equation for the profit maximization problem and the welfare maximization problems. They find: "Thus the efficient solution is the solution a monopoly whose costs are reduced by ... the static monopoly profit, would choose." From this it follows immediately in the model that a tax on vacant seats is a sufficient policy instrument for inducing the profit maximizing airline to choose the welfare maximizing pricing policy. Proposition 3 generalizes this result to a large class of demand functions that includes the exponential functions and the constant elasticity functions.

McAfee and te Velde (2008) consider the case of the model where the consumers' random valuations are distributed according to a constant elasticity function. Their paper studies exclusively the case without any taxes. They compute closed form solutions for the profit maximizing pricing policy and for the welfare maximizing policy and find that the two coincide. However, I show (see proposition 5) that their version of the model, which assumes that the marginal cost of filling a seat is 0, predicts that the vacancy rate will be 0% with probability 1. I calibrate the model so as to reproduce in the absence of any taxes both an observed vacancy rate of 18.8% (the current global average) and the observed marginal cost of filling a seat. I find that a tax on vacant seats that lowers the vacancy rate to 3% is required to achieve maximal social welfare (see section 5.2).

3 The model

The model that I use was introduced by Gallego and van Ryzin (1994). An airline considers a given flight with N seats in isolation and sells tickets over a sales horizon $[0, T]$ for a flight. For each small time interval of duration dt there is a probability λdt that a potential passenger will appear and consider buying a ticket. The passenger's valuation v is defined to be the maximal price that she is willing to pay for the ticket.

Consumers arrive according to a homogeneous Poisson process. Each time a consumer arrives, the person's valuation v is randomly determined. The probability that the person has a valuation of at least y is denoted by $D(y)$. If a person's valuation is at least as high as the ticket price p, then she buys the ticket. Otherwise she does not buy it and disappears. Hence the probability that a potential passenger buys a ticket if faced with the price p is given by $D(p)$. Given this, I will call D the "demand function". The cost caused by the sale of an additional ticket is given by a constant c. This cost consists partly of the increase in fuel consumption due to an additional person on board, which turns out to be around 10 percent of the fixed fuel cost per seat (Borenstein and Rose (2014)).

As a government policy instrument I consider a subsidy of $\varphi(p)$ that is given to the airline if a ticket is sold at the price p. Within the model, a tax on all seats does not have any effect. Hence in particular, in the model a tax on vacant seats is equivalent to a constant subsidy for tickets sold.

The airline maximizes profits, without discounting the revenue it receives[4]. The airline's profit maximization problem reduces to setting a pricing policy $p(n, t)$, which specifies the price posted at time t if n tickets remain at that time.

Let $v_n(t)$ be the expected net revenue from ticket sales (that is the money paid by passengers plus the subsidy minus the cost due to additional passengers) from time t until the end of the sales horizon at time T, given that n tickets are left at time t and that the optimal pricing policy is pursued from that time onward.

The probability that a passenger arrives in a small time interval of length dt is given by λdt[5]. If the price is set at p then this person will with probability $D(p)$ buy a ticket. In that case the net revenue from that sale is given by $p - c + \varphi(p)$, because the airline receives the subsidy $\varphi(p)$ for the ticket and the cost c is incurred for every additional passenger on board. Moreover, in this case there are $n - 1$ tickets left to be sold from time $t + dt$ onward. With probability $(1 - D(p))$ the person does not buy a ticket and in that case there are n tickets left at time $t + dt$. Hence we have:

[4] Neglecting discounting is justified since the sales horizon is less than a year in practice

[5] Here I am assuming that the Poisson arrival rate is constant over time. In the more general case, one can define the "effective time" as a function of the real time such that the arrival rate with respect to this new time is constant. Thereby, one can reduce the model to the case with a constant Poisson arrival date that I am studying here.

$$v_n(t) = \lambda dt \left(max_p D(p) \left(p - c + \varphi(p) + v_{n-1}(t+dt) \right) + (1 - D(p)) v_n(t+dt) \right) + (1 - \lambda dt) v_n(t+dt)$$

Rearranging gives:

$$v_n(t) - v_n(t+dt) = \lambda dt \, max_p D(p) \left(p - c + \varphi(p) - \left(v_n(t+dt) - v_{n-1}(t+dt) \right) \right)$$

Dividing by dt yields:

$$\frac{v_n(t) - v_n(t+dt)}{dt} = \lambda max_p D(p) \left(p - c + \varphi(p) - \left(v_n(t+dt) - v_{n-1}(t+dt) \right) \right)$$

Taking the limit as dt goes to 0 yields:

$$-v_n^{'}(t) = \lambda max_p D(p) \left(p - c + \varphi(p) - \left(v_n(t) - v_{n-1}(t) \right) \right)$$

This equation shows the trade-off that the airline faces at each point in time: Selling a ticket generates net revenue of $p - c + \varphi(p)$. On the other hand, there is a loss in the option value, $v_n(t) - v_{n-1}(t)$, that comes from having one less available seat thereafter.

4 Welfare analysis

Now consider the problem of choosing the prices p over time so as to maximize expected welfare. We obtain the welfare as the sum over all the people taking the flight of their valuation minus the marginal cost c that arises due to each additional person that is taken on board. Let w_n(t) be the expected welfare accruing from time t until the end of the sales period at time T from the sale of tickets during that time interval $[t, T]$. Given that the price is p, the expected amount by which the valuation of a passenger who will buy a ticket at that price exceeds the price is given by:

$\frac{1}{D(p)} \int_{s=p}^{\infty} D(s) \, ds$

Similarly to the profit maximization problem we obtain:

$$w_n(t) = dt\lambda \, max_p D(p) \left(p - c + \frac{1}{D(p)} \int_{s=p}^{\infty} D(s) \, ds + w_{n-1}(t+dt) \right) + (1 - dt\lambda) w_n(t+dt)$$

Rearranging yields:

$$w_n(t) - w_n(t+dt) = dt\lambda \, max_p D(p) \left(p - c + \frac{1}{D(p)} \int_{s=p}^{\infty} D(s) \, ds + w_{n-1}(t+dt) - w_n(t+dt) \right)$$

Dividing by dt and letting dt go to 0 yields:

$$-w_n'(t) = max_p D(p)\left(p - c + \frac{1}{D(p)}\int_{s=p}^{\infty} D(s)\,ds + w_{n-1}(t) - w_n(t)\right)$$

The boundary conditions are identical to those in the profit maximization problem:

$$w_n(T) = 0$$

$$w_0(t) = 0$$

We write the two Bellman equations to see the analogy:

$$-v_n^{'}(t) = max_p D(p)(p - c + \varphi(p) - (v_n(t) - v_{n-1}(t)))$$

$$-w_n'(t) = max_p D(p)\left(p - c + \frac{1}{D(p)}\int_{s=p}^{\infty} D(s)\,ds - (w_n(t) - w_{n-1}(t))\right)$$

We observe that if $\varphi = \frac{1}{D(p)}\int_{s=p}^{\infty} D(s)\,ds$ then the two Bellman equations are identical. We also note that the value function arising from welfare maximization satisfies the same boundary condition as the value function of the profit maximization problem, namely $w_n(T) = 0 \forall n$ and $w_0(t) = 0$. From this we can deduce that if $\varphi = \frac{1}{D(p)}\int_{s=p}^{\infty} D(s)\,ds$ then the value function arising from welfare maximization is the same as the value function arising from profit maximization and also that the set of pricing policies (typically a singleton) that maximize profit is equal to the set of pricing policies that maximize welfare. In other words: the social optimum can be achieved by setting $\varphi(p)$ equal to the consumer surplus of a consumer conditional on the event that her valuation exceeds the price p and then letting the airline pursue profit maximization. The consumer surplus is always positive. Hence this subsidy is positive for all prices.

I have assumed that the distribution of the valuations of the passengers considering at time t to buy a ticket does not change with the time t. Interestingly, if $D(p,t)$ is a function of time, then by setting $\varphi(p,t) = \frac{1}{D(p,t)}\int_{s=p}^{\infty} D(p,t)\,ds$, the profit maximizing policy is again welfare-maximizing. Let us write these general observations as a proposition:

Proposition 1. *Let $D(p,t)$ be the probability that a customer appearing at time t has a valuation of at least p. If a subsidy of $\varphi(p,t) = \frac{1}{D(p,t)}\int_{s=p}^{\infty} D(s,t)\,ds$ is given to the airline for each ticket that it sells at time t for the price p then this perfectly aligns the airline's profit maximization with welfare maximization.*

Example 1. Exponential demand function: $D(p) = e^{-ap}$

This demand function is commonly assumed in the literature on dynamics pricing in the airline industry (see e.g. McAfee and te Velde (2006)). Assuming this functional form, we compute:

$$\frac{1}{D(p)}\int_{s=p}^{\infty} D(s)\,ds = \frac{1}{a}$$

We observe that $\frac{1}{a}$ is the static monopoly price, i.e. the price that maximizes $pD(p) = pe^{-ap}$.

From our preceding discussion it follows that if a constant subsidy of $\frac{1}{a}$ for every ticket sold is given then profit maximization is equivalent to welfare maximization. Such a fixed subsidy is equivalent to a tax on the vacant seats at the rate $\frac{1}{a}$. We now state this result:

Corollary 1. *If the valuations are exponentially distributed with $D(p) = e^{-ap}$ then the socially optimal dynamic pricing policy is profit maximizing if a tax on vacant seat kilometers is introduced at a rate that equals the static monopoly price $\frac{1}{a}$.*

To prepare the analysis for constant elasticity demand functions, we now prove:

Lemma 1. *Suppose $\kappa > 0$. Then the subsidy $\varphi(p)$ achieves the same incentives for the pricing strategy as the subsidy $(\kappa - 1)(p - c) + \kappa\varphi(p)$.*

Proof. The airline is faced with the problem

$$max_{\rho} E(\sum_{i\in I(\rho)} p_i(\rho) - c + \varphi(p_i(\rho)))$$

Where ρ is the pricing policy, $I(\rho)$ is the set of seats sold and $p_i(\rho)$ is the price at which seat i is sold (both are random variables). c is the marginal cost of filling a seat. Since any maximization problem is preserved under multiplication by a positive constant κ , this maximization problem is equivalent to

$$max_{\rho} \kappa E(\sum_{i\in I(\rho)} p_i(\rho) - c + \varphi(p_i(\rho)))$$

Which can be rewritten as

$$max_{\rho} E(\sum_{i\in I(\rho)} p_i(\rho) - c + (\kappa - 1)(p_i(\rho) - c) + \kappa\varphi(p_i(\rho)))$$

□

Example 2. Constant demand elasticity: $D(p) = p^{-\varepsilon}$

We compute the consumer surplus, conditional on a valuation exceeding p : $\frac{1}{D(p)}\int_{r=p}^{\infty} D(r)\,dr = \frac{p}{\varepsilon - 1}$

Proposition 2. *Suppose that valuations are distributed so as to give rise to constant demand elasticity. Then a constant tax on the vacant seats equal to* $\frac{1}{\varepsilon}c$ *achieves that welfare is maximized.*

Proof. By proposition 1 we know that a constant tax on the vacant seats equal to $\frac{1}{D(p)}\int_{r=p}^{\infty} D(r)\,dr = \frac{p}{\varepsilon-1}$ achieves that welfare is maximized. By Lemma 1, we know that the same incentives, and as a result the same welfare, are achieved with subsidy on the occupied seats of

$$\kappa\frac{p}{\varepsilon-1} + (\kappa-1)(p-c)$$

where κ can be any positive real number.

Now let us choose κ such that the p disappears, i.e. let us pick κ such that $0 = \kappa\frac{1}{\varepsilon-1} + (\kappa-1)$, which just means $\kappa = \frac{\varepsilon-1}{\varepsilon}$. For this value we obtain a subsidy on occupied seats of

$$\left(\frac{\varepsilon-1}{\varepsilon} - 1\right)(-c) = \left(1 - \frac{\varepsilon-1}{\varepsilon}\right)c = \frac{1}{\varepsilon}c$$

Equivalently, this is also achieved through a tax on the vacant seats at that same value $\frac{1}{\varepsilon}c$. □

In order for the constant demand elasticity model to be adequate, the elasticity ε has to be greater than 1, since otherwise expected profits could be increased without bounds by simply charging higher and higher prices. Therefore, the preceding proposition 2 implies that the optimal tax rate on vacant seats is smaller than the marginal cost for filling seats.

We can also deduce directly from the proposition 2 that if $c = 0$ then welfare is maximized without any taxes or subsidies. This fact is also proved by McAfee and te Velde (2008). They do this by explicitly finding the Markovian pricing policy for both the profit maximization problem and the welfare-maximization problem and showing that the two policies are identical. In appendix A.3I extend their calculations. This allows me to prove that under their assumption of 0 marginal costs the model predicts an expected vacancy rate of 0. Thus clearly the model with $c = 0$ is not appropriate for assessing whether a tax on vacant seats or other instruments affecting the vacancy rate can improve welfare. In the appendix A.3 I also calibrate the model for this case of constant elasticity.

We have seen above that for both the constant elasticity and the exponential demand functions the tax on vacant seats suffices as an instrument to achieve the equivalence of the profit maximization and the welfare maximization problem. These two classes of demand functions happen to be the ones that are most frequently used in the literature on dynamic pricing. The question arises as to whether there are any other demand functions with this property. The next proposition shows that there are in fact further such demand functions:

Proposition 3. *For the following demand functions the tax on vacant seats is a sufficient instrument to achieve the perfect alignment between profit maximization and welfare maximization:*

demand function	optimal tax on vacant seats
$(a+p)^{-b}$ with $a \geq 0, b > 0$	$\chi = \frac{a+c}{b}$
e^{-gp}, with $g > 0$	$\chi = \frac{1}{g}$
$(max(a-p,0))^{b}$ with[6] $a > c, b > 0$	$\chi = \frac{a-c}{b}$

Proof. see appendix A.1. □

These demand functions cover a large class. Linear demand functions are contained as a special case of the third class $(max(a-p,0))^b$. For the numerical simulations, we will focus on the first two specifications, since they are the ones used in the prior literature, the conclusions of which this paper purports to challenge.

To interpret the size of the optimal tax rates, it is helpful to compare them to the static monopoly price in both cases. In the case of the first specification, $D(p) = (a+p)^{-b}$, the static monopoly price is $\frac{a+cb}{b-1}$, so the ratio of the optimal tax rate on vacant seats to the static monopoly price is $\frac{a+c}{a+cb}\frac{(b-1)}{b}$. Since $b > 1$, this is always smaller than 1. It is close to 0 if b is close to 1 or if b is large.

In the case of the second specification, $D(p) = exp(-ap)$, the static monopoly price is a, so the ratio of the optimal tax rate on vacant seats to the static monopoly price is 1.

Thus the predictions about the optimal tax on vacant seats (relative to the static monoply prices) is sensitive to the choice of specification. However, it appears that conclusions about what the optimal vacancy rate will be under the optimal tax on vacant seats are more robust. This is suggested by the calibrations I will present in the next section.

5 Numerical calibrations

We have seen that for the class of demand functions identified in proposition 3 the tax on vacant seat is a sufficient instrument to achieve that welfare is maximized. The two classes of demand functions that have been studied in the literature on the welfare economics of aviation taxation, exponential and constant elasticity, are included in this class. It is for these functional forms that we now calibrate the model to compute the optimal tax rate on vacant seats. For these calibrations we assume that at the status quo there are no aviation taxes. Since in reality existing taxes are very low this assumption should not substantially distort the results.[7]

5.1 Exponential distribution: $D(p) = e^{-ap}$

In this case the following result shows that we can directly deduce the optimal load factor (i.e. proportion of occupied seats) from the load factor that is

[7] It is clear that proposition 3 still holds if there are taxes on occupied seats in place: In the model, a ticket tax is equivalent to a subsidy on vacant seats. In fact, proposition 3 also still holds in the presence of sales taxes. This is shown in appendix A.4.

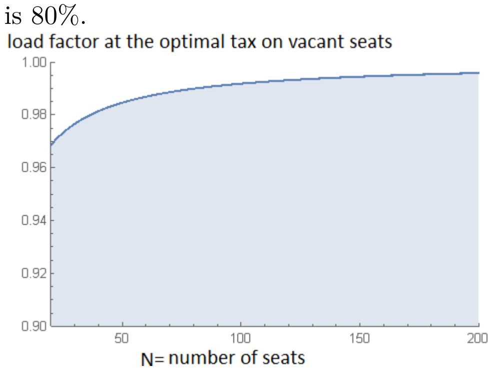
observed in the absence of any tax:

Proposition 4. *Suppose the demand function is exponential, $D(p) = e^{-ap}$. As always, denote by N the total number of seats on the plane. Let Z and Z^* be the expected proportion of seats occupied on the flight without any tax and with the optimal tax on vacant seats, respectively. Z^* can be deduced from Z according to the following formula* $Z^* = \frac{q(Z)e\frac{\sum_{k=0}^{N-1}\frac{(q(Z)e)^k}{k!}}{\sum_{k=0}^{N}\frac{(q(Z)e)^k}{k!}}}{N}$, *where $q(z)$ is implicitly defined by* $Z = \frac{q(Z)\frac{\sum_{k=0}^{N-1}\frac{q(Z)^k}{k!}}{\sum_{k=0}^{N}\frac{q(Z)^k}{k!}}}{N}$.

Proof. see in appendix A.2. □

Results from calibration: Suppose that in the absence of any taxes there is a vacancy rate of 20% (which is approximately the global average according to IATA). Then we obtain the following results:

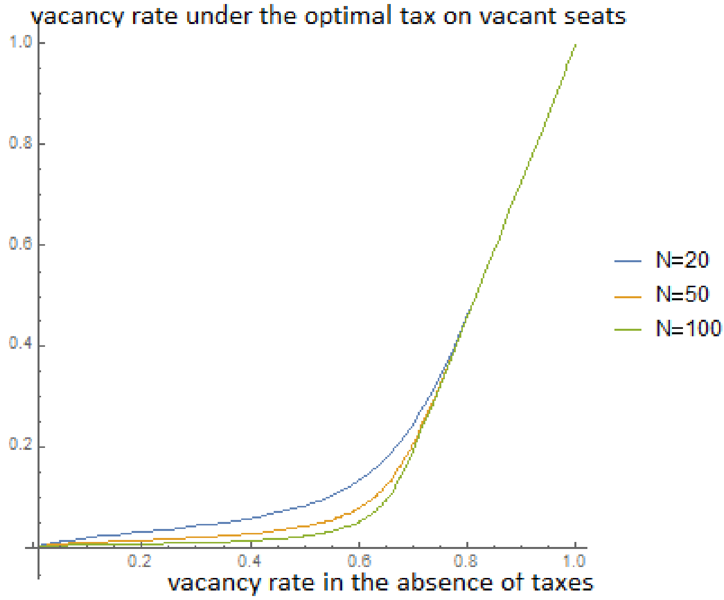
Figure 1: Optimal load factor if the load factor observed in the absence of taxes is 80%.



For example, suppose the number of seats is $N = 50$. Then at the optimal tax policy the optimal vacancy rate is 1.5%. In other words: According to the model with the exponential distribution of valuations the government should introduce a tax on vacant seats at a level that reduces the vacancy rate to 1.5%. $N = 50$ is the lower end for commercial passenger planes. For larger planes the optimal vacancy rate is even lower.

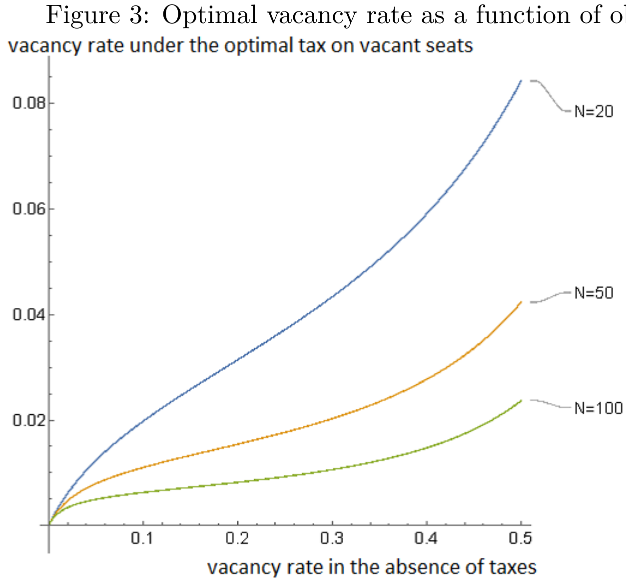
Commercial airlines today have between 20 and 900 seats. The following graph is obtained by applying proposition 4:

Figure 2: Optimal vacancy rate as a function of observed vacancy rate



Zooming into this graph gives:

Figure 3: Optimal vacancy rate as a function of observed vacancy rate



5.2 Constant elasticity distribution: $D(p) = p^{-\varepsilon}$

I solved this version of the model numerically and calibrated it as explained in A.3.2.

Results from calibration: Suppose the parameters ϵ and c are chosen so as to produce the observed ratio of marginal costs of filling a seat to average ticket price and the observed vacancy rate (20 percent as global average) assuming the absence of taxes. Then for all realistic values of the number of seats per flight we obtain that at the socially optimal tax policy the vacancy rate is around 3 percent. Social welfare increases by 3.2 percent as a result of the introduction of the optimal tax on vacant seats.

6 Robustness

We have seen that in the absence of specific tax/subsidy policies profit maximization will in general not lead to welfare maximization. Moreover, in the cases where the distribution of valuations belongs to the family identified in proposition 3, we have found that what is required to align profit maximization incentives with the objective of welfare maximization is precisely a tax on vacant seats (or, equivalently in our model, a subsidy on occupied seats or a constant subsidy on ticket sales).

For other distributions of valuations more complicated instruments would be needed, as shown in proposition 1. However, the question arises as to what the optimal tax on vacant seats would be under different distributions of valuations under the restriction of only using this simple instrument of a tax on vacant seats (thus excluding the price dependent subsidy that would generally be required to achieve maximal welfare). A natural question is then whether the optimal tax rate on vacant seats is always positive. It turns out that it can be negative. In fact, we even have in proposition 5 a stronger result. It states that as long as there is a strictly positive probability that all tickets are sold out, one can always rationalize a policy change that consists of lowering the tax on vacant seats. In particular, this proposition implies that one can always rationalize subsidizing vacant seats.

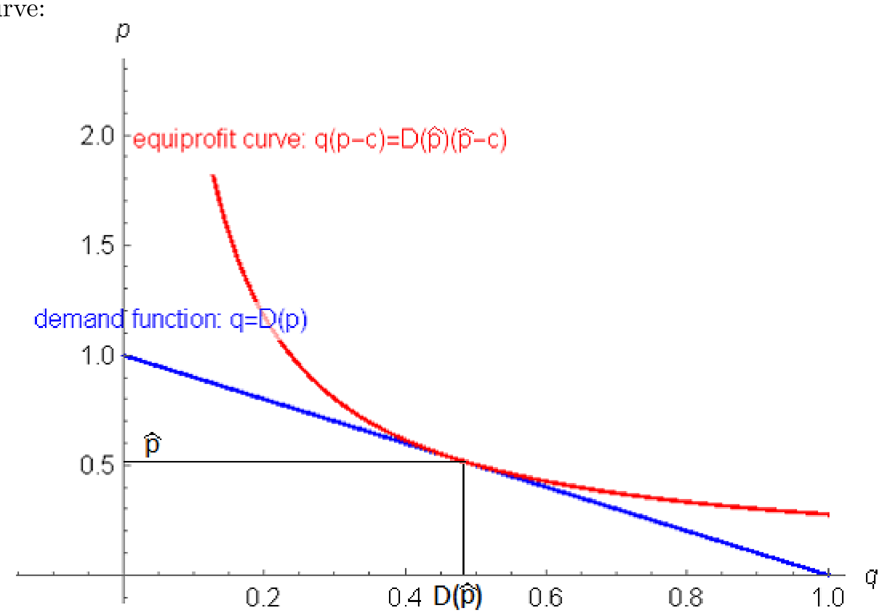
Proposition 5. *Consider a continuously differentiable demand function D and a tax rate α for the tax on the vacant seats. Suppose that under α and D there is a strictly positive probability that all tickets will be sold. Then for any $\alpha' < \alpha$ there exists a demand function $D^{\#}$ that satisfies the following conditions:*

1) the model with $D^{\#}$ implies that welfare will be strictly increased if α is lowered to α'

2) the model with $D^{\#}$ generates the same predictions as D for both the tax rate α and the tax rate α'. (i.e. the profit maximizing pricing policies are the same for $D^{\#}$ and D under both α and α').

Sketch of the proof:

Roughly speaking, in the proof of proposition 5 the demand function $D^{\#}$ is obtained from D by thickening the upper tail of the distribution of valuations without making it profitable for the airline to increase prices. To illustrate this, consider a discrete one period version of the model with just one seat, where the demand function $D(p)$ now denotes the probability that there is a customer willing to buy the ticket, given that the price is p. Let $\hat{p}$ be an expected-profit maximizing price. The red curve in the following diagram shows the equiprofit curve:



If the demand function was actually the equiprofit curve, $q = \frac{D(\hat{p})(\hat{p}-c)}{p-c}$, then by definition $\hat{p}$ would still be expected-profit maximizing. However, the expected valuation for a consumer conditional on his valuation exceeding $\hat{p}$ would then be infinite, since it is given by $\frac{1}{D(\hat{p})} \int_{p=\hat{p}}^{\infty} \frac{D(\hat{p})(\hat{p}-c)}{p-c} = (\hat{p}-c)[log(p-c)]_{p=\hat{p}}^{\infty} = \infty$.

Building on this observation, I show in in appendix A.5 that also in the full dynamic model that is the object studied in this paper, one can thicken the upper tail further and further such that the expected valuation of a potential passenger goes to infinity, whilst the predictions of the model are unchanged for given tax rates. It thus becomes an overwhelming priority to make sure that no person is ever refused the possibility to buy a ticket. Subsidizing vacant seats is one way to induce the airline to make sure that such refusals are less likely to happen.□

Proposition 5 shows that it is possible for the introduction of a tax on vacant seats to reduce social surplus if the distribution of valuations is sufficiently thick so that in expectation a lot of surplus is lost when the flight runs out of tickets and people are refused the possibility to buy one. However, there are other public policy instruments that could be used to reduce the probability of such refusals. For example, a government could enable a secondary market for tickets (see Love (2019)). It could impose by regulation that all airline tickets must be transferable[8]. If people with very high valuations appear, they would then be able to buy tickets from other customers with lower valuations.

One can analyze such a regulation to make all tickets transferable in extensions of the model used above. Consider the following sketch of a version of such an extension: Suppose that once a consumer has appeared and has his valuation realized, this valuation does not change. Thus if given the chance to sell his ticket at a price exceeding his valuation, he will want to do so. Suppose there is no friction in the resale market and all mutually beneficial trades are realized. Typically, the unique socially optimal dynamic pricing policy for the airline to use to sell its tickets would be to always sell tickets at their marginal cost. If and when the airline runs out of tickets, the resale market would ensure that the consumers with the highest valuations would get them.

In such a model the social optimum could be implemented by combining the obligation to make all tickets transferable with a sufficiently high tax on vacant seats and a prohibition to sell tickets below their marginal cost. If introduced without an accompanying tax on vacant seats, the obligation to make all tickets transferable might increase the incentives for airlines to raise their dynamic pricing policy: If the airline sells tickets at a low price then consumers with low valuations might later resell their tickets, thus undermining the price that the airline can charge then. The results from sections 4 and 5 suggest that this could lower welfare, as more seats will be left empty as a result. However, the introduction of sufficiently high tax on vacant seats would counteract this effect. We have no guarantee that the introduction of the obligation to make all tickets transferable will increase welfare (by the arguments just given) nor that the introduction of a tax on vacant seats will increase welfare (by proposition 5). However, in the model just sketched, the joint introduction of these two government policies (combined with a prohibition of sales below marginal cost) allows for the social optimum to be implemented.

A proper analysis will have to take into account that people make decisions with lock-in when they purchase flight tickets: They decide on when to take days off work, book accommodation etc., which are decisions that are potentially costly to reverse. In a richer model taking into account such effects it will no longer be straightforward to find socially optimal sales mechanisms, let alone government regulations that could cause such socially optimal sales mechanisms to be implemented. However, the simplistic analysis just given should at least prevent us from drawing too pessimistic conclusions from proposition 5: This proposition should not lead us to conclude that taxes on vacant seats are not promising instruments for increasing welfare. Instead, proposition 5 should motivate us to pursue the following research question: How can we best design regulations (such as an obligation to make tickets transferable) as complements to a tax on vacant seats in such a way that more seats will be filled whilst still ensuring that people with high valuations will always get a ticket?

[8] Currently, only a small fraction of tickets is transferable. $7 billion worth of tickets remain unused per year.

7 Implications for the optimal design of international agreements on aviation taxes

In 2016, the member states of the International Civil Aviation Organization (ICAO) adopted the Carbon Offsetting and Reduction Scheme for International Aviation (CORSIA). 81 States, representing 76.63% of international aviation activity, intend to voluntarily participate in CORSIA from its outset. Starting in 2021, the participating countries will oblige airlines to offset a part of their emissions by buying certificates from projects in developing countries that have been certified to have reduced emissions.

However, since the depth of this agreement is expected to end up being quite shallow (the carbon price that airlines will face might correspond to only about 0.3% of ticket prices (see Stern (2019) for a discussion based on Warnecke et al. (2019)), the EU is considering taking further unilateral action to achieve higher carbon pricing. In 2016 the EU held a 'consultation on market-based measures to reduce the climate change impact from international aviation'. A common suggestion was to tax all international flights to and from countries that do not have equivalent taxes in place. Moreover, many submissions suggested to allocate a part of the tax revenue to a Global Public Good Institution (GPGI)[9] for climate change mitigation such as the Green Climate Fund. Based on these suggestions, one can define the following proposal (analyzed in Stern (2019)) for a club mechanism that the EU could initiate[10]:

Definition 1. The CORSIA+ mechanism

The CORSIA+ mechanism for a given agreed-upon Global Public Good Institution (GPGI) for climate change mitigation defines a club through the following two obligations that participating countries have to comply with:

the taxation obligation (version 1): Each participating country is required to levy a distance-based air passenger tax with rate τ per kilometer for all outgoing international flights and also for incoming international flights arriving from countries that do not participate.

the allocation obligation: Each participating country must allocate a proportion $1 - r$ of the tax revenue that it collects to the agreed-upon GPGI for climate change mitigation. It can retain the remaining proportion r for itself.

Let us now explore alternative versions of CORSIA+ where we leave the allocation obligation as above but where we modify the taxation obligation. One alternative version of the taxation obligation could be the following:

the taxation obligation (version 2): Each participating country is required to levy a tax with rate τ on carbon emissions for all outgoing international

[9] A GPGI is defined to be any international institution that uses its available budget to contribute to a particular global public good. See Stern (2020) for a more precise definition and for a list of all existing GPGIs.

[10] In Stern (2019) I propose two alternative mechanisms. Simulation results reported there suggest that these alternative mechanisms would achieve greater global welfare gains than CORSIA+. For ease of exposition, I discuss below the different versions of the taxation obligation for the case of CORSIA+. However, the discussion applies analogously to the alternative mechanisms.

flights and also for incoming international flights arriving from countries that do not participate.

A further alternative version of the taxation obligation could be the following:

the taxation obligation (version 3): Each participating country is required to impose taxes and/or emissions trading schemes covering all flights departing from country i and all flights arriving from a non-participating country. Moreover, the aggregate tax /quota purchase burden thus imposed on airlines has to be at least τ times the aggregate emissions from the corresponding flights.

Under version 3 of the taxation obligation countries would be allowed to use any tax. By using a carbon tax with rate τ a country can automatically satisfy this version of the taxation obligation. However, it could also use a tax on vacant seat emissions, meaning that each airline would for each flight have to pay a tax proportional to the proportion of vacant seats times the emissions of the flight. Indeed, the results from the current paper suggest that countries would reap welfare gains by using such a tax.

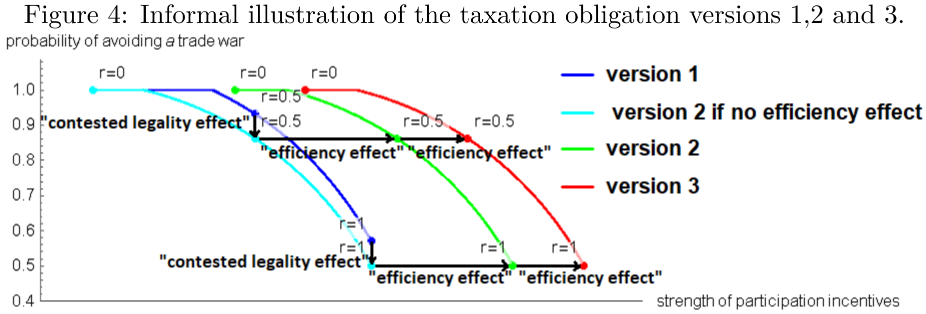
There is an important consideration weighing in favor of version 1 of the taxation obligation, as explained by Larsson et al. (2019): Distance-based air passenger taxes) are the only taxes on international aviation that do not pose legal problems. Larsson et al. (2019) point out: "a tax on jet fuel for international aviation is not permitted under current international agreements (ICAO, 1993). These regulations have not only hindered a tax on jet fuel but even 'next best' options, such as per-plane taxes and carbon-related landing charges.

The UK Government announced in 2010 that it wanted to replace the per-passenger tax with a per-plane tax based on the weight of the aircraft, which would give airlines stronger incentives to fly with full aircraft. But in 2011, the UK Minister of Finance announced; 'We have tried every possible option, but have reluctantly had to accept that all are currently illegal under international law"'. The UK could have gone ahead to introduce the per-plane tax anyway, but then it might have faced opposition from other countries. This is in fact what happened in 2012 when the EU included all international flights in its emissions trading scheme. The US, China, India, Russia, and Saudi Arabia all opposed the scheme and partly prohibited their airlines from complying with it (Murphy et al 2016). The Chinese ambassador to the EU suggested that Chinese airlines would no longer buy Airbus planes. Some observers warned of the risk of the issue triggering a trade war. In response to this opposition, the EU modified after only one year (in November 2012) the system to a 'reduced scope', which means that only flights between and within EU member states are included.

Thus, other things equal, the CORSIA+ mechanism with version 1 of the taxation obligation seems less likely to cause opposition by non-participating countries than the CORSIA+ mechanisms with versions 2 or 3 of the taxation obligation. However, the likelihood of opposition to CORSIA+ by non-participating countries would likely be greatly reduced if the retention rate parameter r is set at a low value: To the extent that non-participating countries benefit from the GPGI that CORSIA+ would finance, they would have less incentives to try to get the participating countries to abandon it. Moreover, the legitimacy of the mechanism would be greater than in the case of the 2012 inclusion of the international flights in the EUTS, when all of the permit auction revenue was retained by the EU member states. The retention rate parameter r should thus be set as low as possible, whilst still generating sufficient incentives for countries to join the mechanism. The results obtained in the current paper suggest that under version 3 of the taxation obligation participating countries would be able to reap welfare gains[11] from using taxes on vacant seat emissions and that therefore participation incentives would be strongest. This in turn would mean that r could be set at lower level than under version 1 and 2 of the taxation obligation and still lead to sufficient participation incentives. Thereby the legitimacy problems would be mitigated.

This discussion can be illustrated informally with the following diagram:

Figure 4: Informal illustration of the taxation obligation versions 1,2 and 3.



Consider the blue line which represents version 1 of the taxation obligation. For each $r \in [0, 1]$ a corresponding point is plotted. For sufficiently small values of the retention rate parameter there is no risk of the mechanism triggering a trade war. This assumption is based on the fact that non-participants would have less incentives to oppose the mechanism if more money is raised for GPGIs. However, for sufficiently large r there is a risk of the mechanism triggering a trade war. Moreover, this risk increases in r. On the hand, the higher the r, the larger are the participation incentives.

Now if version 2 of the taxation obligation was used instead of version 1, then the risk of non-participants opposing the mechanism would be increased due to the contested legality of emissions taxes (as pointed out by Larsson et al. (2019)). If this was the only effect arising from switching from version 1 to version 2 for the taxation obligation, then version 2 would be be represented by the line in cyan: For any value of r, the probability of avoiding a trade

[11]There are of course also welfare gains from using emissions taxes rather than ticket taxes that are due to the greater incentives to harness emissions reductions opportunities fro e.g. engine efficiency and other technical and operational margins (see Anjaparidze (2019). However, these benefits due to reduced climate change are globally dispersed and therefore do not affect individual participation incentives as much as the welfare gains from increased load factors that are reaped by the countries themselves.

war would be decreased. This “contested legality effect” is represented in the diagram by the downward arrows.

However, two further effect would arise from switching to version 2 of the taxation obligation: Firstly, there would be greater emissions reductions, as incentives would be created for reducing emissions, e.g. by increasing engine efficiency. The benefits from the resulting reduction in climate change would accrue to all countries. The participating countries would pay the burden of higher prices if more expensive and more energy efficient planes are used. Thus it is unclear how this effect would affect participation incentives.

However, there is a second effect that would result from switching from version 1 to version 2 of the taxation obligation: Load factors on flights arriving and departing from participating countries would increase. The results from section 4 and 5 suggest that this would generate welfare gains for the participating countries. Hence their participation incentives would increase. This “efficiency effect” is represented in the above diagram by the rightward arrows. The diagram illustrates the case where the “efficiency effect” dominates the “contested legality effect” in the sense that the green curve representing version 2 lies to the right of the blue curve representing version 1.[12]

The results from section 4 and 5 suggest that there is a further efficiency effect reaped if version 2 is replaced by version 3 for the taxation obligation. This effect weighs in favor of using version 3.[13]

8 Conclusion

For a class of demand functions that includes the constant elasticity and the exponential functions this study has shown in a monopolistic dynamic pricing model that a tax on vacant seats is required to achieve maximal social surplus. The illustrative calibrations suggest that at the optimal tax on vacant seats the load factor might be substantially higher than it is today.

A useful next step could be to extend the analysis for to oligopolistic models with multiple competing airlines. A natural solution concept for such models

[12] A formal version of this model is available from the author on request. Within this formal version one can assess the relative size “efficiency effect” using the calibrated models presented in this paper.

[13] When comparing version 2 and version 3, the greater simplicity of version 2 weighs in its favor. However, it seems useful to allow countries to meet the taxation obligation using emissions trading schemes, as some countries appear to have preferences for the latter. (This is also why Nordhaus (2015) proposes to define carbon pricing obligations (in his case for economy wide carbon pricing) in a way that allows countries to use emissions trading scheme to meet them.) To allow for countries meeting their obligation through emissions trading schemes, the complexity of version 3 is required anyway. A disadvantage of version 3 as stated above would be that to the extent that countries choose to use other taxes than carbon taxes, the amount of emissions reductions might be less. However, this could be mitigated by restricting the set of tax instruments that countries would be allowed to use. For example, taxes on vacant seat kilometers could be prohibited. Instead, airlines could be allowed to use taxes on vacant seat emissions which would for each flight be proportional to its emissions times its vacancy rate. In this way, there would both be mitigation incentives and incentives to increase load factors.

could be Markov Perfect Equilibrium, since airlines can estimate competitors remaining vacant seats in real time from seat maps (see e.g. Williams (2018)). An important trade-off that is captured in the monopolistic model of this study would again be present in the oligopolistic models: For social welfare, it is valuable if additional people take flights. On the other hand, leaving more seats empty reduces the expected number of people who will not be able to take the flights even though they have high valuations. The present study suggests that monopolistic airlines will leave too many seats empty. This effect could be called the "load factor distortion". Importantly, this distortion is distinct from the usual "output level distortion" that occurs as monopolists restrict output below the socially optimal level. In particular, whilst increasing the number of firms will mitigate the standard output distortion, the arguments demonstrating this result do not establish that the "load factor distortion" will disappear with sufficiently many firms. Further modeling work is required to find out how the "load factor distortion" is affected by the number of competing airlines.

Further work could explore how to use complementary government policies to soften the trade-off mentioned above. In other words: what government policies could ensure that people with high valuations will always be able to get a ticket even in the presence of a large tax on vacant seats? Proposition 5 shows that this is potentially very important. One candidate government policy for ensuring that people with high valuations can always get a ticket could be to oblige all airlines to make their tickets transferable (Love (2019)). This could enable the emergence of a resale market in which people with low valuations or more flexibility would be able to sell tickets to people with high valuations. To adequately analyze such regulations, one would have to model the fact that people make decisions when they plan a trip such as taking days off work and booking accommodation. These decisions are costly to reverse. An interesting mechanisms design question arises from this: What sales mechanism would be socially optimally given that purchase timing will be endogenous? Moreover, which government policies would induce airlines to use socially optimal sales mechanisms?

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A Appendix

A.1 proof of proposition 3 for the sufficiency of the tax on vacant seats

In order for profit maximization to be equivalent to welfare maximization under the tax on vacant seats with rate χ it is sufficient that there exists some $\mu > 0$ such that $D(p)$ satisfies on the domain $[0, sup\{p : D(p) > 0\})$:

$$\mu (p - c + \chi) = E_t (v|v \geq p) - c$$

This is because maximizing a function is equivalent to maximizing a positive multiple of that function.

Rearranging this condition yields:

$$E_t (v - p|v \geq p) = (1 - \mu) c + (\mu - 1) p + \mu\chi$$

We have

$$E_t (v - p|v \geq p) = \frac{\int_{x=p}^{\infty} D(x)\, dx}{D(p)}$$

Plugging this in yields:

$$\frac{\int_{x=p}^{\infty} D(x)\, dx}{D(p)} = (1 - \mu) c + (\mu - 1) p + \mu\chi$$

Multiplying through by $D(p)$ and then differentiating yields:

$$-D(p) = ((1 - \mu) c + \mu\chi + (\mu - 1) p) D'(p) + (\mu - 1) D(p)$$

Again rearranging:

$$-((1 - \mu) c + \mu\chi + (\mu - 1) p) D'(p) = \mu D(p)$$

Using separation of variables yields:

$$\frac{dD}{D} = -\frac{\mu}{((1 - \mu) c + \mu\chi + (\mu - 1) p)} dp$$

Now we can distinguish the possible cases:
Case 1: $\mu = 1$
In this case we get:

$$\frac{dD}{D} = -\frac{1}{\chi} dp$$

$$D = Bexp\left(-\frac{1}{\chi} p\right)$$

where B is a positive constant. This is the exponential demand function.

Case 2: $\mu > 1$

$$\frac{dD}{D} = -\frac{\mu}{((1-\mu)c + \mu\chi + (\mu-1)p)} dp$$

$$log(D) = -\frac{\mu}{(\mu-1)} log((1-\mu)c + \mu\chi + (\mu-1)p) + const$$

$$D = B((1-\mu)c + \mu\chi + (\mu-1)p)^{-\frac{\mu}{(\mu-1)}}$$

where $B > 0$ is a constant. Denoting $\hat{B} := B(1-\mu)^{-\frac{\mu}{(\mu-1)}} > 0$, we can write this as:

$$D = \hat{B}\left(-c + \frac{\mu}{(\mu-1)}\chi + p\right)^{-\frac{\mu}{(\mu-1)}}$$

The set of pairs (μ, χ) such that $-c + \frac{\mu}{(\mu-1)}\chi = 0$ corresponds to the constant elasticity demand functions.

However, we now also see another class of demand functions. Denoting $a = -c + \frac{\mu}{(\mu-1)}\chi$ and $b = \frac{\mu}{(\mu-1)}$, we have:

$$D = \hat{B}(a+p)^{-b}$$

To find the optimal tax rate on the vacant seats in this case we solve:

$$a = -c + b\chi$$

To obtain:

$$\chi = \frac{a+c}{b}$$

If $a > 0$ then $D(p) = \hat{B}(a+p)^{-b}$ defines a continuous decreasing function on the entire domain $p \geq 0$. We note that in this case $\chi > 0$.

If, on the other hand, $a = -c + \frac{\mu}{(\mu-1)}\chi$ were negative, then the expression $D = B(a+p)^{-\frac{\mu}{(\mu-1)}}$ would have a vertical asymptote at $p = -a$. This means in particular that such a function defined by the expression $D(p) = B(a+p)^{-\frac{\mu}{(\mu-1)}}$ on the domain where $p > -a$ cannot be continuously extended to the domain defined by $p \geq 0$.

Case 3: $\mu < 1$

$$\frac{dD}{D} = -\frac{\mu}{((1-\mu)c + \mu\chi + (\mu-1)p)} dp$$

$$\frac{dD}{D} = -\frac{\frac{\mu}{(1-\mu)}}{\left(c + \frac{\mu}{(1-\mu)}\chi - p\right)} dp$$

Let us define $g = \frac{\mu}{(1-\mu)}$

$$\frac{dD}{D} = -\frac{g}{(c + g\chi - p)} dp$$

$$log\left(D\right) = glog\left(c + g\chi - p\right) + const$$

$$D = B\left(c + g\chi - p\right)^{g}$$

Where $B > 0$ is a constant. We see that if $c + g\chi > 0$ then we obtain a demand function that is defined by $D\left(p\right) = B\left(c + g\chi - p\right)^{g}$ on the domain $[0, c + g\chi]$ and $D\left(p\right) = 0$ for $p > c + g\chi$. Denoting $a = c + g\chi$ implies $\chi = \frac{a-c}{g}$.

This means that for $B > 0, a > 0$ the function that is defined by $D\left(p\right) = B\left(a - p\right)^{g}$ on the domain $[0, a]$ and $D\left(p\right) = 0$ for $p > a$ is a demand function such that with the tax on the vacant seats with tax rate of $\chi = \frac{a-c}{g}$ the profit maximization problem is transformed into the welfare maximization problem. If $a < c$ then the airline could only sell tickets at a loss. Thus $a \geq c$ is the only relevant case. In that case the optimal tax rate on the vacant seats, $\chi = \frac{a-c}{g}$, is always positive. □

A.2 Analytical solution for the case of an exponential distribution of valuations

Now let us consider the special case with exponential distribution of valuations:

$$D\left(p\right) = e^{-ap}$$

As I showed in proposition 1, the constant tax rate of $\varphi\left(p\right) = \frac{1}{a}$ achieves that profit maximization is equivalent to welfare maximization.

We need to solve the model for the two cases: Firstly, we need to solve it without any tax, i.e. with $\varphi\left(p\right) = 0$, so that we can calibrate it to predict the current vacancy rate and the current ratio of the marginal cost of taking an additional passenger on board to the average ticket price. Secondly, we need to solve for the case of the tax rate on vacant seats that achieves that profit maximization leads to welfare maximization, $\varphi\left(p\right) = \frac{1}{a}$. Let us denote by $c^{*} = c - \varphi\left(p\right)$ for the two cases, which is hence always a constant. In other words, $c^{*} = c$ in the case without any tax and $c^{*} = c - \frac{1}{a}$ in the case where the optimal tax rate has been implemented, but we will use the notation c^{*} to do the computations for the two cases at once.

The general equation

$$-v_{n}^{'}\left(t\right) = max_{p} D\left(p\right)\left(p - c^{*} - \left(v_{n}\left(t\right) - v_{n-1}\left(t\right)\right)\right)$$

Now becomes

$$-v_{n}^{'}\left(t\right) = max_{p} \lambda e^{-ap}\left(p - c^{*} - \left(v_{n}\left(t\right) - v_{n-1}\left(t\right)\right)\right)$$

Gallego and van Ryzin (1994) provide the solution to this system of differential equations, which I list below. It is straightforward to verify that the functions provided below do indeed solve the system of differential equations. An inductive derivation of these solutions is available upon request.

Let us denote $\beta := \lambda exp\left(-1 - ac^*\right)$.

Lemma 2. *(Gallego and van Ryzin (1994))* $v_n = \frac{1}{a} log(\sum_{k=0}^{n} \frac{\beta^k (T-t)^k}{k!})$

Let us define $B_n\left(t\right) \coloneqq \sum_{k=0}^{n} \frac{\beta^k (T-t)^k}{k!}$

Corollary 2. *(Gallego and van Ryzin (1994))*$p_n^*\left(t\right) = \frac{1}{a} log\left(\frac{\lambda B_n(t)}{\beta B_{n-1}(t)}\right)$

Let $g(t)$ denote the expected number of seats sold at time t.

Corollary 3. *(McAfee and te Velde (2006))*$g(t) = \beta t \frac{B_{N-1}(0)}{B_N(0)}$

In section 5 I already stated the following result:

Proposition. *4 As always, denote by N the total number of seats on the plane. Let Z and Z^* be the expected proportion of seats occupied on the flight without any tax and with the optimal tax, respectively. Z^* can be deduced from Z according to the following formula* $Z^* = \frac{q(Z)e \frac{\sum_{k=0}^{N-1} \frac{(q(Z)e)^k}{k!}}{\sum_{k=0}^{N} \frac{(q(Z)e)^k}{k!}}}{N}$*, where $q(z)$ is implicitly defined by* $Z = \frac{q(Z) \frac{\sum_{k=0}^{N-1} \frac{q(Z)^k}{k!}}{\sum_{k=0}^{N} \frac{q(Z)^k}{k!}}}{N}$

proof of proposition 4:

Using corollary 3, we obtain:

$$Z = \frac{\beta T \frac{B_{N-1}(0)}{B_N(0)}}{N}$$

Now using that $B_n\left(0\right) \coloneqq \sum_{k=0}^{n} \frac{\beta^k T^k}{k!}$ we obtain:

$$Z = \frac{\beta T \frac{\sum_{k=0}^{N-1} \frac{\beta^k T^k}{k!}}{\sum_{k=0}^{N} \frac{\beta^k T^k}{k!}}}{N}$$

Let $\beta^* = \lambda exp\left(-1 - ac^*\right)$ denote β under the optimal tax on vacant seats. We have:

$$Z^* = \frac{\beta^* T \frac{\sum_{k=0}^{N-1} \frac{\beta^{*k} T^k}{k!}}{\sum_{k=0}^{N} \frac{\beta^{*k} T^k}{k!}}}{N}$$

By Lemma 6 we know that there is a unique welfare maximizing dynamic pricing policy. We also know by Lemma 9 that the load factor at the welfare maximizing pricing policy, z^*, must strictly increase in T. Moreover $lim_{T \to 0} z^* = 0$ and $lim_{T \to \infty} z^* = 1$. In particular, this holds for the expected load factor, Z^*. We have thereby established the following algebraic fact: The function

$q \mapsto \frac{q \frac{\sum_{k=0}^{N-1} \frac{q^k}{k!}}{\sum_{k=0}^{N} \frac{q^k}{k!}}}{N}$ defined on the domain $[0, \infty)$ is strictly increasing and has as its image $[0, 1)$.

This algebraic fact, applied to the analogous expression $Z = \frac{\beta T \frac{\sum_{k=0}^{N-1} \frac{\beta^k T^k}{k!}}{\sum_{k=0}^{N} \frac{\beta^k T^k}{k!}}}{N}$ implies that the function $q(z)$ is well-defined on $[0, 1)$ via the condition $Z = \frac{q(Z) \frac{\sum_{k=0}^{N-1} \frac{q(Z)^k}{k!}}{\sum_{k=0}^{N} \frac{q(Z)^k}{k!}}}{N}$. Thus for any load factor from $[0, 1)$ there is a unique value for βT such that the model predicts that load factor.

We have:

$$\beta = \lambda exp\left(-1 - ac\right)$$

Now since by corollary 1 the optimal tax rate on vacant seats is $\frac{1}{a}$, we get:

$$\beta^* = \lambda exp\left(-1 - a\left(c - \frac{1}{a}\right)\right) = \beta e$$

This implies that $Z^* = \frac{q(Z)e \frac{\sum_{k=0}^{N-1} \frac{(q(Z)e)^k}{k!}}{\sum_{k=0}^{N} \frac{(q(Z)e)^k}{k!}}}{N}$. □

A.3 Appendix 2: The case of constant elasticity distributions of valuations

A.3.1 proofs

In the case of constant elasticity with

$$D\left(p\right) = p^{-\epsilon}$$

The general equation

$$-v_n^{'}\left(t\right) = max_p \lambda D\left(p\right)\left(p - c^* - \left(v_n\left(t\right) - v_{n-1}\left(t\right)\right)\right)$$

now becomes:

$$-v_n^{'}\left(t\right) = max_p \lambda p^{-\epsilon}\left(p - c^* - \left(v_n\left(t\right) - v_{n-1}\left(t\right)\right)\right)$$

Proposition 6. *(McAfee and te Velde (2006))Suppose $c = 0$. Then the value function is given by $v_n\left(t\right) = \beta_n\left(T - t\right)^x$ where $\beta_0 = 0$ and β_n is given inductively by*

$$\beta_n\left(\beta_n - \beta_{n-1}\right)^{\epsilon-1} = \lambda \frac{\epsilon^{\epsilon+1}}{\left(\epsilon - 1\right)^{\epsilon-1}}$$

Moreover, the profit maximizing pricing policy is given by $p_n(t) = \beta_n^{-\frac{1}{\epsilon-1}}(\lambda(T-t))^{\frac{1}{\epsilon}}$.

Corollary 4. *Let $\eta_m(t)$ denote the probability that there are m vacant seats at time t given that there were N vacant seats at time t. We have $\eta_m(t) = \left(\frac{T-t}{T}\right)^{\beta_N^{\frac{\epsilon}{\epsilon-1}}}$, where β_N is determined as in proposition 6.*

Proof. Now let us find the time evolution of the vacancy rate. We have for $m \leq N-1$:

$$\frac{d}{dt}\eta_m = -\eta_m \lambda D\left(p_m\left(t\right), t\right) + \eta_{m+1}\lambda D\left(p_{m+1}\left(t\right), t\right)$$

And for $m = N$:

$$\frac{d}{dt}\eta_N = -\eta_N \lambda D\left(p_N\left(t\right), t\right)$$

Using the solution from proposition 6, we obtain:

$$\frac{d}{dt}\eta_N = -\eta_N \lambda \left(\beta_N^{-\frac{1}{\epsilon-1}}\left(\lambda(T-t)\right)^{\frac{1}{\epsilon}}\right)^{-\epsilon}$$

$$\frac{d}{dt}\eta_N + \eta_N \beta_N^{-\frac{\epsilon}{\epsilon-1}}\left(T-t\right)^{-1} = 0$$

We compute the integrating factor:

$$\int \beta_n^{\frac{\epsilon}{\epsilon-1}}\left(T-t\right)^{-1} dt = -\beta_n^{\frac{\epsilon}{\epsilon-1}} log\left(T-t\right)$$

$$exp\left(\int \beta_n^{\frac{\epsilon}{\epsilon-1}}\left(T-t\right)^{-1} dt\right) = \left(T-t\right)^{-\beta_n^{\frac{\epsilon}{\epsilon-1}}}$$

so we get:

$$\frac{d}{dt}\left(\eta_N\left(T-t\right)^{-\beta_n^{\frac{\epsilon}{\epsilon-1}}}\right) = 0$$

$$\eta_N\left(T-t\right)^{-\beta_n^{\frac{\epsilon}{\epsilon-1}}} = b$$

Where b is some constant. But we have the initial condition $\eta_N\left(0\right) = 1$, so we have:

$$\eta_N = \left(\frac{T-t}{T}\right)^{\beta_N^{\frac{\epsilon}{\epsilon-1}}}$$

□

We note in particular that $\eta_N\left(T\right) = 0$ which is the key observation allowing us to prove:

Corollary 5. *With constant demand elasticity and 0 marginal cost per occupied seat the flight sells out completely with probability 1.*

Proof. Let $f(t)$ be the random variable that is the number of free seats at time t. Let $\tilde{t}$ be the time of the first ticket sale. Since $\eta_N(T) = 0$ by corollary 4, we know that $\tilde{t} \in [0, T]$ with probability 1.

The proposition is equivalent to the statement that the expected number of vacant seats is 0 at time T. We use the law of iterated expectations:

$E(f(T)|f(0) = N) = E_{\tilde{t}}(E(f(T)|f(0) = N \, and \, \tilde{t}))$

But since we are dealing with a Poisson process we know that

$E(f(T)|f(0) = N \, and \, \tilde{t} = t) = E(f(T)|f(t) = N - 1)$

Denote $h(t, n) := E(f(T)|f(t) = n)$. With this notation, we obtain:

$h(0, N) = E_{\tilde{t}}(h(\tilde{t}, N - 1))$

But by definition we have $h(t, 0) = 0 \forall t \in [0, T]$. Hence we deduce that $h(0, n) = 0 \forall n$. □

A.3.2 Numerical calibration

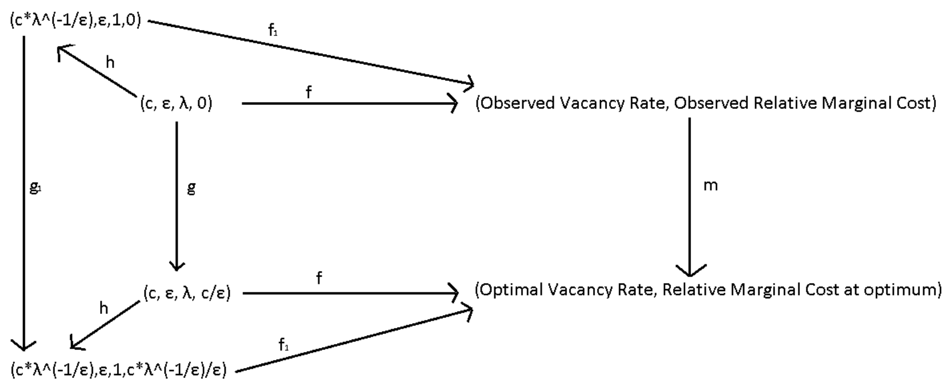
Without loss of generality we can normalize $T = 1$. There are three parameters in the model, namely the marginal cost per occupied seat c , the elasticity ϵ and the intensity λ of the Poisson arrival process. It seems a natural approach to try to calibrate the model using the observed vacancy rate and the relative marginal cost, i.e. the ratio between the additional cost incurred through an additional seat being occupied to the average ticket price paid by passengers. It turns out that for the constant elasticity case it is actually possible to calibrate the model solely with these two numbers, despite the fact that there are three parameters.

Definition 2. Let us denote by f the correspondence that assigns to the quadruple $(c, \epsilon, \lambda, s)$ of marginal cost per occupied seat c, elasticity ϵ , intensity λ of the Poisson arrival process and subsidy for sold tickets s the pairs (V, C) of vacancy rates V and relative marginal costs C (defined as the ratio between the marginal cost per occupied seat and the average ticket price).

Lemma 3. $f(\gamma c, \epsilon, \gamma^{\epsilon}\lambda, \gamma s) = f(c, \epsilon, \lambda, s)$

Proof. We first note that for any given Markovian pricing policy if we multiply it by some factor γ , multiply the marginal cost c and the subsidy s by that same factor γ and multiply λ by the factor γ^{ϵ} then the resulting demand processes are isomorphic and, the resulting profit is multiplied by the factor γ . Hence multiplying c and s by any positive factor γ and multiplying λ by the factor γ^{ϵ} results in analogous optimal pricing policies, simply scaled by γ, yielding identical flows of ticket sales. Hence in particular the resulting vacancy rates are identical. Moreover, since both c and the average ticket price are multiplied by the factor γ , their ratio, i.e. the relative marginal cost, is also unaffected. □

A solution to the 'calibration problem' should be defined to be a map m such that the inner rectangular diagram below commutes for all $(c, \epsilon, \lambda) \in (0, \infty) \times (1, \infty) \times (0, \infty)$.



f maps any combination of the parameter values to the values of the vacancy rate and the relative marginal cost that result

g is the function that introduces the optimal tax rate on vacanct seats

h is a scaling function defined by (c,ε,λ,s) ->(c*λ^(-1/ε),ε,1,s*λ^(-1/ε))

f_1 is the restriction of f to the set of parameter values such that λ=1

g_1 is the restriction of g to the set of parameter values such that λ=1

Lemma 4. *The inner rectangular part of the diagram commutes iff the outer part commutes.*

Proof. The inner diagram commutes iff

$$f^{\circ} g = m^{\circ} f$$

By Lemma 3 we have that $f = f_1\ {}^{\circ} h$ so this is equivalent to

$$f_1^{\circ} h^{\circ} g = m^{\circ} f_1^{\circ} h$$

But algebraically we can see that $g_1^{\circ} h = h^{\circ} g$ so this is equivalent to

$$f_1^{\circ} g_1^{\circ} h = m^{\circ} f_1^{\circ} h$$

Hence we see that if the outer diagram commutes (i.e. if $f_1^{\circ} g_1 = m^{\circ} f_1$), then the inner diagram commutes (i.e. $f_1^{\circ} g_1^{\circ} h = m^{\circ} f_1^{\circ} h$).

Conversely, if the inner diagram commutes then the outer diagram must commute, since it is a restriction of the inner diagram to the set of parameter values such that $\lambda = 1$. (In category language, we could draw another diagram where h is replaced by an inclusion map going into the opposite direction and then we can deduce the result as above through identities of maps.) □

Lemma 4 implies that the calibration is equivalent to finding a map m such that the outer diagram commutes.

Conjecture 1. *f_1 is injective.*

If conjecture 11 is true then the calibration problem simply amounts to inverting f_1 because once we have found such a function k that satisfies $k^\circ f_1 = identity$ then we can define $m := f_1^\circ g_1^\circ k$. This task is carried out by a Matlab program downloadable here, which computes m assuming conjecture 11. It is done by approximating the continuous time problem by the discrete time analogue, which we describe there.

In order to carry out the calibration we need an estimate for the relative marginal cost. To get this, we first note that from Gillen et al. (1990) we have to very good approximation:

$$totalcost = fixedcost + numberofpeopleonboard * c$$

Where c is the marginal cost. Taking the expectation of this equation yields:

$$averagetotalcost = fixedcost + averagenumberofpeopleonboard * c$$

Borenstein and Rose (2014)makes the rough estimate that "80% of costs are assumed to be invariant to changes in the load factor" . This means

$$\frac{fixedcost}{averagenumberofpeopleonboard * c} = \frac{0.8}{0.2} = 4$$

$$fixedcost = 4 * averagenumberofpeopleonboard * c$$

So that we get:

$$averagetotalcost = 5 * averagenumberofpeopleonboard * c$$

Let us assume that profits are zero so that $averagetotalcost = averagerevenue$. Using this assumption we obtain:

$$averagetotalcost = averagerevenue = averagenumberofpeopleonboard * averageprice$$

Putting these two equations together yields:

$$5 * averagenumberofpeopleonboard * c = averagenumberofpeopleonboard * averageprice$$

$$5 * c = averageprice$$

$$relativemarginalcost := \frac{c}{averageprice} = 0.2$$

Globally, the average vacancy rate is around 20% (see IATA). Using this number together with the relative marginal cost estimate of 0.2 we compute the optimal vacancy rate for planes with 50 seats using a backward induction algorithm. It turns out to be 0.0295.

To summarize: Under the assumption of constant elasticity for the distribution of valuations a tax on vacant seats can perfectly align profit maximization with welfare maximization. If the observed vacancy rate in the absence of the tax is 20% then the optimal tax rate on vacant seats will lower the vacancy rate to about 3% .

A.4 Optimal tax rates on vacant seats in the presence of a sales tax

Now we consider the case where there is a sales tax in place[14] that is a fixed proportion of ticket prices.

Lemma 5. *For the monopolistic profit maximization problem the tax policy pair* (η, χ)*, meaning a sales tax rate of* η *and a tax on vacant seats of* χ*, is equivalent to the tax policy pair* $\left(0, \chi - \frac{\eta}{(1-\eta)}c + \frac{\eta}{(1-\eta)}\chi\right)$.

Proof. Consider the case where there is a sales tax at rate η and a fixed subsidy of χ per occupied seat. The net revenue generated for the airline through a sale of a ticket at price p is given by

$$p\,(1-\eta) - c + \chi$$

We can rewrite this as follows:

$$p\,(1-\eta) - c + \chi = (1-\eta)\left(p - c + \chi - \frac{\eta}{(1-\eta)}c + \frac{\eta}{(1-\eta)}\chi\right)$$

But since an optimization problem is not changed under multiplication by a positive constant, this creates the same incentive for the airlines as when the net revenue generated through a sale of a ticket at price p is

$$p - c + \chi - \frac{\eta}{(1-\eta)}c + \frac{\eta}{(1-\eta)}\chi$$

This is case when there is no sales tax and a subsidy on occupied seats (or equivalently, a tax on vacant seats) of $\chi - \frac{\eta}{(1-\eta)}c + \frac{\eta}{(1-\eta)}\chi$. □

Lemma 5 implies that Lemma 3 still holds in the presence of a sales tax.

We see from Lemma 5 that the effect of a change in the sales tax rate on the vacancy rate depends on the rate of the tax on the vacant seats that we are at. When $\chi > c$, i.e. when the tax on the vacant seats exceeds the marginal cost, then the sales tax increases the incentives for decreasing the vacancy rate.

Corollary 6. *Let* $\chi^*(\eta)$ *denote the optimal tax rate on vacant seats, given that the sales tax rate is* η *. Then we have*

$$\chi^*(\eta) = \chi(0) + \eta\,(c - \chi(0))$$

[14]This is the case for flights between the US, Mexico and in Canada.

Proof. Lemma 5 establishes the equivalence of (η, χ) and $\left(0, \chi - \frac{\eta}{(1-\eta)}c + \frac{\eta}{(1-\eta)}\chi\right)$, so in particular we have

$$\chi(0) = \chi^*(\eta) - \frac{\eta}{(1-\eta)}c + \frac{\eta}{(1-\eta)}\chi^*(\eta) = -\frac{\eta}{(1-\eta)}c + \frac{1}{(1-\eta)}\chi^*(\eta)$$

which implies

$$\chi^*(\eta) = \chi(0) + \eta(c - \chi(0))$$

□

Corollary 7. *For the case of constant demand elasticity we have*

$$\chi^*(\eta) = \frac{1}{\varepsilon}c + \eta\frac{\varepsilon - 1}{\varepsilon}c$$

Proof. This follows directly from 2 and 6since

$$\chi^*(\eta) = \frac{1}{\varepsilon}c + \eta\left(c - \frac{1}{\varepsilon}c\right) = \frac{1}{\varepsilon}c + \eta\frac{\varepsilon - 1}{\varepsilon}c$$

□

Corollary 8. *For the case of exponential demand we have*

$$\chi^*(\eta) = \frac{1}{a} + \eta\left(c - \frac{1}{a}\right)$$

Proof. This follows directly from 1 and 6 □

We see that the optimal tax rate on vacant seats is a linear function of the sales tax rate. However, whereas in the constant elasticity case the optimal tax rate on the vacant seats is an increasing function of the sales tax rate, this relationship holds in the exponential case only if $a < 1$. If $a > 1$, on the other hand, then the optimal tax rate on vacant seats is a decreasing function of the sales tax rate.

A.5 Proof of proposition 5

To prepare the proof of proposition 5, we will here prove some results that are also of some interest in themselves.

Lemma 6. *Suppose that the demand function D is differentiable and $D'(p) > 0 \forall p$. Then there exists a unique pricing policy that maximizes welfare.*

Proof. Consider the Bellman equation:

$$-w_n'(t) = max_p D(p) \left(p - c + \frac{1}{D(p)} \int_{s=p}^{\infty} D(s)\, ds - (w_n(t) - w_{n-1}(t)) \right)$$

The first order condition for the optimality of p is:
$D'(p)\,(p - c + (w_n(t) - w_{n-1}(t))) + D(p) - D(p) = 0$
As long as $D'(p) > 0 \forall p$, this has the unique solution given by:
$p = c - (w_n(t) - w_{n-1}(t))$
We also have $w_n(T) = 0 \forall n$. Thus by the continuous analogue of backward induction a unique welfare maximizing pricing policy is determined. □

Assumption 1. *Given any tax policy, there exists a unique profit maximizing dynamic pricing policy.*

Justification:

This assumption holds for "regular" demand functions, as defined and discussed in Gallego and van Ryzin (1994). It simplifies the exposition of the results and proofs leading up to proposition 5. From the proof provided below it will become clear that an appropriately reformulated version of proposition 5 still holds if assumption 1 is relaxed.□

Definition 3. A complete history h is specified by the arrivals over time of passengers characterized by their valuation for the flight. For example, one history is specified by saying that a first consumer arrived at time t_1 and had valuation v_1, a second arrived at time t_2 and had a valuation of v_2 and so on.

Definition 4. Let $f(t, n, \rho, h)$ be the number of seats that will remain empty at departure if at the time t the airline has n seats left and given that the airline chooses from then onwards the pricing policy ρ.

Since we are assuming that the probability measure on the set of histories is such that it gives rise to a Poisson arrival process of potential passengers we can throughout assume that the airline chooses a Markovian pricing policy. We have:

Lemma 7. *Let h be any complete history from time t onward and let ρ be a Markovian pricing policy. Then $f(t, n, \rho, h)$ is increasing in n .*

Proof. Let $m(n, \rho, s, h)$ be number of remaining seats at time s, given that at time t there were n remaining seats.

Let $t^* := sup\{s : m(n+1, \rho, s, h) > m(n, \rho, s, h)\}$. If $t^* = T$ then $f(t, n+1, \rho, h) = m(n+1, \rho, T, h) > m(n, \rho, T, h) = f(t, n, \rho, h)$. If $t^* < T$ then we have: $m(n+1, \rho, s, h) = m(n, \rho, s, h)\ \forall t^* \in [s, T]$, so $f(t, n+1, \rho, h) = m(n+1, \rho, T, h) = m(n, \rho, T, h) = f(t, n, \rho, h)$ □

Corollary 9. *Suppose that the demand function D is differentiable and $D'(p) > 0 \forall p$. Let z^* denote the load factor (i.e. the proportion of occupied seats) at the unique (by Lemma 6) welfare maximizing dynamic pricing policy. Then z^* is a strictly increasing function in T with $lim_{T \to 0} z^* = 0$ and $lim_{T \to \infty} z^* = 1$.*

Proof. Suppose $\tilde{T} > T$. Let $\tilde{\rho}$ denote the welfare maximizing pricing policy when the time horizon is $\tilde{T}$ and let ρ denote the welfare maximizing pricing policy when the time horizon is T. From time $t = \tilde{T} - T$, $\tilde{\rho}$ must be equivalent to ρ applied to the time horizon remaining then. Hence the result follows by Lemma 7. □

Definition 5. Let $F(t, n, \rho)$ denote the expected number of seats that will remain empty, given that at time t there are n remaining tickets and given that the airline chooses the complete pricing policyρ from time t onward.

Lemma 8. *Suppose that $D(p)$ is bounded above. Then $F(t, n, \rho)$ is a strictly increasing function in n for any ρ.*

Proof. From Lemma 7 we deduce that $F(t, n, \rho)$ is increasing. We also know that there is a strictly positive probability of at least $exp(-sup_p D(p)T)$ that no ticket is ever sold, which implies that $F(t, n, \rho)$ is strictly increasing in n. □

Definition 6. Let $p(t, n, \alpha)$ denote the price that the (profit maximizing) airline sets for time t, given that the rate of the tax on vacant seats is α and given that n seats are vacant at the time t.

Proposition 7. [15]

Suppose $D(p)$ is continuously differentiable with $D'(p) < 0 \forall p$. Then $p(t, n, \alpha)$ is strictly decreasing in α for all n and t.

Proof. In section 4, we assumed that the subsidy on tickets sold are paid to the airline immediately at the time of sale. In the model, a constant subsidy on tickets sold is equivalent to a tax on vacant seats that is paid at time T. It is this latter perspective that will turn out to be the most convenient for the purposes of this proof.

Let $R(t, m, \rho)$ be the expected sales revenue net of marginal costs that the airline will make from period $i + 1$ until departure if it employs the complete pricing strategy ρ, given that there are m tickets left at the beginning of period $i + 1$. Let $V(t, m, \rho, \alpha)$ be the expected net revenue that the airline will make from time t until departure, given that it employs the pricing strategy ρ and given that there is a tax on vacant seats with rate α. It follows from the definitions that

$$V(t, m, \rho, \alpha) = R(t, m, \rho) - \alpha F(t, m, \rho)$$

[15]This result parallels proposition 3 from Gershkov and Moldovanu (2009), which establishes that under the assumption that$-\frac{F'(y)}{F(y)}$ is increasing, we have: for any time t and any number n of remaining tickets, the welfare maximizing price policy is lower than the profit maximizing price policy. In a sense, "the changes in the pricing policy induced by the tax on vacant seats point in the right direction". This suggest the conjecture that the optimal tax on vacant seats is always positive if $-\frac{F'(y)}{F(y)}$ is increasing. Proposition 5 does not refute this conjecture, since the demand functions constructed violate the condition that $-\frac{F'(y)}{F(y)}$ be increasing.

Let $v_n(t,\alpha)$ the airline's value function at time t, given that n seats remain then and given that there is a tax on vacant seats with rate α. By definition, we have:

$v_n(t,\alpha) = sup_\rho V(t,m,\rho,\alpha)$

Let $\rho(\alpha)$ denote the profit maximizing pricing policy, given the tax rate α on the vacant seats. By the envelope theorem, we have:

$\frac{d}{d\alpha}(v_n(t,\alpha)) = \frac{d}{d\alpha}V(t,m,\rho(\alpha),\alpha) = -F(t,m,\rho(\alpha))$

As in section 3 we can derive the Bellman equation: [16]:

$$-v_n^{'}(t) = \lambda max_p D(p)(p-c-(v_n(t,\alpha)-v_{n-1}(t,\alpha)))$$

Let us denote: $\pi(p,\alpha) := D(p)(p-c-(v_n(t,\alpha)-v_{n-1}(t,\alpha)))$ and $p^*(\alpha) = argmax_p\pi(p,\alpha)$.

$\frac{\partial}{\partial\alpha}(\frac{\partial\pi}{\partial p}) = \frac{\partial}{\partial\alpha}(D'(p)(p-c-(v_n(t,\alpha)-v_{n-1}(t,\alpha))) + D(p))$

$= D'(p)(F(t,n,\rho(\alpha)) - F(t,n-1,\rho(\alpha)))$

where $\rho(\alpha)$ denotes the optimal pricing policy, given the tax on vacant seats with rate α. By Lemma 8, we can deduce that $\frac{\partial}{\partial\alpha}(\frac{\partial\pi}{\partial p}) < 0$. Now the result follows from the strict monotonicity theorem 1 from Edlin and Shannon (1998) . □

Lemma 9. *Consider the probability distribution $H(\alpha,t)$ over the number of seats remaining empty at time t , given that the tax rate on vacant seats is α and given that the airline chooses the expected profit maximizing dynamic pricing policy. Then if $\alpha^{'} > \alpha$ then $H\left(\alpha^{'},t\right)$ can be obtained from $H(\alpha,t)$ by shifting probability weight downward from higher numbers of seats remaining empty to lower ones. Moreover, if $D(p)$ is continuously differentiable with $D'(p) > 0 \forall p$ then the probability weight shifted downwards is strictly positive.*

Proof. We proceed by a sample path argument. Given a history h , we can compare what happens in the case with $\alpha^{'}$ to that with α . We start in both cases with N available seats at time 0 . By proposition 7 we know that the price charged at time 0 cannot be higher under the policy α' . Now there are two possibilities: Either in both cases the same sales occur or at some point there is a divergence in the sense that a sale that does not occur under α does occur under α' . From this point onward there exists the possibility that a sale that occurs under α does not occur under α' , since by theorem 1 from Gallego and van Ryzin (1994) the price under α' might be higher. The probability of there ever being more tickets sold under α at any point in time is 0. Hence all that could happen is that we revert back to the case where under both policies the same number of tickets have been sold, so we are back to the case discussed before. This establishes the fact that for each sample path an increase in the tax on vacant seat kilometers decreases the number of seats remaining empty

[16] Note that $v_n(t,\alpha)$ here includes the fact that at time T the tax on vacant seats will have to be paid. In other words, there is a negative scrap value for the airline. This timing is different in section 3, where the subsidy is paid to the airline when the sales are made rather than at the end of the sales horizon.

at any point in time. In particular, the probability distribution $H(\alpha)$ over the number of seats remaining empty at time t is obtained from $H(\alpha, t)$ by shifting probability weight downward from higher numbers of seats remaining empty to lower ones. □

Lemma 10. *The expected number of tickets sold converges to* 0 *as* $\alpha \rightarrow -\infty$ *.*

Proof. From Lemma 9 it follows that the expected number of tickets sold increases in α. Thus as $\alpha \rightarrow -\infty$, the expected number of tickets sold has to decrease. Suppose it does not converge to 0. Then there is some number $f > 0$ such that the expected number of tickets sold exceeds f for arbitrarily negative values for α. But the revenue that can be extracted from consumers is finite, so this would mean that the airline would for some values α make negative profits at its optimal policy, in contradiction to the fact that not selling any tickets and making 0 profit is an option. □

Definition 7. Let $z(\alpha)$ be the expected number of potential passengers arriving during the period $[0, T]$ that face a plane that is not entirely filled.

Lemma 11. $lim_{\alpha->-\infty} z(\alpha) = \lambda T$ *and* $lim_{\alpha->\infty} z(\alpha) = \lambda T$*. Moreover,* $z(\alpha)$ *is increasing on* $[sup\{\alpha : z(\alpha) > 0\}, inf\{\alpha : z(\alpha) < \lambda T\}]$ *and if* $D(p)$ *is continuously differentiable with* $D'(p) > 0 \forall p$ *then* $z(\alpha)$ *is strictly increasing on* $[sup\{\alpha : z(\alpha) > 0\}, inf\{\alpha : z(\alpha) < \lambda T\}]$ *.*

Proof. By definition $z(\alpha) = \int_{r=0}^{T} \lambda Probability\,(someseats\,left\,at\,time\,r)\,dr$

It follows from Lemma 9 that $Probability\,(some\,seats\,left\,at\,time\,r)$ is increasing in α for every r and strictly increasing if $D(p)$ is continuously differentiable with $D'(p) > 0 \forall p$.

The fact that $lim_{\alpha->-\infty} z(\alpha, 0) = 0$ follows from Lemma 10 □

Proof of proposition 5:

Suppose that there is a tax on vacant seats with tax rate α. Suppose that under α and D there is a strictly positive probability that all tickets will be sold. Now consider any $\alpha' < \alpha$. By 11, there will be strictly fewer refusals, i.e. $z(\alpha') > z(\alpha)$

Consider the situation under α': Let us denote by $p_{n,t}$ the airline's optimal price at time t given that there are n tickets left to be sold. Let as before $V_n(t)$ denote the value function for the airline and $\Delta_{n,t}V := V_n(t) - V_{n-1}(t)$. From theorem 1 from Gallego and van Ryzin (1994) we know that $\Delta_{n,t}V$ is decreasing in n. Also, it is clear that $\Delta_{n,t}V$ is decreasing in t: It can only be valuable to have more time to sell tickets. Thus in particular we have $\Delta_{1,0}V \geq \Delta_{n,t}V\ \forall n, t$. Also from theorem 1 from Gallego and van Ryzin (1994) we know that $p_{n,t}$ is decreasing in n and t. In particular, we have $p_{1,0} \geq p_{n,t} \forall n, t$.

Fix a small $\delta > 0$. Fix a $\bar{p} > p_{1,0}$ and consider the demand function $D^{\#}$ defined by $D^{\#}(p) = D(p)$ for $p \leq p_{1,0}$ and $D^{\#}(p) := \frac{D(p_{1,0})(p_{1,0}-c+s)}{(p(1+\delta)-\delta p_{1,0}-c+s)}$ for $p \in (p_{1,0}, \bar{p}]$ and $D^{\#}(p) = 0$ for $p \in (\bar{p}, \infty)$. Then we have:

$$D^{\#}(p)(p-c+s-\Delta_{n,t}V) = \frac{D(p_{1,0})(p_{1,0}-c+s)}{(p(1+\delta)-\delta p_{1,0}-c+s)}(p-c+s-\Delta_{n,t}V)$$

Hence for $p \geq p_{1,0}$we have:

$$D^{\#}(p)(p-c+s-\Delta_{n,t}V) < D(p_{1,0})(p_{1,0}-c+s-\Delta_{n,t}V)$$

which implies that no price $p \geq p_{1,0}$ can ever be chosen at a profit maximizing pricing policy. Hence the profit maximizing pricing policy is unchanged by the modification.

Now let us compute the expected value of the valuation of a consumer:

$$E_{D^{\#}}[v] = \int_{v=0}^{\infty} D^{\#}(v)\,dv = \int_{v=0}^{p_{1,0}} D(v)\,dv + \int_{v=p_{1,0}}^{\infty} D^{\#}(v)\,dv$$

$= \int_{v=0}^{p_{1,0}} D(v)\,dv + \int_{v=p_{1,0}}^{\bar{p}} \frac{D(p_{1,0})(p_{1,0}-c+s)}{(v(1+\delta)-\delta p_{1,0}-c+s)}dv$
$= \int_{v=0}^{p_{1,0}} D(v)\,dv + D(p_{1,0})\frac{p_{1,0}-c+s}{1+\delta} log\left(\frac{\bar{p}(1+\delta)-\delta p_{1,0}-c+s}{p_{1,0}-c-s}\right)$

We see that this converges to infinity as $\bar{p} \to \infty$.

Now consider the welfare consequences. Let $w\left(D^{\#},\alpha'\right)$ denote the welfare given that the demand function is $D^{\#}$ and given that the pricing policy is chosen so as to maximize profits under α'. Since $D^{\#}$ is constructed so as to lead to the same profit maximizing pricing policy under α' as D, we have:

$$w\left(D^{\#},\alpha'\right) = w(D,\alpha') + z(\alpha')(E_{D^{\#}}[v] - E_D[v])$$

$w\left(D^{\#},\alpha'\right) - w\left(D^{\#},\alpha\right) = w(D,\alpha') + z(\alpha')(E_{D^{\#}}[v] - E_D[v]) - w(D,\alpha) - z(\alpha)(E_{D^{\#}}[v] - E_D[v]) = w(D,\alpha') - w(D,\alpha) + (z(\alpha') - z(\alpha))(E_{D^{\#}}[v] - E_D[v])$

Since $z(\alpha') > z(\alpha)$ we conclude that $w\left(D^{\#},\alpha'\right) > w\left(D^{\#},\alpha\right)$ as long as $\bar{p}$ is sufficiently large. □