Optimal Case Detection and Social Distancing Policies to Suppress COVID-19*

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Abstract

This paper studies how to optimally combine case detection and social distancing to halt the transmission of a new infectious disease. The crucial trade-off for the policymaker is not lives vs. livelihoods but the intensity of social distancing vs. the time it needs to stay in place. Theoretically, I characterize the optimal dynamic policy as a simple function of observables, which eases its implementation. Sufficiently stringent social distancing ensures consistently decreasing case numbers, such that case detection can gradually take over the control of the disease. The total cost of suppression depends critically on the efficiency of case detection since it determines how fast economic restrictions can be relaxed. Calibrating the model to Italy in May 2020, I find that suppressing COVID-19 without efficient contact tracing costs 11 % of annual GDP. Digital tracing reduces this cost to 0.4 %, which is by one order of magnitude lower than the cost for reaching herd immunity estimated in the literature.

Keywords: COVID-19, suppression strategies, elimination strategies, Zero COVID.

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1 Introduction

What is the optimal response to a rapidly spreading and deadly infectious disease when no vaccine or effective medication is yet widely available? Due to the COVID-19 pandemic, it has become very urgent to answer this policy question. The epidemiological literature (see Ferguson et al., 2020) defines two possible response strategies: mitigation and suppression. On the one hand, mitigation strategies control, but do not reverse, the spread of the disease to build up herd immunity in the population.[1] On the other hand, suppression strategies push the growth rate of infected below zero to eliminate human-to-human transmission or, if that is not possible, keep transmissions at a low and controlled level.[2] Contrary to mitigation, suppression strategies halt the pandemic within a country before the population acquires herd immunity. During the COVID-19 pandemic, some countries successfully adopted these strategies (Baker, Wilson, and Blakely (2020) mention China, Taiwan, and New Zealand). Several economists and health specialists advocated for their wider adoption sometimes calling them "Zero COVID" or "No COVID" (see Aghion, Artus, Oliu-Barton, and Pradelski, 2021; Baumann et al., 2021; Horton, 2021; Philippe and Marques, 2021; and Priesemann et al., 2021).

This paper shows how to optimally suppress an infectious disease within a country and compares the cost of suppression to the cost of mitigation strategies studied in the literature. The policymaker has two tools to control transmissions: social distancing and case detection. Social distancing reduces the disease's growth rate by reducing social contacts between all individuals in the population. Case detection, for instance, with the help of tests and contact tracing, reduces growth by actively finding infectious individuals and isolating them from the susceptible population. The optimal policy minimizes the sum of economic and health costs.

The first contribution of the paper is theoretical. I analytically characterize the optimal suppression strategy to derive concrete policy implications. Methodologically, I exploit the fact that when the number of infectious is low, the number of susceptible is approximately constant. Therefore, an exponential process approximates the dynamic behavior of the

[1] A population reaches herd immunity when a large enough fraction is immune to infection, i.e., not susceptible. The fraction is large enough when, on average, one infected individual meets and transmits the disease to less than one susceptible individual.

[2] Baker et al. (2020) further distinguish the subset of suppression strategies that eliminate human-to-human transmissions within the borders of a country as elimination strategies.

disease. I find that in the optimum, the intensity of social distancing decreases when prevalence (i.e., the number of infectious in the population) decreases. This simple property offers important guidance for policy design. Suppose a policymaker discovers an outbreak of a new infectious disease. In the optimum, she immediately implements strong enough social distancing measures to reverse its spread. As the number of infectious decreases, the detection rate (i.e., the number of daily detectable cases relative to the overall number of infectious) increases. Case detection gradually takes over the control of the disease, and social distancing can progressively be relaxed. I find that it is not optimal to "smooth in" social distancing, wait before imposing measures, or apply them cyclically (i.e., stop and go). Any initial hesitation or delay in using firm enough measures worsens economic and health outcomes. The reason is that a weak initial response allows the disease to grow further, which increases the number of casualties and the time and intensity of necessary measures later on. For the same reason, easing of social distancing too soon, which allows case numbers to grow again, is not optimal. In case a policymaker discovers signs of rebounding case numbers - such as an increase in the flow of confirmed cases, symptomatic patients, hospitalizations, or death - a swift increase in social distancing is the optimal response to regain control.

I find that the critical trade-off when eliminating the disease is between the intensity of social distancing and the time it needs to stay in place. Too extreme measures rapidly reduce the number of infections in the population; however, they have very high instantaneous costs because even the most fundamental economic activities are on hold. Too weak measures have low instant costs; however, they need to stay in place for a very long time to eliminate transmissions. The optimal policy trades off these two margins at every point in time. I characterize it by a simple formula of two sufficient statistics: first, the instantaneous growth rate of the disease, and second, the instant flow of costs from distancing measures and health outcomes.[3] A policy at a certain point in time is optimal if the elasticity of the current flow-cost to the current growth rate is equal to one. The condition gives specific and straightforward guidance on how to stay on the optimal elimination path over time and, in particular, on how fast to relax social distancing measures. The policymaker only needs to evaluate a measure's relative impact on the current flow of costs and the disease's growth rate to decide upon relaxing it. If the percentage reduction in cost is larger than the percentage increase in growth, a measure should be relaxed.

[3]For a real-time estimation of these costs, see Adams-Prassl, Boneva, Golin, and Rauh (2020); Aspachs, Durante, Graziano, Mestres, Montalvo, and Reynal-Querol (2020); and Chetty et al. (2020).

The combination of case detection and social distancing is crucial to limit the overall cost of elimination. Relying only on social distancing poses two problems. First, social distancing affects all individuals in a population, making it very costly. Second, because the decrease in prevalence follows an exponential decay process, reducing prevalence by social distancing becomes very inefficient when prevalence is low. For example, it takes the same time and intensity of social distancing to reduce the number of infected from 20,000 to 10,000 as reducing it from 20 to 10. Mathematically, the cost of reducing prevalence by one unit goes to infinity as prevalence goes to zero.

Case detection offers a solution to these problems. Its direct cost is born only by detected individuals and not the whole population. Moreover, the lower the number of undetected cases, the greater the number of available resources a country can employ to detect a single case. Consequently, the detection rate, and therefore the efficiency of detection, is at its maximum when prevalence is low. Thus, social distancing and case detection complement each other. A key cost-determinant of the optimal elimination strategy is the maximal detection rate at low prevalence. On the one hand, if the maximal detection rate is larger than the disease's uncontrolled growth rate, optimal social distancing measures can be entirely removed in the long run. As a consequence, the cost of reducing prevalence by one unit is bounded even when prevalence goes to zero. Moreover, if a small inflow of infections from abroad cannot be avoided, case detection alone can control the disease at a low case number. I call this case efficient detection. On the other hand, if the maximal detection rate is lower than the disease's uncontrolled growth rate, case detection needs to be complemented with social distancing until the disease dies out. As in the case with no case detection, the time of reducing prevalence by one unit goes to infinity as prevalence goes to zero. Therefore, also the cost of reducing prevalence by one unit goes to infinity. Moreover, if inflows of infections from abroad cannot be avoided, case detection needs to be complemented by social distancing until a vaccine arrives. I call this case inefficient detection.

The second contribution of the paper is quantitative. I calibrate the model to the COVID-19 pandemic using data from Italy and South Korea. I compare the total cost and time of eliminating the virus under the use of different detection technologies. Moreover, I compare the cost of optimal elimination strategies to cost-estimates of optimal mitigation strategies in the literature. As an initial condition, I use a prevalence of 0.07% - an estimate for Italy at the end of the first lockdown on May 10th 2020. I consider three different detection scenarios. In the first scenario, Italy uses fast and efficient digital contact tracing like South Korea. Ferretti et al. (2020) find that the maximal detection rate of digital contact tracing is higher than the uncontrolled growth rate of COVID-19. Therefore, it provides an example for efficient detection. In the second scenario, Italy uses slower and less efficient manual tracing. The detection rate of manual tracing is lower than uncontrolled growth. Hence, it provides an example of inefficient detection. In the third scenario, Italy detects cases at a constant and low rate observed on May 10th, providing an example for elimination without the benefits of an increasing detection rate.[4]

I find that the total cost of eliminating COVID-19, using digital tracing, is only 0.4% of annual GDP. The strategy allows for a fast and continuing reduction of social distancing. The virus is entirely under control, and social activity is back to normal after three months. The number of additional casualties under this scenario would be 3,300. Under the second scenario, when using manual contact tracing, I find that the total cost of elimination is 1.7% of annual GDP. Manual tracing is not efficient enough to allow for a fast return to normality. In the medium term, some degree of social distancing needs to stay in place; however, its flow-cost amounts to only 0.1% of daily GDP. The virus dies out after 15 months. The number of additional casualties under this scenario would be 4,200. In stark contrast, the total cost of elimination in the third scenario is 11% of annual GDP. The reason is that, in this case, optimal social distancing is very close to constant. Its flow-cost is 19% of daily GDP. The cost accrues until the virus becomes extinct after eight months. Additionally, if new cases are imported after extinction, the pandemic restarts. Therefore, meticulous border controls need to stay in place until a vaccine arrives. The number above does not take into account their costs. The number of additional casualties would be 2,900. The quantitative results suggest that the use of contact tracing, and, in particular digital contact tracing, has enormous benefits. However, their use raises important concerns related to privacy.

[4] Digital contact tracing uses mobile phone data to identify and inform the past contacts of a confirmed infectious individual. It is particularly fast and efficient. Its maximal detection rate is 35% per day (Ferretti et al., 2020). Manual contact tracing relies on teams of tracing personal who question confirmed infectious and find their contacts manually. Its maximal detection rate is 10% per day. See Ferretti et al. (2020) for an extensive discussion. Currently, Italy detects 3% of cases per day. The uncontrolled growth rate of the original variant of COVID-19 is 14% per day.

In comparison, the literature estimates the economic and health costs of optimal mitigation strategies from 7% of annual GDP (Gollier, 2020), to 7-14% (Acemoglu, Chernozhukov, Werning, and Whinston, 2020), to around 30% (Alvarez, Argente, and Lippi, 2020). These numbers stand in stark contrast to the low cost of elimination under efficient contact tracing. Acemoglu et al. (2020) estimate the additional number of casualties in the most optimistic mitigation scenario to around 0.2% of the population. In the case of Italy, these are around 120,000 additional casualties. The relatively low estimates in Acemoglu et al. (2020) and Gollier (2020) rely on the very optimistic assumption that it is possible to shelter the most vulnerable part of the population. Even in that case, the number of casualties of an optimal mitigation strategy stands in stark contrast to the additional casualties of optimal suppression strategies. Moreover, mitigation strategies bear the risk that immunity vanishes over time or that the virus mutates. In both cases, the virus may become endemic, and the strategy fails. None of the cited estimates take this risk into account. The comparison suggests that suppression is the most cost-efficient strategy. Moreover, the strategy leads to a much lower number of casualties.

Related literature This paper contributes to the economic literature on optimal disease control. A large and recent literature studies mitigation policies. See Acemoglu et al. (2020), Atkeson (2020), Atkeson (2021), Atkeson, Kopecky, and Zha (2021), Azzimonti, Fogli, Perri, and Ponder (2020), Berger, Herkenhoff, and Mongey (2020), Brotherhood, Kircher, Santos, Tertilt et al. (2020), Eichenbaum, Rebelo, and Trabandt (2020a), Eichenbaum, Rebelo, and Trabandt (2020b), Farboodi, Jarosch, and Shimer (2020), Favero, Ichino, and Rustichini (2020), Gonzalez-Eiras and Niepelt (2020), Hornstein (2020), Jones, Philippon, and Venkateswaran (2020), Kaplan, Moll, and Violante (2020), Krueger, Uhlig, and Xie (2020), Makris and Toxvaerd (2020), Miclo, Spiro, and Weibull (2020), Obiols-Homs (2020), and Toxvaerd (2020). This list is far from exhaustive. Assenza et al. (2020), Garibaldi, Moen, and Pissarides (2020), Rachel (2020), and Toda (2020) characterize the theoretical properties of optimal mitigation policies. Chari, Kirpalani, and Phelan (2020), Eichenbaum, Rebelo, and Trabandt (2020b), and Brotherhood, Kircher, Santos, Tertilt et al. (2020) study the impact of testing on mitigation. This literature uses variants of the standard SIR model augmented with economic interactions. In this model, eliminating the disease is not possible. Even if control measures reduce the mass of infected to almost zero, the pandemic restarts if the population did not reach herd-immunity and measures are removed. This undesirable model feature is called atto-fox problem (see Mollison (1991) and Moll (2020) for a discussion). I contribute to this literature by studying suppression policies, which, as noted by Ferguson et al. (2020), are distinct from mitigation policies. To this end, I explicitly introduce the possibility for extinction into the standard SIR model.

A smaller part of the literature studies suppression using simulations. Gollier (2020), Scherbina (2020), and Ugarov (2020) simulate the impact of a uniform lockdown. Dorn et al. (2020) simulate the effects of various control scenarios using a detailed economic and epidemiological model. Contreras et al. (2020) simulate suppression in an epidemiological model. Wang (2020) simulates the effect of mass testing and shows that it can lead to suppression before herd immunity. I contribute to this literature by explicitly characterizing the optimal time-variable suppression strategy.

Closest to my paper are Gerlagh (2020), Piguillem and Shi (2020), Bethune and Korinek (2020), and Alvarez et al. (2020) who study optimal suppression strategies. Gerlagh (2020) provides a closed from solution for the optimal suppression policy when case-detection is constant and ineffective, and assumes elimination is not possible. I explicitly model decreasing returns to scale in case-detection and extinction. The later three papers are quantitative. Piguillem and Shi (2020) solve for the optimal suppression strategy in a SIR model with social distancing and random testing. Bethune and Korinek (2020) solve for the optimal suppression strategy under two polar assumptions on the planner's information set: the planner exactly knows who is infected, and the planner has no information at all on who is infected. Alvarez et al. (2020) solve for the optimal strategy with tracing. They explicitly assume a functional form for tracing. I contribute to this literature by analytically characterizing the optimal suppression strategy as the solution of two simple sufficient statistics. I derive its properties and policy implications for general functional forms and parameter values. Moreover, I study how the optimal policy depends on the unknown parameters. In particular, I show that the optimal suppression strategy and its cost depend crucially on well-defined properties of the detection technology. The tracing function used by Alvarez et al. (2020) is infinitely efficient in the limit. This property is at odds with the epidemiological literature on case detection; see Ferretti et al. (2020). My paper's theoretical results show that this property leads to overly optimistic estimates for the cost of suppression. Quantitatively, I contribute to this literature by comparing the optimal suppression policy under realistic detection technologies. I use epidemiological estimates (Ferretti et al., 2020) to calibrate the detection technologies.

Pueyo (2020) gives an extensive informal discussion of possible policy responses.

2 The Model

In a first step, I introduce the standard SIR model without policy interventions. Assume the population has a mass of one and consists of three groups. I_t denotes the fraction of infectious individuals at time t. S_t denotes the fraction of the population susceptible to getting infected by the disease when meeting an infectious. R_t denotes the number of recovered individuals. Infectious and susceptible meet randomly, and the disease transmits at a certain rate. When there are no active control measures, the infection rate is β^0. The recovery rate, once infected, is γ. The initial fraction of infectious and susceptible is denoted by I_0 and S_0. Three differential equations describe the uncontrolled dynamics of the three groups:

$$\dot{I}_t = (\beta^0 S_t - \gamma) I_t; \tag{1}$$

$$\dot{S}_t = -\beta^0 S_t I_t; \tag{2}$$

$$\dot{R}_t = \gamma I_t, \tag{3}$$

where the dot superscript denotes the time derivative. Denote the mass of one individual in the population as I^ϵ. I assume that the disease dies out when less than one individual in the population is infectious:

$$\text{If } I_T < I^\epsilon \text{ it follows that for all } t > T \; I_t = 0. \tag{4}$$

I call I^ϵ the extinction threshold. This assumption introduces the possibility of extinction in the deterministic and continuous SIR model. It circumvents the need for a less tractable discrete and probabilistic model. The assumption is common in the economics literature (see Gollier, 2020 and Piguillem and Shi, 2020).[5] For parts of the analysis, it is useful

[5] Without Assumption 4, the deterministic and continuous SIR model has the unrealistic feature that at a low mass of infected I, a fraction of an individual is infected. This feature implies that the disease never dies out since even if there is constant and negative growth, in the limit, an infinitesimal fraction of one individual is infected. In reality, the infection process is discrete. At each instant of time, a discrete number of individuals are infected. New infections and the exit from the state of infectiousness are probabilistic. In particular, if I is sufficiently low, there is a positive probability that all infected recover without infecting a new susceptible individual. In this case, the disease dies out. Assumption 4 is a convenient way to introduce extinction in the more tractable deterministic and continuous model. I leave a generalization to probabilistic extinction for future research. Note that all results in this paper hold for arbitrarily small extinction thresholds.

to define herd-immunity. As soon as the number of susceptible is lower than the herd-immunity threshold, the number of infectious goes to zero without the application of control measures by the policymaker. The herd immunity threshold $\overline{S}$ is:

$$\overline{S} = \frac{\gamma}{\beta^0}. \tag{5}$$

It is straightforward to see that the uncontrolled dynamics in Equation 1 to 3 give $\dot{I}_t < 0$ when $S_t < \overline{S}$.

A policymaker can apply control measures to alter the dynamic behavior of the disease. As noted by Ferguson et al. (2020), controlling the dynamics allows following two different strategies: mitigation and suppression. The two strategies differ in the way the pandemic halts. When following a mitigation strategy, the pandemics halts because the population reaches herd immunity: $S_\infty < \overline{S}$. The policymaker controls the spread of the disease to achieve herd immunity in a cost-efficient way. When following a suppression strategy, the policymaker halts the pandemic without the population reaching herd immunity: $S_\infty > \overline{S}$. To this end, the policymaker applies control measures to push the number of infectious below the extinction threshold. After extinction, the policymaker protects the population from reinfection by using meticulous border controls or, if that is not possible, by keeping the number of infectious at a very low level. In this paper, I study suppression strategies, and I compare them to mitigation strategies studied in the literature.

In the next step, I simplify the dynamics of the model. When the mass of infected I_t is small compared to the mass of susceptible S_t, the change in the mass of susceptible $\dot{S}_t$ is small compared to S_t. Therefore, S_t is approximately constant: $S_t \approx S$. As a consequence, the dynamic behavior of the infectious is approximately described by an exponential growth process:

$$\dot{I}_t = r^0 I_t, \tag{6}$$

where $r^0 = \beta^0 S - \gamma$. Note that I_t is small compared to S_t at the beginning of the pandemic and after an extended period of effective control measures.[6] In particular, when suppressing

[6]For the quantitative results, I calibrate the model to Italy, using as an initial condition the situation at the end of the first lockdown on May 10th, 2020. This date is three months after the onset of the pandemic in Italy. The country was hit hard by the virus, and it went through a period of strict lockdown. The mass of infectious on that date is equal to 0.07%, and the mass of susceptible is equal to 96%. It shows that the number

the disease, the number of infectious converges to zero, while the number of susceptible converges to a constant. Therefore, at some point, the mass of infected is small compared to the mass of susceptible. I will use the approximation to study suppression, which has the advantage of simplifying the analysis considerably. It allows me to solve for the optimal suppression strategy analytically. Note that the approximation abstracts from immunization by infection. Therefore, it is unsuitable for studying mitigation strategies.

The following two sections introduce the control measures available to the policymaker.

2.1 Case Detection

Case detection allows for quarantining a mass X of infectious individuals at each instant of time. I assume infectious individuals in quarantine do not infect susceptible individuals. $X(I)$ is the flow of detected cases into quarantine, which is a function of the mass of infectious I. Intuitively, it is the speed of detection. When there is case detection the mass of infectious over time follows

$$\dot{I}_t = r^0 I_t - X(I_t). \tag{7}$$

For a derivation of this equation from the SIR model, see appendix A.1.2. I assume $X(0) = 0$; none are detectable if there are no infectious. $X'(I) > 0$; the speed of detection increases with the number of infectious, because when there are more infectious, it is easier to detect them. Next, I assume that $X"(I) < 0$; the increase in detection speed is decreasing in the number of infectious. This property models a detection technology that becomes overwhelmed when there are too many infected, i.e., decreasing returns to scale. In the limit, if I goes to infinity, $X'(I)$ goes to zero. A discussion of this assumption is in appendix A.1.1. It follows when the total resources for case detection in a country are fixed and constant over time. Note that, in the case of COVID-19, it has been difficult for many countries to increase the detection capacity in the short term. For this reason, the main part of the paper focuses on the simple case where resources for detection are constant. An extension to the case where resources for detection are a choice variable is in appendix A.3.2.

of infectious may be small compared to the number of susceptible well after the onset of a pandemic. The quantitative exercise in Section 4 shows that the change in the mass of susceptible on the optimal suppression path is smaller than 0.7%, and therefore negligible. The results suggest that the mistake from using the approximation in Equation 6 is small.

Define the detection rate as

$$\frac{X(I)}{I}. \tag{8}$$

Intuitively, it gives the percentage of overall cases that are detected daily. Under the above assumptions, the detection rate is decreasing in I. The proof of this statement is in Lemma 3 in the appendix. It is the largest at zero. The detection rate at zero is a key parameter for the analysis. Denote it as ξ_0:

$$\xi_0 = \lim_{I \to 0} \frac{X(I)}{I} = X'(0). \tag{9}$$

There are two distinct cases:

Lemma 1. .

1. *If $\xi_0 \leq r^0$, the rate of case detection is never larger than the uncontrolled growth rate of the disease. Therefore, detection alone cannot suppress the pathogen.*

2. *If $\xi_0 > r^0$, there exists a level of infectious $I^* > 0$, such that for all $I < I^*$, it holds that $\dot{I}(I) < 0$. Therefore, if $I_t < I^*$, detection alone suppresses the pathogen. I^* is the point where $r^0 I^* = X(I^*)$.*

The proof is in appendix A.1.3.

2.2 Social Distancing

Assume the policymaker has access to a continuum of social distancing measures indexed by p, where p is an element in the interval $[0, 1]$. Each policy p has an infinitesimal impact on the growth rate of the disease $dr(p)$ and on the social cost $dc(p)$. Without loss of generality, assume policies are indexed such that the cost benefit ratio $\frac{dc(p)}{dr(p)}$ is increasing. Also, assume that $\frac{dc(0)}{dr(0)} = 0$ and $\frac{dc(1)}{dr(1)} = \infty$. Denote by p_t the fraction of policies applied by the policymaker at time t, i.e., applying p_t means applying all measures in the set $[0, p_t]$. Applying p_t has a growth impact

$$r(p_t) = \int_0^{p_t} r'(p)dp, \tag{10}$$

and a cost impact

$$c(p_t) = \int_0^{p_t} c'(p)dp. \tag{11}$$

For example, p can be interpreted as the economy's sectors and applying p_t means closing a fraction p_t of the economy. More generally, p includes any policy that reduces contagions, e.g., mask mandates, curfews, mobility restrictions, etc. Assume that there are enough policies available such that at $r(1) >> r^0$. Strict enough measures allow pushing the growth rate of the disease below zero, i.e., exponential decay. When there is social distancing, the mass of infectious over time follows

$$\dot{I}_t = (r^0 - r(p_t))I_t. \tag{12}$$

For a derivation from the SIR model, see appendix A.1.2. If $r(p_t) > r^0$ the process follows an exponential decay.

Physically, for any initial level of infections I_0, the suppression of the disease is possible by keeping $r(p_t) > r^0$. However, $\dot{I}_t$ goes to zero as I_t goes to zero. The smaller I_t, the slower the suppression is advancing. In the limit, the process becomes infinitely slow. This property has important consequences for the cost of suppression, which I discuss in Section 3.4.

To summarize, the functions $r(p)$ and $c(p)$ have the following properties: they are increasing and zero at zero, $\frac{c'(p)}{r'(p)}$ is increasing and zero at zero, $r(1) >> r^0$, and $c(1) = \infty$. For some derivations, it is more convenient to express the flow-cost as a function of r instead of p:

$$c(r) = c(p(r)). \tag{13}$$

It follows that $c(r)$ is increasing and convex: $c'(.) > 0$ and $c''(.) > 0$. The cost, as well as the marginal cost, is zero in the origin: $c(0) = 0$ and $c'(0) = 0$. Note that this is an abuse of notation. I use the same letter for two different functions. Which function is meant will be clear from the context.

2.3 Health Costs

Assume that each new infection has a health cost of v. Note that infectious exit the state of infectiousness at rate γ. When they exit they recover or die. $(r^0 - r(p_t))I_t$ is the net flow of infections. Therefore, the flow of new infections is $(r^0 - r(p_t) + \gamma)I_t$. For the detailed derivation see appendix A.1.4. Denote the probability that the outcome of an infection is death by δ. Denote the value of a statistical life by vsl. The flow-cost per infection v is equal to $v = \delta\, vsl$.[7]

3 The Optimal Policy

I assume the policymaker chooses the optimal intensity of social distancing to minimize the economic cost of social distancing and the health costs caused by infections:

$$\min_{p_t} C(p_t) = \int_0^\infty c(p_t) + vI_t(r^0 - r(p_t) + \gamma)dt \tag{14}$$

where

$$\dot{I}_t = (r^0 - r(p_t))I_t - X(I_t), \tag{15}$$

and

$$I_t = 0 \text{ for all } t > T \text{ if } I_T < I^\epsilon. \tag{16}$$

First, note that Problem (14) neglects discounting. This assumption simplifies the problem considerably. Time discounting is not important when a pandemic moves fast. The relevant time frame is days, and daily interest rates are low. The probabilistic arrival of an effective cure or mass vaccine for a disease can be modeled in the same way as discounting. Again, discounting is not important if daily arrival probabilities are low. I solve the general case with time discounting and a random vaccine or treatment arrival in appendix A.3.1. I find that, as long as the discount rate and arrival-probability are low enough, the qualitative results in this section do not change. Second, since the costs of case detection are small compared to the costs of social distancing, I neglect them in the paper's main part. See appendix A.3.2 for a generalization in this direction. Third, Problem (14) assumes that, after extinction, the population can effectively be protected against reinfection from abroad. I discuss the case when an inflow of cases from abroad cannot be avoided in appendix A.3.3.

[7] v may be interpreted more broadly as containing all other costs caused by an infection, such as the disutility of being sick and chronic damages caused by the disease. The cost can be generalized to a nonlinear cost in I, accounting for congestion effects in the health care sector.

The solution to Problem (14) is an optimal control function p_t with the following property:

Lemma 2. .
In the optimum, the mass of infectious decreases over time: $\dot{I}_t < 0$ *for all* t. *Over time, the mass of infectious reaches the extinction threshold: there exists a time* T *such that* $I_T = I^\epsilon$, *and* $I_t = 0$ *for all* $t > T$.

The result in Lemma 2 is very intuitive. The only way to keep the total cost in Problem (14) bounded is to eliminate the disease over a finite time horizon. To this end, it is optimal to decrease the number of infectious over time. The proof of the lemma is in appendix A.2.1. To further characterize the optimal policy, I restrict the analysis to control functions that exhibit the properties in Lemma 2. I use the monotonicity and the boundary values to change variable in the minimization problem (14) which eliminates the time variable:

$$\min_{p(.)} C(p(.)) = \int_{I_0}^{I^\epsilon} \frac{c(p(I)) + vI(r^0 - r(p(I)) + \gamma)}{\dot{I}(I)} dI, \tag{17}$$

where

$$\dot{I}(I) = (r^0 - r(p(I)))I - X(I). \tag{18}$$

The solution to the problem is a control function $p(.)$. It is the solution to a simple pointwise minimization of the above integral. Its solution gives the main result of the paper:

Proposition 1. .
For each amount of currently infectious I, *the optimal policy* $p(I)$ *solves:*

$$\frac{c'(p) - vIr'(p)}{c(p) + vI(r^0 - r(p) + \gamma)} = \frac{r'(p)}{r(p) + \frac{X(I)}{I} - r^0}. \tag{19}$$

An optimal solution $p(.)$ *exists and is unique.*

The proof is in appendix A.2.2. Note that the denominator on the left of Condition (19), $c(p) + vI(r^0 - r(p) + \gamma)$, is the instantaneous flow-cost from suppression measures and health outcomes, while the denominator on the right, $r(p) + \frac{X(I)}{I} - r^0$, is the instantaneous negative growth rate of the disease. The respective enumerators are the marginal impacts of a policy change on these two variables. This fact gives rise to the following corollary:

Corollary 1. .

A policy $p(I)$ is optimal if at each point in time, its relative effect on the flow-cost is equal to its relative effect on the viral growth rate:

$$\frac{d\ \log\left(c(p) + vI(r^0 - r(p) + \gamma)\right)}{d\,p} = \frac{d\ \log\left(r(p) + \frac{X(I)}{I} - r^0\right)}{d\,p}. \tag{20}$$

Corollary 1 has concrete policy implications, which I discuss in Section 3.2. Alternatively, one can express Condition (19) as an elasticity:

Corollary 2. .

A policy is optimal if at each point in time, the elasticity of the flow-cost to the negative growth rate is equal to one:

$$\frac{d\ \log\left(c\left(-g + r^0 - \frac{X(I)}{I}\right) + vI\left(g + \frac{X(I)}{I} + \gamma\right)\right)}{d\ \log(-g)} = 1. \tag{21}$$

Note that Corollary 2 follows from the fact that, for each level of infectious I, there is a one to one mapping from policy p to the instantaneous negative growth rate g:

$$-g(p, I) = r(p) + \frac{X(I)}{I} - r^0. \tag{22}$$

Therefore, one can change variable in (19) from p to $-g$.

3.1 The Intuition Behind Proposition 1

To better understand the intuition behind the optimality condition (19), it is useful to recall each mathematical step in the derivation intuitively:

The first step is the change in variable from t to I. Integrating over time means summing the flow costs at each point in time. Integrating over I means summing the flow-cost for each reduction in I. The policymaker would like to reduce new infections from I_0 to 0. As a consequence of the extinction threshold, the policymaker only needs to reduce infections to $I^\epsilon > 0$. It is useful to think about the reduction as of a distance to cover. In particular, partition the distance into many small and constant intervals ΔI. The minimization problem consists of minimizing the cost for each of these intervals. The cost to reduce new infections at I to $I - \Delta I$ depends on the flow cost and the time it takes to cross the interval:

$$\underbrace{\left(c(p) + vI(r^0 - r(p) + \gamma)\right)}_{\textit{flow cost}} \times \underbrace{\Delta t(I,p)}_{\textit{crossing time}} . \tag{23}$$

Note that the crossing time is a function of p and I:

$$\Delta t = \frac{\Delta I}{(r(p) - r^0)I + X(I)}. \tag{24}$$

The flow-cost is increasing in the intensity of applied policies p while the crossing time is decreasing p. Note that for small I, vI is only marginally relevant. The following corollary summarizes this finding:

Corollary 3. .
The key trade-off for suppressing the disease is between the cost of social distancing, $c(p)$*, and the time it needs to stay in place,* $\Delta t(p)$*. The optimal policy trades off these two margins at every instant of time.*

To find the optimal policy $p(I)$, take the logarithm of the above expression and perturb the current policy p by a small amount Δp to derive a change in cost ΔC:

$$\Delta C = \left(\frac{c'(p) - vIr'(p)}{c(p) + vI(r^0 - r(p) + \gamma)} + \frac{\frac{\partial \Delta t(I,p)}{\partial p}}{\Delta t(I,p)} \right) \Delta p. \tag{25}$$

A policy is optimal if there exits no policy perturbation that reduces the cost. It is the case when the expression in brackets is equal to zero.

Instead of using Δt in the condition above, it is possible to express the same condition as a function of the growth rate. Define the growth rate g as $\frac{\dot{I}}{I}$. The crossing time Δt is inversely proportional to the growth rate:

$$\Delta t(I,p) = \frac{\Delta I}{I} \frac{1}{-g(I,p)}. \tag{26}$$

Therefore, the change in cost ΔC as a function of the growth rate is:

$$\Delta C = \left(\frac{c'(p) - vIr'(p)}{c(p) + vI(r^0 - r(p) + \gamma)} - \frac{\frac{\partial -g(I,p)}{\partial p}}{-g(I,p)} \right) \Delta p. \tag{27}$$

Using the definition of the growth rate gives the expression in Proposition 1:

$$\frac{c'(p) - vIr'(p)}{c(p) + vI(r^0 - r(p) + \gamma)} = \frac{r'(p)}{r(p) + \frac{X(I)}{I} - r^0}. \tag{28}$$

3.2 The Policy Implications of Corollary 1

Two simple and observable sufficient statistics characterize the optimal policy: the current flow cost and the disease's current growth rate. The policymaker only needs to consider the relative change of these two observables to a policy change to evaluate the current policy's optimality. Optimality solely depends on present variables and not on the value function. This property is somewhat surprising. The problem is a dynamic optimization problem, and, to be optimal, a decision at a certain point in time needs to account for its effects on the whole future. However, the two observables summarize all relevant dynamic information.

The optimality condition gives specific guidance to organize a de-confinement after an extended lockdown. For relaxing a certain confinement measure, the policymaker only needs to evaluate its relative impact on the current social cost and growth rate. If the relative reduction in cost is larger than the relative increase in growth, a measure should be relaxed. For instance, a policymaker may want to evaluate reopening a particular sector of the economy, such as construction. To make an optimal decision, the policymaker only needs information on how many percentage points such a measure would ease the current cost of the confinement and how many percentage points it would increase the disease's current growth rate.

Note that how to reopen, which is which policy to reverse first, is determined by the ratio $\frac{dc(p)}{dr(p)}$. Policies with a high ratio should be relaxed first. Proposition 1 tells how fast to reopen. The optimality condition is robust to complementaries between policies, both, in cost and in growth impact. The optimal decision only depends on the marginal impact of the most efficient policy at a certain point in time.

3.3 Properties of the Optimal Policy

To derive the optimal policy's properties, it is simpler to use r as a control variable instead of p. Note that such a change in the variable is without loss of generality. The optimal policy is characterized by a function $r(I)$.

Proposition 2. .

1. *In the optimum, social distancing measures are always positive, and increasing in the number of infectious. Social distancing is the largest at the beginning when* $I = I_0$*. It is gradually released, when the number of infectious decreases:*

$$r(I) > 0, \textit{ for all } I > 0, \textit{ and } r'(I) > 0. \tag{29}$$

2. *In the limit, at* I^ϵ*, the optimal policy* $r(I^\epsilon)$ *has the following properties:*
   - *If* $\xi_0 \geq r^0$*, social distancing goes to zero:* $\lim_{I^\epsilon \to 0} r(I^\epsilon) = 0$;
   - *If* $\xi_0 < r^0$*, social distancing goes to a constant:* $\lim_{I^\epsilon \to 0} r(I^\epsilon) = 2(r^0 - \xi_0) > 0$.
3. *Under the optimal policy, the growth rate of* I *is negative:* $g(I) < 0$*. In the limit it is equal to* $\lim_{I^\epsilon \to 0} g(I^\epsilon) = -|\xi_0 - r^0|$*. In particular, the growth rate goes to* $-\infty$ *if* $\xi_0 = \infty$.

The proof is in appendix A.2.3. Note that for the case $\xi_0 < r^0$, I assume a quadratic cost. The proposition underlines the crucial role of ξ_0, i.e., the detection rate at zero. It governs the amount of time it takes to suppress the disease and the optimal policy in the limit. If detection is efficient enough, social contacts go gradually back to the pre-crisis level. The same is not true when case detection is inefficient. Note that the efficiency of detection is characterized by the derivative of the flow of detections in zero. With inefficient detection, some social distancing needs to stay in place until the extinction limit I^ϵ is reached. If this limit is infinitely small, suppression takes infinitely long. However, the efficiency of detection still matters in this case. It determines the level of necessary social distancing in the limit. The necessary level may consist of mild measures such as washing hands, wearing masks, and forbidding mass events. In the next step, I study the cost of suppression at the optimum.

3.4 The Cost of the Optimal Policy

In the optimum, the total cost of suppressing a mass I_0 of infectious is

$$C = \int_{I^\epsilon}^{I_0} \frac{c(r(I)) + vI(r^0 - r(I) + \gamma)}{\left(r(I) + \frac{X(I)}{I} - r^0\right) I} dI, \tag{30}$$

where $r(I)$ denotes the optimal policy. The unit cost of suppression, intuitively, the cost to suppress one more infectious, is equal to

$$\frac{dC}{dI} = \frac{c(r(I))}{\left(r(I) + \frac{X(I)}{I} - r^0\right) I} + \frac{v(r^0 - r(I) + \gamma)}{\left(r(I) + \frac{X(I)}{I} - r^0\right)}. \tag{31}$$

It consists of two parts: an economic unit cost, the first summand, which comes from the costs of the taken suppression policies, and a health unit cost, the second summand, which comes from the health-consequences of infections.

Proposition 3. .
Case 1, $\xi_0 > r^0$:

- *As I converges to zero, the economic unit cost of suppression converges to zero, and the health unit cost of suppression converges to a constant. If $\xi_0 = \infty$, also the health unit cost converges to zero.*
- *The total cost of suppression is bounded, even if $I^\epsilon = 0$.*

Case 2, $\xi_0 < r^0$:

- *As I converges to zero, the economic unit cost of suppression converges to infinity, and the health unit cost of suppression converges to a constant.*
- *If the extinction limit I^ϵ goes to zero, the total cost of suppression goes to infinity.*

The proof is in appendix A.2.4. The case $\xi_0 < r^0$ assumes a quadratic cost. The proposition underlines the importance of the properties of case detection when I goes to zero. If the detection rate is high enough, social contacts gradually go back to normal, which bounds the total cost. If the rate is not high enough, the total cost goes to infinity if I^ϵ goes to zero. When $\xi_0 < r^0$, optimal social distancing does not go to zero in the limit;

therefore, its cost does not go to zero. On top of that, the time to suppress the disease goes to infinity if I^ϵ goes to zero. It follows that the total cost of suppression goes to infinity if I^ϵ goes to zero. However, this does not mean suppression is not optimal. The necessary long-run measures may be very mild and therefore worth enduring. Even if $\xi_0 < r^0$, its size still matters because it determines the amount of social distancing necessary in the long run. It may still be less costly to suppress the disease than to use another solution, such as herd immunity. Especially, suppression minimizes the risk that the disease mutates and becomes endemic. I further discuss these issues in the next section.

4 Quantitative Results

The results discussed so far are theoretical and hold for general parameter values. In this section, I calibrate the model to the COVID-19 pandemic in Italy, which allows me to study two crucial questions: What are the economic and health costs of the optimal suppression strategy for COVID-19 when using realistic detection technologies? And, how do these costs compare to the costs of optimal mitigation policies? To calculate the total cost of suppression, it is necessary to know the functions $c(p)$, $r(p)$ and $X(I)$, as well as the parameters r^0 and v. Therefore, due to the high parameter uncertainty, a precise answer to the above questions is impossible. However, the calibration exercise in this section gives an approximate answer.

4.1 Data Sources

I use frequently updated epidemiological data from the Institute for Health Metrics and Evaluation (IHME) at the University of Washington.[8] They provide a time series of confirmed cases and estimates for the real number of daily infections for many countries. Their estimates are based on Murray et al. (2020). I use data from Italy and South Korea.

4.2 Calibration

4.2.1 Parameters Literature

I use the following parameters from the literature as a starting point for my calibration:

[8] I downloaded the data from http://www.healthdata.org on May 12th, 2020.

Parameter	Symbol	Value	Source
Mortality rate	δ	0.01	Alvarez et al. (2020)
Time of contagiousness	$\frac{1}{\gamma}$	5 days	Fernández-Villaverde and Jones (2020)
Value of statistical life	vsl	$20\ \frac{\text{GDP}}{\text{capita}}$	Alvarez et al. (2020)
Uncontrolled growth rate	r^0	0.14	Ferretti et al. (2020)
Detection rate digital tracing	ξ_0^d	0.35	Ferretti et al. (2020)
Detection rate manual tracing	ξ_0^m	0.1	Ferretti et al. (2020)
GDP loss strict lockdown	c_{LD}	0.5	Gollier (2020)

Table 1: Parameters Literature

4.2.2 The Cost Function

I use a direct relation between the cost of social distancing, measured as lost GDP, and the reduction in the viral growth rate r. I assume the function is iso-elastic:

$$c(r) = \zeta_0 r^{\zeta_1}, \tag{32}$$

where $\zeta_0 > 0$ and $\zeta_1 > 1$. Neglecting the value of lives lost as well as tracing, by Corollary 4 in the appendix, it holds that the optimal r solves

$$\zeta_1 = \frac{r}{r - r^0}. \tag{33}$$

From March 10th until April 26th, the Italian government imposed a nationwide lockdown. A lockdown is a strict form of social distancing. Any no-essential social contacts are forbidden. A large part of the population is forced to stay at home. Going outside is permitted only if essential. To calibrate ζ_1, I assume the strict lockdown in Italy was close to optimal. Note that Italy did not use much tracing during the time of the lockdown. The value of lives lost is small compared to the lost GDP. The assumption of optimality is certainly strong. However, in many countries such as France, Spain, the UK, and Germany, we have seen very similar lockdown intensities. This is consistent with Equation (33). Note that the optimal intensity of r does not depend on the level of infections. It only depends on ζ_1, which parametrizes the convexity of the cost. Once a country discovers an outbreak, it should hit hard to reduce new infections. If tracing is infeasible in the short term, and the number of death is relatively small, Equation (33) is a good approximation for the optimal policy. The intensity of r does only depend on the convexity of the cost ζ_1. Note that the convexity should be similar between countries. The more convex the cost $c(.)$, the more it costs to implement a strict lockdown. The similarly intense lockdowns between countries suggest that governments approximately followed the optimal lockdown strategy. A different interpretation of the optimality assumption is that it makes the simulated costs consistent with the strict lockdown. Assume the government acted optimally, given its best estimate of the cost function. It follows that the implied simulated policies and costs are consistent with the estimate.

Using the epidemiological data from Murray et al. (2020), I estimate the growth rate under the Italian lockdown at $g_{LD} = -0.032$. I use the estimated number of new infections form the peak on March 11th until May 10th. Together with an uncontrolled growth rate of $r^0 = 0.14$ (see Ferretti et al. (2020)), the estimated growth reduction during the lockdown is $r_{LD} = r^0 - g_{LD} = 0.172$. Using Equation (33), it implies an elasticity of $\zeta_1 \approx 5$. Following Gollier (2020), I assume a strict lockdown implies a daily GDP loss of around $c_{LD} = 50\%$.[9] The implied $\zeta_0 \approx 3300$.

4.2.3 The Case Detection Function

I use the following case detection function:

$$X'(I) = \left(\xi_0^{-\frac{1}{\alpha}} + \xi_1 I\right)^{-\alpha}, \tag{34}$$

and $X(0) = 0$. The function fulfills the necessary properties of a detection function specified above. $\xi_0 > 0$ is the value of the function for $I = 0$. Note that it is equal to $\lim_{I\to 0} \frac{X(I)}{I}$, i.e., the relative speed of tracing at zero. The parameter $\xi_1 > 0$ controls the function's behavior for large values of I. $\alpha \geq 0$ controls how fast $X'(I)$ goes from ξ_0 to its behavior for large I. This function is quite general and contains some intuitive tracing functions as special cases. For $\alpha = 0$ it reduces to a constant returns to scale tracing function: $X(I) = \xi_0 I$. In particular, if ξ_0 is equal to the daily flow of tests, it is equal to tracing under random testing. When ξ_0 goes to infinity, the function reduces to a power function as used in Alvarez et al. (2020). The disadvantage of a power function is that $X'(0) = \infty$ by assumption. This assumption is unrealistic. It makes tracing overly efficient at the end of suppression.

To calibrate the parameters, I distinguish two cases: digital tracing and manual tracing.

[9] The number in Gollier (2020) is based on GDP estimates from the "National Institute of Statistics and Economic Studies" in France (https://www.insee.fr/fr/statistiques/4485632).

I use micro estimates to calibrate the function for both cases. I use results from Ferretti et al. (2020). This epidemiological paper estimates how much optimal contact tracing can reduce daily new infections. They compare digital contact tracing with manual contact tracing. Ferretti et al. (2020) estimate that, under optimal conditions, digital contact tracing can find infectious individuals at a rate of $\xi_0^d = 35\%$ per day. It means that the stock of currently infectious can be reduced by 35% in one day. Manual contact tracing is much slower. The authors argue that optimal manual contact tracing achieves a rate of $\xi_0^m = 10\%$ per day because of unavoidable delays. I use these estimates as values for ξ_0 in the two cases. I assume that if a country uses its full resources to find the last cases, tracing achieves its optimal rate. However, as soon as the caseload grows, the system becomes overwhelmed, and the efficiency of tracing decreases.

I calibrate ξ_1 such that at a prevalence level of 10%, i.e., 10% of the population is infected at the same time, $X'(I) = \xi_0/10000$, which is close to zero. It means that at a prevalence level of 10%, the system is so overloaded that any further increase in the number of infected will not lead to more traced cases. To calibrate α, I use estimates of the fraction of traced cases from Italy and South Korea. I use data from Murray et al. (2020), and I use the date with the maximal detection rate in both countries. I estimate that Korea, using digital tracing, at a prevalence of 56 infected per million, found 21 % of total cases daily. I assume the number of confirmed cases is equal to the number of traced cases. Note that the estimated rate is not too far from the theoretical limit of 35%. It implies that, for digital tracing, $\alpha_d = 1.2$. For manual tracing, I use the same procedure using data from Italy. On May 10th, at an estimated prevalence of 700 per million, Italy manages to confirm 3% of the total cases daily. It implies that, for manual tracing, $\alpha_m = 1.5$.

4.2.4 Remaining Parameter Values

As already mentioned, I use $r^0 = 0.14$ as in Ferretti et al. (2020).

To estimate the current prevalence in Italy, I use the data from Murray et al. (2020). I use May 10th as a starting date. Murray et al. (2020) estimate daily new infections. I use new infections to calculate the current stock of infectious by summing the infections over the 5 preceding days. I assume an infected is infectious for $1/\gamma = 5$ days, following Fernández-Villaverde and Jones (2020). On May 10th, I find a level of prevalence of

$I_0 = 0.0007$ and a level of susceptible of $S = 0.96$. It shows that the number of infectious is indeed small compared to the number of susceptible. Therefore, the assumption that the number of susceptible is constant is approximately true. I will confirm the assumption ex-post in Section 4.3.1.

To estimate the health cost, I assume that an infectious dies with probability $\delta = 0.01$, following Alvarez et al. (2020). Following Alvarez et al. (2020), I use a value of statistical life of 20 times the annual output per capita. It follows that $v = vsl * 365 * \delta = 73$. My results are insensitive to the assumptions related to mortality because of the low prevalence level.

4.2.5 Summary Relevant Parameters

Parameter	Symbol	Value	Matched Moment or Source
Factor cost function	ζ_0	3300	GDP loss lockdown
Cost-elasticity	ζ_1	5	Lock-down intensity
Maximal rate digital tracing	ξ_0^d	0.35	Ferretti et al. (2020)
Maximal rate manual tracing	ξ_0^m	0.10	Ferretti et al. (2020)
Scalability digital tracing	α_d	1.2	Confirmed cases Korea
Scalability manual tracing	α_m	1.5	Confirmed cases Italy
Initial Prevalence	I_0	0.07%	Estimate for Italy May 11th
Flow value of casualties	v	73	Alvarez et al. (2020)
Uncontrolled growth rate	r^0	0.14	Ferretti et al. (2020)

Table 2: Relevant Parameters

4.3 Results

As an initial condition, I take the prevalence in Italy on May 10th, 2020. I quantify the optimal suppression policy for three different tracing scenarios:

1. Italy isolate infectious at a constant rate of 3% per day like on May 10th. I refer to this case as no tracing.
2. Italy adopts an optimal manual contact tracing strategy.
3. Italy adopts an optimal digital contact tracing strategy like South Korea.

To compare the three scenarios, I examine the intensity of the optimal social distancing measures, their implied flow-costs, the time it takes to reach certain thresholds in daily cost, and the total cost.

Cost or intensity measure	No tracing	Manual tracing	Digital tracing
Optimal intensity r on May 10th	0.14	0.14	0.12
Implied daily cost [daily GDP]	19 %	19 %	8.7%
Time until cost drops to 1% of daily GDP	never	3.4 months	39 days
Daily cost in the limit	16%	0.1%	0
Time to reach extinction threshold 1 ppm	7.8 months	15 months	3.0 months
Total cost until extinction [annual GDP]	11 %	1.7%	0.44%
Total additional death until extinction	2,900	4,200	3,300

Table 3: Comparison Scenarios

The optimal reduction in social distancing intensity is such that r goes from 0.17 under the lockdown to 0.14 (no tracing/manual tracing) and 0.12 (digital tracing). Some degree of easing is optimal. The reason is that identifying a fraction of the contagious takes over some of the burdens to keep viral growth at an optimal level. This modest reduction in social distancing already has an essential impact on economic cost. It reduces from 50% of daily GDP under the lockdown to 19 % under no tracing/manual tracing and 8.7% under digital tracing. Said differently, on May 10th, it is possible to ease the lockdown by around half, measured in lost daily GDP. An immediate switch to the Korean strategy would allow for an easing of a factor of almost 6.

The cost of social distancing drops over time because the number of infectious reduces, and, in the optimum, social distancing is gradually relaxed. Under digital tracing, the cost drops below 1% of daily GDP after 39 days already. Under manual tracing, it takes 3.4 months to reach this point. However, under no tracing, this point is never reached. The optimal intensity is almost constant and stays close to 0.14. A cost close to 19% of daily GDP needs to be paid until the virus disappears. In the long run, the daily cost reduces to 0.1% for manual tracing and 0% for digital tracing. The numerical results confirm the theoretical results. Only efficient tracing with $\xi_0 > r^0$ allows the society to go back to a normal activity level. If tracing is inefficient, i.e., $\xi_0 << r^0$ relatively strong and costly social distancing measures need to stay in place until the virus disappears. Mild efficiency implies that measures have to stay in place in the long run; however, they are mildly intense and not very costly. For instance, this may correspond to the case where society only imposes restrictions on mass events and general hygiene measures such as mask-wearing.

Next, to compare the different strategies' total cost, following Piguillem and Shi (2020), I assume the virus disappears when prevalence falls below an extinction threshold of 1 infectious per million inhabitants. Piguillem and Shi (2020) use a threshold of 10 per million. I use a more conservative threshold because South Korea already reached a prevalence of 6 per million, and the virus did not get extinct.

The differences in cost between the different strategies are enormous. No tracing takes 7.8 months and costs 11% of annual GDP. Note that this cost is in addition to the already incurred cost due to the strict lockdown. Also, under this policy, the virus causes around 2,900 additional victims. The suppression with the help of manual tracing takes 15 months. Compared to no tracing, manual tracing is slower but much less costly. The total cost is only 1.7% of annual GDP. The reason is that social distancing is gradually relaxed in the optimum. It reaches a limit where its cost is only 0.1% of daily GDP. Because the medium-term flow cost is low, it is optimal to apply weaker measures for a longer time. The total number of additional casualties in the manual tracing scenario is around 4,200. Note that it is larger than in the no tracing scenario. The reason is that the health cost has only a very small influence on the optimal policy. It is optimal to apply weaker measures for a longer time. Additionally, more control is exerted with the help of tracing. Contrary to social distancing, tracing does not directly reduce health costs because individuals are traced after infection. Both factors contribute to a higher number of casualties. The total cost in the manual tracing scenario is still substantial. The most economical option is digital tracing. The virus disappears after three months. Social distancing is relaxed quickly and substantially, well before that date. The total cost is only 0.44 % of annual GDP. The number of additional casualties is around 3,300. For all strategies, after extinction, social distancing is completely relaxed. In case there is an import of new cases, the pandemic restarts under no tracing and manual tracing, but not under digital tracing. Digital tracing alone can keep small new outbreaks under control. I do not count the cost to avoid or control new outbreaks when digital tracing is not used. In case a country decides to suppress the virus using no tracing or optimal manual tracing, the policy needs to be complemented by meticulous border controls until a vaccine arrives. I leave the quantification of these additional costs for future research.

Note that the total cost of suppression with digital tracing is by one order of magnitude smaller than estimates for the total cost under optimal mitigation strategies. Acemoglu et al. (2020) and Gollier (2020) evaluate mitigation strategies with age-depended social distancing measures. Age-depended policies give the most optimistic estimates for total costs and casualties of mitigation policies. They find a total cost of mitigation in the range of 7 to 13 % of annual GDP. The death toll of optimal mitigation strategies in the most optimistic scenarios is 0.2% of the population. In Italy's case, these are around 120,000 casualties, which stand in stark contrast to the 3,300 additional casualties of an optimal suppression policy.

4.3.1 The Evolution of the Number of Susceptible

I find that the change in the mass of susceptible ΔS is small under all three scenarios:

No tracing	Manual tracing	Digital tracing
0.49%	0.69%	0.48%

Table 4: Change in the Number of Susceptible ΔS

The initial mass of susceptible is $S_0 = 96\%$, and therefore, it is indeed very close to constant.

5 Conclusion

This paper characterizes the optimal policy to suppress COVID-19. I find that the elimination is possible at a reasonable economic cost of 0.4% of annual GDP. A simple function of observables, the optimal policy is easily implementable. However, some crucial questions are still uncertain. In particular, in the case of COVID-19, is it more efficient to use mitigation or suppression?

Remember, mitigation controls the spread of the virus until contagions stop because the population achieves herd immunity. If the current number of infectious individuals is sufficiently low and case detection is efficient enough, the answer to this question is undoubtedly suppression. The same is true if the value of lives lost is large enough. However, for all other cases, it becomes much harder to make an optimal decision. Moreover, a decision needs to be made. The two strategies dictate a very different optimal time path of infections. A mitigation strategy lets infections grow because the virus needs to reach a large enough part of the population. Optimal suppression never allows infections to grow. The policymaker stands at a crossroads and needs to decide which path to take. The total cost of either of them is still very uncertain. It depends crucially on: the cost and viral growth impact of social distancing policies, the speed of tracing, especially at low infection levels, the statistical value of life, and the capacity of the health care system and its impact on mortality rates. All of these variables are highly uncertain. Only the precise estimates of the mentioned unknowns can give a definite answer to the question. However, the calibration exercise in this paper can give rough guidance on how to answer the question. It suggests that suppression is the most cost-efficient strategy. It is certainly the strategy with the lower number of casualties.

Curiously, it is easier to find the exact optimal amount of social distancing at each time point when following a suppression strategy than to decide which strategy to take. The policymaker can turn to the econometrician and the epidemiologist - they can estimate the local impact of a policy change on the flow cost and the viral growth rate - and apply the condition derived in this paper.

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A Appendix

A.1 The Model

A.1.1 The Detection Technology

$\tilde{X}(I,z)$ denotes the flow of detected cases. It is a production function with two inputs: the number of undetected cases I and the amount of resources allocated to case detection z. I assume $\tilde{X}(I,z)$ fulfills the following standard properties of a production function: $\tilde{X}(0,z) = 0$, $\tilde{X}(I,0) = 0$, $\frac{\partial \tilde{X}(I,z)}{\partial I} > 0$, $\frac{\partial \tilde{X}(I,z)}{\partial z} > 0$, $\frac{\partial^2 \tilde{X}(I,z)}{\partial I^2} < 0$, $\frac{\partial^2 \tilde{X}(I,z)}{\partial z^2} < 0$, $\lim_{I\to\infty} \frac{\partial \tilde{X}(I,z)}{\partial I} = 0$. In particular, the concavity in I comes from the fact that, if resources are fixed, the resources per undetected case I decrease as I increases. An additional detection is carried out with fewer resources. Therefore, it is slower, and $\frac{\partial^2 \tilde{X}(I,z)}{\partial I^2} < 0$.

Assume the overall resources a country can allocate to case detection are fixed and constant: $z = \hat{z}$. Define $X(I) = \tilde{X}(I,\hat{z})$. It follows that $X(0) = 0, X'(I) > 0, X''(I) < 0$ and $\lim_{I\to\infty} X'(I) = 0$.

Lemma 3. *If $X''(I) < 0$ and $X'(I) > 0$ for all I, it follows that $\frac{X(I)}{I}$ is decreasing for all I.*

PROOF:
If $X''(.) < 0$ it follows that

$$\int_0^I (X'(\tilde{I}) - X'(I))d\tilde{I} > 0, \tag{35}$$

because $X'(.)$ is a decreasing and positive function. It follows that $X(I) - X'(I)I > 0$, which implies

$$\frac{d\,\frac{X(I)}{I}}{d\,I} = \frac{X'(I)I - X(I)}{I^2} < 0. \tag{36}$$

qed.

A.1.2 The Derivation of Equation (7) and (12) from the SIR Model

The mass of not quarantined infectious is I_t. Denote by J_t the overall mass of infected and by Q_t the mass of quarantined. It follows that $J_t = Q_t + I_t$ and $\dot{J}_t = \dot{Q}_t + \dot{I}_t$. The flow of new infections follows

$$\dot{J}_t = (\beta_0 S - \beta(p_t)S)I_t - \gamma J_t, \tag{37}$$
$$\dot{Q}_t = X(I_t) - \gamma Q_t, \tag{38}$$

where $\beta(p_t)$ is the reduction in viral transmission due to application of policy $p_t \in [0, 1]$. Denote $\beta_0 S - \gamma = r^0$ and $\beta(p_t)S = r(p_t)$ to get

$$\dot{I}_t = (r^0 - r(p_t))I_t - X(I_t),$$

which gives Equation (7) if $p_t = 0$, and Equation (12) if detection is not used (i.e., $X(I) = 0$ for all I).

A.1.3 Proof of Lemma 1

$\frac{X(I)}{I}$ is strictly decreasing from ξ_0 to 0. It follows that there exits an I^* such that $\frac{X(I)}{I} = r^0$. For all $I < I^*$, $r^0 - \frac{X(I)}{I} < 0$. Therefore, $\dot{I} = \left(r^0 - \frac{X(I)}{I}\right) I < 0$.
qed.

A.1.4 The Flow of New Infections

By Equation (37), the flow of new infections is $\beta_0 S - \beta(p_t)S$. Since $\beta_0 S - \gamma = r^0$ and $\beta(p_t)S = r(p_t)$, it is equal to $r^0 - r(p_t) + \gamma$.

A.2 Proofs Section 3

A.2.1 Proof Lemma 2

Change the policy variable from p_t to r_t and use the Hamiltonian to derive necessary conditions for the solution of Problem (14):

$$c'(r_t) - vI_t = \lambda_t I_t, \tag{39}$$
$$\dot{\lambda}_t = (X'(I_t) + r_t - r^0)\lambda_t - v(r^0 - r + \gamma), \tag{40}$$
$$\dot{I}_t = (r^0 - r_t)I_t - X(I_t), \tag{41}$$
$$\lim_{t\to\infty} \lambda_t = 0. \tag{42}$$

It follows that

$$\dot{r}_t = \frac{\left(X(I_t)/I_t - X'(I_t)\right) c'(r_t) - v\left(\gamma + X'(I_t)I_t\right)}{c''(r_t)}, \tag{43}$$

$$\dot{I}_t = (r^0 - r_t)I_t - X(I_t). \tag{44}$$

Note that $\dot{r}_t < 0$ for all t, since $X(I_t)/I_t - X'(I_t) > 0$.

Lemma 4. *If on the optimal path there exists a time* t_1*, such that* $\dot{I}_{t_1} \geq 0$*, then* $\dot{I}_t > 0$ *for all* $t > t_1$.

Proof by contradiction:
Suppose there exists a time $t_2 > t_1$ such that $\dot{I}_{t_2} \leq 0$. Take the smallest such t_2. It follows that for all t in (t_1, t_2), $\dot{I}_t > 0$. Therefore, $I_{t_1} < I_{t_2}$. Since $\dot{r}_t < 0$ and $\frac{X(I)}{I}$ is decreasing, it follows that

$$\frac{\dot{I}_{t_2}}{I_{t_2}} = r^0 - r_{t_2} - \frac{X(I_{t_2})}{I_{t_2}} > r^0 - r_{t_1} - \frac{X(I_{t_1})}{I_{t_1}} = \frac{\dot{I}_{t_1}}{I_{t_1}} \geq 0, \tag{45}$$

which gives a contradiction.
qed.

Proof of Lemma 2, by contradiction:
Suppose on the optimal path there exits a time t_1, such that $\dot{I}_{t_1} \geq 0$. By Lemma 4, $\dot{I}_t > 0$ for all $t > t_1$. Note that $r_t \leq r^0 + \gamma$, since it is not possible to reduce social contacts below zero. The total cost of this policy is

$$C = \int_0^\infty c(r_t) + (r^0 - r_t + \gamma)I_t v \, dt > \int_{t_1}^\infty (r^0 - r_{t_1} + \gamma)I_{t_1} v \, dt = \infty. \tag{46}$$

There exist policies, for example $r_t = r^0 + \gamma/2 - X(I_t)/I_t$, for which the total cost is finite. Therefore, the above policy cannot be optimal.

Suppose there exists an optimal policy such that $I_t > I^\epsilon$ for all t. It follows that the total cost is

$$C = \int_0^\infty c(r_t) + (r^0 - r_t + \gamma)I_t v \, dt > \int_0^\infty (r^0 - r_0 + \gamma)I_\epsilon v \, dt = \infty. \tag{47}$$

By the same argument as above, the policy cannot be optimal.

A.2.2 Proof Proposition 1

The minimum of the integral

$$\min_{p(.)} C(p(.)) = \int_{I^{\epsilon}}^{I_0} -\frac{c(p(I)) + vI(r^0 - r(p(I)) + \gamma)}{(r^0 - r(p(I)))I - X(I)} dI \tag{48}$$

is at the point-wise minimum of each integrand. Note that I swapped the bounds. Change policy variable from p to r. For each I, the integrand is equal to

$$\frac{c(r) + vI(r^0 - r + \gamma)}{\left(r - r^0 + \frac{X(I)}{I}\right) I}. \tag{49}$$

Note that $\dot{I} < 0$. Therefore, the denominator has to be positive, which is the case when $r > r^0 - \frac{X(I)}{I}$. Also, $r \geq 0$ by definition. Note that $r \leq r^0 + \gamma$. At this bound social contacts are zero, and therefore, $c(r^0 + \gamma) = \infty$. There are two cases:
First, if $r^0 - \frac{X(I)}{I} > 0$ it holds that $r \in \left(r^0 - \frac{X(I)}{I}, r^0 + \gamma\right]$.
Second, if if $r^0 - \frac{X(I)}{I} \leq 0$ it holds that $r \in [0, r^0 + \gamma]$.

Note that the integrand is finite, positive, and continuous for any interior r.

Lemma 5. *There exists a minimum of the integrand, and it is interior.*

PROOF:

Case 1, $r^0 - \frac{X(I)}{I} > 0$:
It follows that $r \in \left(r^0 - \frac{X(I)}{I}, r^0 + \gamma\right]$. If r goes to the left limit, the integrand goes to infinity. If r goes to the right limit, the integrand goes to infinity as well. The integrand is finite, positive, and continuous for any interior r. It follows that there exists an interior minimum.

Case 2, $r^0 - \frac{X(I)}{I} < 0$:
It follows that $r \in [0, r^0 + \gamma]$. At the left boundary, the integrand is equal to $\frac{v(r^0+\gamma)}{\frac{X(I)}{I} - r^0}$. The

minimum cannot be at zero, because the integrand is strictly decreasing in zero:

$$\frac{(c'(0)-vI)\left(0-r^0+\frac{X(I)}{I}\right)I-(c(0)+vI(r^0-0+\gamma))\,I}{\left(0-r^0+\frac{X(I)}{I}\right)^2 I^2}=\frac{-v\left(\frac{X(I)}{I}+\gamma\right)}{\left(-r^0+\frac{X(I)}{I}\right)^2}<0. \tag{50}$$

If r goes to the right limit, the integrand goes to infinity. The integrand is finite, positive, and continuous for all r. It follows that there exists an interior minimum.

Case 3, $r^0-\frac{X(I)}{I}=0$:
It follows that $r\in[0,r^0+\gamma]$. As case 1. The integrand goes to infinity at both boundaries.

qed.

Any interior extremum fulfills the first order condition:

$$\frac{c(r)+vI(r^0-r+\gamma)}{\left(r-r^0+\frac{X(I)}{I}\right)I}\left(\frac{c'(r)-vI}{c(r)+vI(r^0-r+\gamma)}-\frac{1}{r+\frac{X(I)}{I}-r^0}\right)=0 \tag{51}$$

Cancel the left factor and change the choice variable back from r to p to get the optimality condition in Proposition 1.

Each interior extremum is a strict minimum. To see that, rearrange the first derivative of the integrand to:

$$\frac{1}{\left(r-r^0+\frac{X(I)}{I}\right)^2 I}\left(-c(r)-vI(r^0-r+\gamma)+(c'(r)-vI)\left(r+\frac{X(I)}{I}-r^0\right)\right) \tag{52}$$

Take the derivative to get the second order condition. Note that it is equal to $a'b+ab'$ where a is the first factor above and b is the second factor. If the FOC holds, b is zero. Also, a is always positive. The sign of the SOC only depends on the sign of b'. $b'=c''(r)\left(r+\frac{X(I)}{I}-r^0\right)$, which is strictly greater than zero.

Since each minimum is a strict minimum, and the function is continuous, there can only be one minimum. In particular, it fulfills the first-order condition.

qed.

A.2.3 Proof Proposition 2

Lemma 6. .
Consider the case where $\lim_{I\to 0}\frac{X(I)}{I}\geq r^0$. *It follows that:*
1) The optimal policy $r(I)$ *converges to zero as* I *converges to zero:*

$$\lim_{I\to 0} r(I) = 0. \tag{53}$$

2) For small I *the optimal policy* $r(I)$ *is approximately equal to*

$$r(I) \approx -\left(\frac{X(I)}{I} - r^0\right) + \sqrt{\left(\frac{X(I)}{I} - r^0\right)^2 + 2\frac{v}{c''(0)} I \left(\frac{X(I)}{I} + \gamma\right)}. \tag{54}$$

In particular, $r(I) > 0$ *for* $I > 0$.
3) For small I *the growth rate under the optimal policy* $g(I)$ *is approximately equal to*

$$g(I) \approx -\sqrt{\left(r^0 - \frac{X(I)}{I}\right)^2 + 2\frac{v}{c''(0)} I \left(\frac{X(I)}{I} + \gamma\right)}. \tag{55}$$

4) Under the optimal policy $r(I)$ *the growth rate converges to*

$$\lim_{I\to 0} g(I) = -(x^0 - r^0). \tag{56}$$

In particular, for

$$\lim_{I\to 0}\frac{X(I)}{I} = \infty, \text{ it holds that } \lim_{I\to 0} g(I) = -\infty. \tag{57}$$

The decay of the virus is accelerating as I *approaches zero.*

PROOF:
The optimality condition with r as the policy variable writes

$$\frac{c'(r) - vI}{c(r) + vI(r^0 - r + \gamma)} = \frac{1}{r + \frac{X(I)}{I} - r^0}. \tag{58}$$

Taylor approximate the function $c(r)$ in the origin:

$$c(r) \approx \frac{1}{2}c''(0)r^2. \tag{59}$$

Use the approximation in the optimality condition to solve for Equation (54), which proofs point 2). Note that $r(I)$ is the solution of a quadratic equation. The second solution can be discarded as it violates $\dot{I} < 0$. Point 1) follows from taking the limit in Equation (54). Point 3) follows from using the definition of the growth rate. Point 4) follows from taking the limit in Equation (55).
qed.

Lemma 7. .
Consider the case where $\lim_{I\to 0}\frac{X(I)}{I} = \xi_0 < r^0$. *Assume that the cost function is quadratic:* $c(r) = \frac{1}{2}c''(0)r^2$ *It follows that:*
1) As I *converges to zero, the optimal policy* $r(I)$ *converges to:*

$$\lim_{I\to 0} r(I) = 2(r^0 - \xi_0). \tag{60}$$

In particular, if there is no test and trace $\xi_0 = 0$, *and*

$$\lim_{I\to 0} r(I) = 2r^0. \tag{61}$$

3) The optimal policy $r(I)$ *is equal to*

$$r(I) = r^0 - \frac{X(I)}{I} + \sqrt{\left(r^0 - \frac{X(I)}{I}\right)^2 + 2\frac{v}{c''(0)}I\left(\frac{X(I)}{I} + \gamma\right)}. \tag{62}$$

3) The implied optimal growth rate $g(I)$ *is equal to*

$$g(I) = -\sqrt{\left(r^0 - \frac{X(I)}{I}\right)^2 + 2\frac{v}{c''(0)}I\left(\frac{X(I)}{I} + \gamma\right)}. \tag{63}$$

In particular $r(I) > 0$ *for all* I.
4) Under the optimal policy $r(I)$ *the growth rate converges to*

$$\lim_{I\to 0} g(I) = -(r^0 - x^0). \tag{64}$$

PROOF:
As above. However, the cost function is quadratic by assumption and not by approximation.
qed.

Lemma 8. .
The optimal policy $r(I)$ is strictly increasing in I:

$$r'(I) > 0. \tag{65}$$

PROOF:
The optimal policy solves

$$\frac{c'(r(I)) - vI}{c(r(I)) + vI(r^0 - r(I) + \gamma)} = \frac{1}{r(I) + \frac{X(I)}{I} - r^0}. \tag{66}$$

Differentiate with respect to I to obtain

$$r'(I) = \frac{v\left(\gamma + \frac{X(I)}{I}\right) - (c'(r(I)) - vI)\frac{d\frac{X(I)}{I}}{dI}}{-g(r(I), I)c''(r(I))}. \tag{67}$$

The expression is positive because $v > 0$, $\frac{d\frac{X(I)}{I}}{dI} < 0$, $g(r(I), I) < 0$, $c''(.) > 0$, and $c'(r(I)) - vI > 0$. The last inequality follows from the FOC.
qed.

Point 1 of Proposition 2 follows directly from Proposition 1 and Lemma 8. Point 2 of Proposition 2 follows from point 1 in Lemma 6 and 7. Point 3 of Proposition 2 follows directly from Proposition 1 and from point 4 in Lemma 6 and 7.
qed.

A.2.4 Proof Proposition 3

Lemma 9. .
If $\xi_0 = \infty$, the total unit cost of suppression goes to zero as the mass of infectious goes to zero.

PROOF:

$\frac{dC}{dI}$ is the unit cost in the optimum. It is smaller or equal to the unit cost under any other policy that satisfies $\dot{I}(I) < 0$. In particular, take the policy $\tilde{r}(I) = 0$ for all $I < I^*/2$. It follows that

$$0 \leq \frac{c(r(I))}{\left(r(I) + \frac{X(I)}{I} - r^0\right) I} + \frac{v(r^0 - r(I) + \gamma)}{\left(r(I) + \frac{X(I)}{I} - r^0\right)} \leq \frac{v(r^0 + \gamma)}{\frac{X(I)}{I} - r^0}. \tag{68}$$

Take the limit on both sides to obtain the result.

qed.

Lemma 10. .

If $\xi_0 > r^0$, the economic unit cost of suppression goes zero, and the health unit cost from the flow of death goes to a constant, as the mass of infectious goes to zero.

PROOF:

Use the same argument as above. In the limit

$$0 \leq \lim_{I \to 0} \frac{c(r(I))}{\left(r(I) + \frac{X(I)}{I} - r^0\right) I} + \frac{v(r^0 + \gamma)}{\xi_0 - r^0} \leq \frac{v(r^0 + \gamma)}{\xi_0 - r^0}, \tag{69}$$

which proves the result.

qed.

Lemma 11. .

If $\xi_0 > r^0$, the total cost of suppression is bounded in the optimum.

PROOF:

The total cost of suppression at the optimum is smaller or equal to the total cost of suppression under any other policy that satisfies $\dot{I}(I) < 0$. In particular, take the policy $\tilde{r}(I) = 0$ for $I \leq I^*/2$ and $\tilde{r}(I) = r^0$ for $I > I^*/2$. It follows that

$$\int_0^{I_0} \frac{c(r(I)) + vI(r^0 - r(I) + \gamma)}{\left(r(I) + \frac{X(I)}{I} - r^0\right) I} dI \leq \int_0^{I^*/2} \frac{v(r^0 + \gamma)}{\frac{X(I)}{I} - r^0} dI + \int_{I^*/2}^{I_0} \frac{c(r^0) + vI\gamma}{X(I)} dI \tag{70}$$

Both integrals exist, which gives the result.

qed.

Lemma 12. .
If $\xi_0 < r^0$, and the cost function is quadratic, the economic unit cost of suppression goes to infinity and the health unit cost goes to a constant as the mass of infectious goes to zero.

PROOF:
Take the definition of the total unit cost and take the limit. Use the results from Lemma 7. Assume that $2(r^0 - \zeta_0) << r^0 + \gamma$. Intuitively, it means the system is far enough from the right limit $r^0 + \gamma$. Note that r close to the limit are at odds with the assumption of a quadratic cost:

$$\lim_{I \to 0} \frac{c(r(I))}{\left(r(I) + \frac{X(I)}{I} - r^0\right) I} + \lim_{I \to 0} \frac{v(r^0 - r(I) + \gamma)}{\left(r(I) + \frac{X(I)}{I} - r^0\right)} = \frac{c(2(r^0 - \xi_0))}{r^0 - \xi_0} \lim_{I \to 0} \frac{1}{I} + \frac{v(r^0 + \gamma - 2(r^0 - \xi_0))}{r^0 - \xi_0}. \tag{71}$$

qed.

Lemma 13. .
If $\xi_0 < r^0$, and the cost function is quadratic, the total cost of suppression goes to infinity if the extinction threshold I^ϵ goes to zero, even in the optimum.

PROOF:
Take the expression for the total cost and use the optimality condition to get

$$C = \int_{I^\epsilon}^{I_0} \frac{c(r(I)) + vI(r^0 - r(I) + \gamma)}{\left(r(I) + \frac{X(I)}{I} - r^0\right) I} dI = \int_{I^\epsilon}^{I_0} \frac{c'(r(I)) - vI}{I} dI \tag{72}$$

The optimal policy is increasing and larger than zero in zero; therefore

$$C \geq \int_{I^\epsilon}^{I_0} \frac{c'(r(0)) - vI}{I} dI \tag{73}$$

Take the limit for I^ϵ going to zero to get the result.
qed.

Lemma 9 to 11 prove the statements on case 1 in Proposition 3. Lemma 12 and 13 prove the statements on case 2.
qed.

A.3 Extensions

A.3.1 Discounting, Vaccine, and Cure

Suppose the planner discounts the future at a positive time-discount rate. Additionally, a vaccine or cure that immediately end the pandemic arrive stochastically at a positive and constant Poisson-rate. Together, the two phenomena give rise to a total discount factor of i. The planner's problem is:

$$\min_{r_t} C(r_t) = \int_0^\infty e^{-it} \left(c(r_t) + vI_t(r^0 - r_t + \gamma)\right) dt, \tag{74}$$

such that,

$$\dot{I}_t = (r^0 - r_t)I_t - X(I_t). \tag{75}$$

Only consider solutions that converge to I^ϵ and $\dot{I}_t < 0$ for all t. Change variable to I. Note that

$$t(I) = \int_{I_0}^{I} \frac{1}{\dot{I}(I)} dI. \tag{76}$$

It follows that

$$\min_{r(I)} C(r(I)) = \int_{I_0}^{I^\epsilon} e^{-i\int_{I_0}^{I} \frac{1}{\dot{I}(\tilde{I})} d\tilde{I}} \left(\frac{c(r(I)) + vI(r^0 - r(I) + \gamma)}{\dot{I}(I)}\right) dI. \tag{77}$$

The solution to this problem is a control function $r(I)$.

Proposition 4. .

1. *For every I, the solution $r(I)$ of Problem (77) fulfills*

$$\frac{c'(r(I)) - vI}{c(r(I)) + vI(r^0 - r(I) + \gamma)} + \\ + \frac{i}{(c(r(I)) + vI(r^0 - r(I) + \gamma)(-g(r(I), I))} C(I) = \frac{1}{-g(r(I), I)}, \tag{78}$$

where $C(I)$ denotes the value function and $g(r(I), I)$ the growth rate.

2. *The optimal policy for discount rate i, denote it by $r_i(I)$, is smaller than the optimal policy for discount rate $i = 0$, denoted by $r_{i=0}(I)$ and characterized in Proposition*

1.

3. *For* i *close to zero or* I_0 *close to* I^ϵ*, the optimal policy for discount rate* i*,* $r_i(I)$*, is close to the optimal policy for discount rate* $i = 0$*,* $r_{i=0}(I)$*. In the limit when* $i = 0$ *or* $I = I^\epsilon$ *the policies are equal.*

4. *For* i *close to zero or* I_0 *close to* I^ϵ *the solution* $r(I)$ *of Problem (77) exists.*

5. *There exist* i*,* $X(.)$*, and* I_0*, such that Problem (74) has local minima where prevalence converges to a constant steady state level* I_{ss}*:* $\lim_{t\to\infty} I_t = I_{ss} \neq I^\epsilon$*.*

6. *For* i *close to zero or* I_0 *close to* I^ϵ*, the solution* $r(I)$ *of Problem (77), converging to* I^ϵ*, is a global minimum of Problem (74).*

7. *For* i *close to zero it holds that optimal social distancing is increasing in prevalence:* $r'(I) > 0$ *for all* $I \in [I^\epsilon, I_0]$*.*

The proposition shows that for small enough discount rates the qualitative results in Proposition 1 and 2 do not change. This is the relevant case. Typically, a pandemic moves fast such that the relevant time discount rate is the daily or weekly rate. This rate is very low. Similarly, the daily or weekly probability for an effective cure or mass vaccine is typically very low. Point 2 shows that quantitatively, social distancing is less intense under discounting. This result is intuitive. Under discounting, costs can be reduced to some extend by deferring them into the future. In order to do that, suppression needs to progress slower, which is why r is smaller. The effect is driven by the second summand on the left in Condition (78).

Interestingly, as Point 5 shows, the suppression solution may not be the only local minimum of the problem. For some I_0, $X(I)$, and i, it does not even exist. One or several other local minima exist where prevalence converges to some steady-state value I_{ss}. In this case, the question of which local minimum is the global minimum is a quantitative question. In some cases, the global minimum is a path that converges to a steady-state level of prevalence. This second solution is the mitigation solution discussed in the conclusion. However, a steady-state level of prevalence is at odds with the initial assumption that the number of susceptible is approximately constant. As a consequence, mitigation needs to be studied in the full SIR model, which has already been done in the literature. Therefore, it is beyond the scope of this paper.

PROOF of Proposition 4:

Point 1:

Take the derivative of (77) with respect to each $r(I)$:

$$\left(\left(c'(r(I)) - vI\right)\frac{1}{\dot{I}(I)} + \left(c(r(I)) + vI(r^0 - r(I) + \gamma)\right)\frac{1}{\dot{I}(I)^2}I\right)e^{-i\int_{I_0}^{I}\frac{1}{\dot{I}(\tilde{I})}d\tilde{I}} + \\ + \int_{I}^{I^\epsilon} e^{-i\int_{I_0}^{\tilde{I}}\frac{1}{\dot{I}(\tilde{\tilde{I}})}d\tilde{\tilde{I}}}\left(-\frac{1}{\dot{I}(I)^2}Ii\right)\left(\frac{c(r(\tilde{I})) + v\tilde{I}(r^0 - r(\tilde{I}) + \gamma)}{\dot{I}(\tilde{I})}\right)d\tilde{I} = 0. \quad (79)$$

The FOC simplifies to

$$\left(c'(r(I)) - vI\right)\frac{1}{-\dot{I}(I)} - \left(c(r(I)) + vI(r^0 - r(I) + \gamma)\right)\frac{I}{\dot{I}(I)^2} + \frac{iI}{\dot{I}(I)^2}C(I) = 0, \quad (80)$$

where $C(I)$ is the value function.

To show that it is a local minimum consider the SOC. Rearrange the FOC to

$$\frac{1}{\left(r - r^0 + \frac{X(I)}{I}\right)^2 I}\left(-c(r) - vI(r^0 - r + \gamma) + iC(I) + (c'(r) - vI)\left(r + \frac{X(I)}{I} - r^0\right)\right) \quad (81)$$

Take the derivative to get the second order condition. Note that it is equal to $a'b + ab'$ where a is the first factor above and b is the second factor. If the FOC holds, b is zero. Also, a is positive. The sign of the SOC only depends on the sign of b'. $b' = c''(r)\left(r + \frac{X(I)}{I} - r^0\right)$, because $\frac{dC(I)}{dr(I)} = 0$. $b' > 0$, therefore, the SOC>0 and the FOC characterizes a local minimum.

Rewrite the FOC to get

$$\frac{c'(r(I)) - vI}{c(r(I)) + vI(r^0 - r(I) + \gamma)} + \frac{i}{(c(r(I)) + vI(r^0 - r(I) + \gamma)(-g(r(I), I))}C(I) = \frac{1}{-g(r(I), I)},$$

which is the condition in the proposition.

Point 2:

Rewrite the FOC to

$$(c'(r) - vI)\left(r + \frac{X(I)}{I} - r^0\right) - (c(r(I)) + vI(r^0 - r + \gamma)) = -iC(I). \tag{82}$$

The right hand side is negative. For $r_{i=0}(I)$ the left hand side is equal to zero. The left hand side is increasing in r. Therefore, $r_i(I) < r_{i=0}(I)$.

Point 3:

$C(I)$ is bounded. If i goes to zero, $iC(I)$ goes to zero and Condition (78) goes to Condition (19). Therefore, $r_i(I)$ goes to $r_{i=0}(I)$. Similarly, when I goes to I^ϵ, $C(I)$ goes to zero. Therefore, $iC(I)$ goes to zero and the same argument applies.

Point 4:

The solution exists if $\dot{I} < 0$ for all $I \in [I^\epsilon, I_0]$. When I_0 is close to I^ϵ or i is close to 0, then r_i is close to $r_{i=0}$. By Proposition 1, the zero discount solution exists. Therefore $\dot{I}(r_{i=0}(I)) < 0)$, which by continuity is also true for all r close to $r_{i=0}$.

Point 5:

Use the Hamiltonian to solve Problem (74):

$$c'(r_t) - vI_t = \lambda_t I_t \tag{83}$$

$$\dot{\lambda}_t = (i + X'(I_t) + r_t - r^0)\lambda_t - v(r^0 - r + \gamma) \tag{84}$$

$$\dot{I}_t = (r^0 - r_t)I_t - X(I_t) \tag{85}$$

$$\lim_{t\to\infty} e^{-it}\lambda_t = 0 \tag{86}$$

It follows that

$$\dot{r}_t = \frac{(i + X'(I_t) - X(I_t)/I_t)\,(c'(r_t) - vI_t) - vI_t\left(\gamma + \frac{X(I)}{I}\right)}{c''(r_t)} \tag{87}$$

$$\dot{I}_t = (r^0 - r_t)I_t - X(I_t) \tag{88}$$

The two equations give two loci. $\dot{I}$ is zero if

$$r = r^0 - X(I)/I. \tag{89}$$

$\dot{r}$ is zero if

$$c'(r) = \frac{vI(\gamma + i + X'(I))}{i - (X(I_t)/I_t - X'(I_t))}. \tag{90}$$

One or several steady states with positive I may exist, dependent on if the above system has a solution, i.e., the two loci cross at least once. Choose i and $X(.)$ such that the two loci cross at least once and I_0 equal to the corresponding steady-state prevalence I_{ss}.

Point 6:

Choose i small enough such that for all $I \in [I^\epsilon, I_0]$ it holds that $i - (X(I_t)/I_t - X'(I_t)) < 0$. Note that such an $i > 0$ exists because $(X(I)/I - X'(I) > 0$ for all $I > 0$. It follows that $\dot{r}_t < 0$ for all t. The only path fulfilling the boundary condition of the Hamiltonian converges to I^ϵ.

If a saddle path from I^ϵ to the steady state I_{ss} exists, denote by $C_{ss}(I^\epsilon)$ the value function to reach it. It is larger than zero. Denote by $C_\epsilon(I_0)$ the value function of reaching I^ϵ from I_0. Point 4 shows that it exists. $C_\epsilon(I_0)$ goes to zero if I_0 goes to I^ϵ. Choose I_0 low enough such that $C_{ss}(I^\epsilon) > C_\epsilon(I_0)$.

Point 7:

The total cost under the optimal policy is

$$C(I) = \int_I^{I^\epsilon} e^{-i\int_I^{\tilde{I}} \frac{1}{\dot{I}(\tilde{\tilde{I}})} d\tilde{\tilde{I}}} \left(\frac{c(r(\tilde{I})) + v\tilde{I}(r^0 - r(\tilde{I}) + \gamma)}{\dot{I}(\tilde{I})} \right) d\tilde{I}. \tag{91}$$

The derivative is

$$C'(I) = \frac{c(r(I) + vI(r^0 - r(I) + \gamma)}{-\dot{I}(I)} -$$
$$- \frac{i}{-\dot{I}(I)} \int_I^{I^\epsilon} e^{-i\int_I^{\tilde{I}} \frac{1}{\dot{I}(\tilde{I})} d\tilde{I}} \left(\frac{c(r(\tilde{I})) + v\tilde{I}(r^0 - r(\tilde{I}) + \gamma)}{\dot{I}(\tilde{I})} \right) d\tilde{I}, \tag{92}$$

which can be written as

$$C'(I) = \frac{c(r(I)) + vI(r^0 - r(I) + \gamma)}{-\dot{I}(I)} - \frac{i}{-\dot{I}(I)} C(I). \tag{93}$$

Together with Condition (78) it gives

$$C'(I) = \frac{c'(r(I)) - vI}{I}. \tag{94}$$

Differentiate the Condition (78) with respect to I and use the result for $C'(I)$ to get

$$r' = \frac{v\left(\gamma + \frac{X(I)}{I}\right) + \frac{c'(r) - vI}{I}\left(\frac{X(I)}{I} - X'(I) - i\right)}{\left(r + \frac{X(I)}{I} - r^0\right) c''(r)}. \tag{95}$$

In general, it is possible that r' is negative. However, if i is small enough r' is always positive. Choose i small enough such that for all $I \in [I^\epsilon, I_0]$ it holds that $X(I)/I - X'(I) - i > 0$. Note that such an $i > 0$ exists, because $(X(I)/I - X'(I)) > 0$ for all $I > 0$. Also, note that, by Condition 83, $c'(r) - vI > 0$.
qed.

A.3.2 Endogenous Choice of Detection

The function $\tilde{X}(I, z)$ is the detection function discussed in Section A.1.1. I is the number of undetected cases and z is the amount of resources spend for detection. The planner's problem is

$$\min_{r(.), z(.)} C(r(.), z(.)) = \int_{I_0}^{0} (c(r(I)) + z(I) + vI(r^0 - r(I) + \gamma)) \frac{1}{\dot{I}(I)} dI, \tag{96}$$

where

$$\dot{I}(I) = I(r^0 - r(I) - \tilde{X}(I, z(I)). \tag{97}$$

Proposition 5. .
For each level of prevalence I, the optimal policy $\{r(I), z(I)\}$ solves

$$\frac{c'(r) - vI}{c(r) + z + vI(r^0 - r + \gamma)} = \frac{1}{r + \frac{\tilde{X}(I,z)}{I} - r^0}, \tag{98}$$

$$\frac{\frac{I}{\frac{\partial \tilde{X}(I,z)}{\partial z}}}{c(r) + z + vI(r^0 - r + \gamma)} = \frac{1}{r + \frac{\tilde{X}(I,z)}{I} - r^0}. \tag{99}$$

PROOF:
As for Proposition 1 in Section A.2.2.

A.3.3 Infections from Abroad

Suppose that after elimination, a small inflow of infections from abroad cannot be avoided. Denote the flow by ψ. The dynamic behavior of infectious changes to

$$\dot{I}_t = (r^0 - r_t)I_t - X(I_t) + \psi. \tag{100}$$

What is the minimal level of social distancing at which the policymaker can control the number of infectious at a low and constant level?

The answer to this question depends on the solution to the following equation:

$$r^0 - \frac{X(I_{lr})}{I_{lr}} + \frac{\psi}{I_{lr}} = 0, \tag{101}$$

where I_{lr} denotes the constant long-run level of infectious. The equation may have zero, one, or two solutions.

Two different cases are possible:

1) Equation (101) has one or two solutions
A necessary but not sufficient condition for this case is efficient detection, i.e., $\xi_0 > r^0$. In this case, the minimal level of social distancing is zero, and the long-run number of infectious is constant at I_{lr}. In the case of two solutions, the minimum of the two is relevant. Note that if ψ is small, it follows that I_{lr} is small because $\lim_{\psi \to 0} I_{lr} = 0$.

2) Equation (101) does not have a solution

It follows that the necessary long-run level of social distancing r_{lr} and the long-run level of infectious I_{lr} are the solution to

$$r^0 - r_{lr} - X'(I_{lr}) = 0 \tag{102}$$

$$(r^0 - r_{lr})I_{lr} - X(I_{lr}) + \psi = 0. \tag{103}$$

The level of infectious can be further characterized by

$$X'(I_{lr})I_{lr} - X(I_{lr}) + \psi = 0. \tag{104}$$

Note that, as above, I_{lr} is small because $\lim_{\psi \to 0} I_{lr} = 0$.

In both cases, the policymaker may want to impose stricter social distancing to reduce the long-run health costs from infections. However, that is not the case if there is some fixed cost of applying social distancing with $c(0) >> vI_{lr}$. Moreover, strictly speaking, when there is an inflow of infections from abroad, the mass of susceptible does not converge to a constant. However, if I_{lr} is small, $\dot{S}_{lr}$ is small, and therefore, S_{lr} is approximately constant.

A.3.4 The Problem of Using Social Distancing Alone

Assume the policymaker would like to minimize only the total economic cost of suppressing the disease. Additionally, assume the only available tool to do so is social distancing:

$$C = \min_{r(.)} \int_{I_0}^{I^\epsilon} \frac{c(r(I))}{\dot{I}(I)} dI, \tag{105}$$

$$\text{where } \dot{I} = (r^0 - r(I))I. \tag{106}$$

Corollary 4. .

1. *When only using social distancing, the optimal cost-minimizing policy is constant over time.*

2. *The optimal policy r is equal to*

$$\frac{c'(r)}{c(r)} = \frac{1}{r - r^0}. \tag{107}$$

3. *Assume $c(r)$ is iso elastic. It follows that the optimal effect of social distancing r is equal to*

$$r = r^0 \frac{\zeta_1}{\zeta_1 - 1}, \tag{108}$$

where $\zeta_1 > 1$ is the cost-elasticity.

4. *The optimal economic unit cost of reducing an infection goes to infinity as I goes to zero.*

Points one to three follow as special cases from Proposition 1. Point 4 follows as a special case from Proposition 3. In particular, point 4 shows the problem of using social distancing alone to suppress the disease. Even in the optimum, the cost-efficiency of social distancing measures decreases as I decreases because the reduction in infectious becomes infinitely slow when I goes to zero. This result is quite intuitive. Given a certain intensity of social distancing, it takes the same time to reduce infections from 10 million to 1 million as reducing them from 10 to 1. Suppressing the disease by social distancing is possible but very costly. One may be tempted to think that point 4 of Corollary 4 is not relevant in practice. The last unit to reduce is at $I = I^\epsilon$ and not at $I = 0$. However, I^ϵ is very small. Therefore, point 4 shows that the costs of reducing the last units close to I^ϵ are very high. Note that the discussed policy is the cost-minimizing policy in an economic sense. When maximizing social welfare, as discussed above, the result becomes even more extreme.