Tax Competition in Presence of Profit Shifting.[1]

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Abstract

The popular view is that governments should crack down on tax avoidance by multinational firms. In this paper, we analyze how anti-profit-shifting policies influence fiscal competition. Governments commit to profit shifting control effort and then set taxes on capital. Equilibrium tax rates are determined by the elasticities of the two components: profit shifting and capital mobility. Anti-profit-shifting policies decrease the elasticity of the first but increase the elasticity of the second, so that the impact of these policies on the equilibrium of the tax game is ambiguous. We show that there are cases in which laxer policies increase all equilibrium tax rates and that the country announcing laxer profit shifting policies may gain. It appears that there is not always a pure strategy equilibrium in such a fiscal competition game. We construct a mixed strategy equilibrium when the pure strategy equilibrium does not exist.

Key Words: Tax competition; Profit shifting; International taxation; Capital mobility

JEL Codes: H87; H25; H26; F38; F23

1 Introduction

Firms can reduce their tax burden in many ways, but international firms have more opportunities to do so, being able to shift profits from high-tax countries to low-tax ones.[1] Profits can be shifted from one country to another by various means: reorganizing debts, altering internal prices, or strategically assigning intangible assets like patents so as to generate lower profits in fiscally undesirable jurisdictions. While these practices allow firms to significantly lower their effective tax rates, however, they erode the ability of high-tax jurisdictions to generate tax revenue. The OECD estimate that up to 10% of the global tax base escapes taxation due to profit shifting. For that reason, the OECD advocates strongly for policies and practices that limit firms' ability to take advantage of international tax differences. Due to the complexity of tax codes and the international dimension, however, monitoring and enforcement of such activities is not straightforward, which in fact explains the prevalence of fiscal leakage. Yet, although the task is acknowledged to be complex, the feeling remains that many countries are unwilling to aggressively address profit shifting practices. In this paper, we look at the question from a different angle, investigating why some profit shifting can be beneficial even for profit-exporting countries. .[2] In this paper, we look at the question from a different angle, investigating why some profit shifting can be beneficial even for profit-exporting countries.

Haufler & Schjelderup (2000), Slemrod & Wilson (2009) and Weichenrieder & Xu (2019) take a negative view of both profit shifting and tax havens. In addition to tax base erosion, the first paper suggests that profit shifting distorts the optimal tax system, the second considers the detrimental effects of enforcement and concealment, while the last paper points out the issues raised by round-tripping investments.

[1]Even firms with subsidiaries in sub-national regions can take advantage of differences in regional tax rates. Delaware is often considered as a "tax haven" within the US

[2]Tørsløv, Wier & G. Zucman (2020) found that high-tax countries engage in limited enforcement of transfer pricing toward tax havens. They instead focus on transactions with other high-tax countries.

Many other authors, however, highlight potential benefits associated with profit shifting. Mintz & Smart (2004) and Desai, Foley & Hines (2006) point out that lower effective tax rates stimulate investments. Peralta et al. (2006), Bucovetsky & Haufler (2008) and Hong & Smart (2010) suggest that profit shifting allows governments to fiscally discriminate between international firms and local firms who are unable to shift profits. Johannesen (2010), Becker & Fuest (2012) and Stoewhase (2013) all argue that profit shifting may incite countries to set higher nominal tax rates. Our goal is to paint a broad picture of the problem and fill some important gaps in our understanding of the forces at play. Profit shifting influences the well-being of a jurisdiction through three different channels. The most obvious one, commonly referred to as the "tax base erosion" channel, consists simply in firms sending profits to be taxed abroad instead of locally. This channel is considered in all the papers mentioned above and unambiguously reduces the welfare of high-tax countries. Since shifting profits abroad enables firms to lower their effective tax rates, high-tax jurisdictions become a more desirable location choice, making it easier for high-tax countries to attract capital and firms. We refer to this second channel as the "capital allocation" channel. Many of the papers above also consider this positive effect. The last and least obvious channel is the "strategic tax setting" effect. The tax competition game is altered by profit shifting, so that equilibrium tax rates increase or decrease. Our goal is to develop a model that considers all three channels simultaneously and where all tax rates, including for a low-tax country, are determined endogenously. We thus depart from the existing literature. Becker & Fuest (2012) abstract from physical capital mobility, in Mintz & Smart (2004) tax rates are exogenous, Slemrod & Wilson (2009), Johannesen (2010) and Hong & Smart (2010) assume the presence of tax havens posting a zero tax rate. Our results are built on observable or conceptually understandable elasticities of both capital movement and tax base erosion. Contrary to Peralta et al. (2006), Johannesen (2010) and Stoewhase (2013), we account for multiple heterogeneous international firms. We abstract from the investment channel as proposed in Haufler & Schjelderup (2000), Mintz & Smart (2004) and Desai, Foley & Hines (2006), by constructing a standard tax competition model where world capital is fixed. We also ignore the merits of discrimination as proposed in Peralta, Wauthy & van Ypersele (2006), Bucovetsky & Haufler (2008) and Hong & Smart (2010), by assuming all firms are international.

We develop a two-country tax competition model with heterogenous firms who decide where to allocate their main and secondary production facilities. Firms face country-specific costs, meaning that in the absence of tax differences, firms have preferences over location decisions. We assume that one country has cost advantages over the other. In addition to location decisions, firms in the high-tax jurisdiction can shift profits to the low-tax one. Monitoring and the design of the tax system can make such activities more or less costly. Countries tax profits to maximize tax revenue. We also allow countries to derive additional benefits from having major production facilities located in their jurisdictions.[3] We solve for equilibrium tax rates and look at how enforcement of anti-profit shifting measures impacts countries' welfare. In most cases, the country with a comparative advantage in attracting capital ends up applying the higher tax rate. Our results depend on two tax bases and four elasticities. We can break down a country's overall tax base into two components: domestic profits generated by capital allocation and per-firm retained profits. When the high-tax country tightens profit shifting control, per-firm retained profit increases. At the same time, firms find it less attractive to locate in the high-tax country as the effective tax rate increases. Depending on whether tax base erosion or capital mobility is more responsive to profit shifting control, the high-tax country's welfare may increase or decrease. Equilibrium tax rates also influence the well-being of a country. In particular, the high-tax jurisdiction gains when the low-tax country increases its tax rate. Equilibrium tax rates tend to be higher when the tax base is less elastic. As profit shifting control increases, per-firm retained profits become less sensitive to the tax differential because shifting is more difficult. At the same time, capital allocation reacts more to the tax differential because effective and nominal tax rates are more similar. If capital mobility elasticity increases by more than tax base erosion elasticity

[3] Peralta, Wauthy & van Ypersele (2006) make a similar assumption and in Hong & Smart (2010) countries care about having more firms as this increases wages.

when profit shifting is made more difficult, tax rates will tend to drop. The size of the tax base also influences a country's willingness to lower its tax rate. When the low-tax country is facing a smaller tax base, it tends to be more aggressive. If the high-tax country loses tax revenue by reducing enforcement, the low-tax country will tend to raise taxes. We derive a condition based on these four elasticities that states when laxer profit shifting control is desirable.

Our paper also makes a technical contribution to the tax competition literature in general. With asymmetric jurisdictions, best-response functions may be discontinuous. Kanbur & Keen (1993) and Mongrain & Wilson (2018) show in a standard tax competition model that best-response functions can be discontinuous because of regional size differences. As the tax differential switches from positive to negative, capital flows in the opposite direction. If the regional endowment of capital is different, best-response functions can jump. Keen & Konrad (2013) point out that the same applies when immobile firms can shift profits. In all three cases, the parameters are such that these discontinuities pose no threat to a pure strategy equilibrium. Here, with both capital and profits being mobile, we can no longer guarantee the existence of a pure strategy equilibrium. In these cases, we propose a mixed strategy equilibrium where the country with a comparative advantage in attracting capital applies a deterministic tax rate, while the less attractive country randomizes between a lower and a higher tax rate. As a consequence, the "small" country may end up as the high-tax jurisdiction.

In the next section, we set out the model and characterize firms' location and profit shifting decisions for a given menu of taxes and regulatory enforcement policies. In section 3, we define governments' best fiscal policy, taking profit shifting control as given. In section 4, we investigate whether less than full effort to control profit shifting may be desirable. We then conclude. All proofs are in the appendix

2 The Model

There exist two countries labeled 1 and 2. Each country sets a source base profit tax at rate t_i. Governments also control the cost of profit shifting via monitoring and well-designed regulations. Parameter $\alpha_i \in [0, 1]$ summarizes the enforcement of profit shifting control by country i. To focus on potential benefits associated with allowing some profit shifting, we assume that monitoring is costless.

The economy is composed of a unitary measure of firms. All firms are multinational and mobile. Firms decide where to locate their main revenue-generating operation, which we refer to as the headquarters. Each firm generates A units of gross profit where the headquarters is located. The firms also operate a foreign subsidiary only for profit shifting purposes and no subsidiary generates any real economic activity.[4] Firms are indexed by a location-specific cost parameter c uniformly distributed on the support $[0, C]$. A firm of type c pays an additional un-observable cost ιc in Country 1 and $\iota(1 - c)$ in Country 2, where ι determines the importance for those un-observable cost. A small cost c suggests a comparative advantage for Country 1 in attracting a given firm. To meaningfully study profit shifting, we assume that $C \in [0.5, 1[$, meaning that , for equal taxes, there are more firms locating in Country 1 and so countries are different.[5]

Let γ_i be the share of taxable profit a firm located in country i decides to shift to the other country. The cost of shifting a proportion γ_i of a firm's total profits to the other country is $g(\alpha_i)\frac{\gamma_i^2}{2}A$. The policy parameter $\alpha_i \in [0, 1]$ is chosen by country i to make profit shifting more or less costly, so $g'(\alpha_i) > 0$.

[4]Normalizing subsidiaries profits to zero does not qualitatively influence the analysis. Making subsidiaries profit $B < A$, would allow for the possibility of shifting profit from the subsidiary firm back to the main country of operation.

[5]Baumann & Friehe (2013) show that there can be incentives to shift profits even if tax rates are equal, as long as regulatory enforcement efforts are different.

2.1 Profit Shifting

Since firms only shift profits from a high tax country to a low tax one, it is important to distinguish whether the tax rate in one country is larger or lower than the one in the other country. Denote the difference in tax rates by $\Delta = t_1 - t_2$. Imagine Δ is positive. A firm located in the high tax country must then decide how much profits to shift to the other country. Such a firm would shift profit according to

$$\gamma_1(\alpha_1, \Delta) = \arg\max_{\gamma_1} \left\{ \left[(1 - t_1) + \Delta\gamma_1 - g(\alpha_1)\frac{{\gamma_1}^2}{2} \right] A \right\} \tag{1}$$

The optimal amount of profit shifting is increasing in the tax difference Δ and decreasing with monitoring α_1. More precisely, $\gamma_1(\alpha_1, \Delta)$ given by

$$\gamma_1(\alpha_1, \Delta) = \frac{\Delta}{g(\alpha_1)}. \tag{2}$$

Firms end up paying a lower effective tax rate by shifting profits. Firms whose headquarters are in Country 1 only pay $\hat{t}_1(\alpha_1, \Delta) = t_1 - \frac{\Delta^2}{g(\alpha_1)}$ per unit of real profit generated. Since profit shifting is costly however, the net effective tax rate faced by firms in the high tax jurisdiction is $\tilde{t}_1(\alpha_1, \Delta) = t_1 - \frac{\Delta^2}{2g(\alpha_1)}$, which is consistent with the approach presented in Slemrod & Wilson (2009) and others who used similar profit shifting technologies. As we will see in the next sub-section, it is the net effective tax rate that influences firms' location decisions.

We define $\varepsilon\big(1 - \gamma_1|\Delta\big)$ as Country 1's per-firm retained profit semi-elasticity with respect tax differential, which is represented by:

$$\varepsilon(1 - \gamma_1|\Delta) = \frac{-1}{1 - \gamma_1(\cdot)} \frac{\partial[1 - \gamma_1(\cdot)]}{\partial\Delta} = \frac{\gamma_1(\cdot)}{\Delta[1 - \gamma_1(\cdot)]} = \frac{1}{g(\alpha_1) - \Delta} > 0. \tag{3}$$

The semi-elasticity is expressed in positive terms for expositional convenience. It is important to remember, however, that an increase in tax rate leads to less retained profits. Dharmapala (2014), in a survey paper, reports estimates ranging from 0.4 to 2.25, including a 0.8 "meta-regression" estimate by Heckemeyer & Overesch (2017). This estimate means that a 10 percentage point drop in nominal tax rate would increase reported income by 8 percent. The elasticity of per-firm retained profit is simply $t_1\varepsilon(1-\gamma_1|\Delta)$.

Lemma 1 *The per-firm retained profit semi-elasticity $\varepsilon(1-\gamma_1|\Delta)$ with respect to taxes differential is decreasing with Country 1's control intensity α_1.*

Stricter control increases the cost of shifting profits. As a consequence, retained profits become less sensitive to variation in tax rates. We can also define the semi-elasticity of retained profits with respect to profit shifting control as $\varepsilon(1-\gamma_1|\alpha_1)$, where

$$\varepsilon(1-\gamma_1|\alpha_1) = \frac{1}{1-\gamma_1(\cdot)}\frac{\partial[1-\gamma_1(\cdot)]}{\partial\alpha_1} = \frac{g'(\alpha_1)\gamma_1(\cdot)^2}{\Delta[1-\gamma_1(\cdot)]} = \frac{g'(\alpha_1)\Delta}{g(\alpha_1)[g(\alpha_1)-\Delta]}. \quad (4)$$

Were Country 2 to be the high-tax country, Δ would be negative and $\gamma_2(\alpha_2,-\Delta)$ would behave similarly.

2.2 Location Decisions

When firms decide where to locate, they compare net profit associated with establishing their headquarters in Country 1 versus Country 2. When $\Delta > 0$, a firm with cost c locating in Country 1 generates the following profits:

$$\Pi_1(c) = [1-\tilde{t}_1(\alpha_1,\Delta)]A - \iota c, \quad (5)$$

where $\tilde{t}_1(\alpha_1,\Delta)$ is the effective tax rate net of profit shifting costs. Note that $t_2 < \tilde{t}_1(\alpha_1,\Delta) < t_1$. Alternatively, if the same firm establishes its headquarters in Country 2, profits are given by:

$$\Pi_2(c) = (1-t_2)A - \iota(1-c) \quad (6)$$

Define $\bar{c}(\alpha_1,\Delta)$ as the cost parameter such that a firm is indifferent between locating in either country. Figure 1 shows the allocation of firms across countries for a case where $\Delta > 0$.

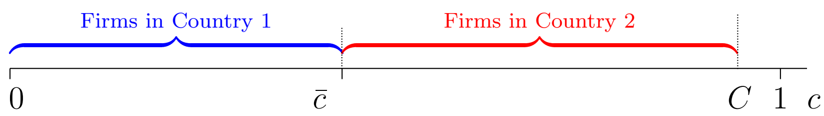

Figure 1: Allocation of Firms

Solving the indifference condition $\Pi_1(c) = \Pi_2(c)$ implies that $\bar{c}(\gamma_1, \Delta)$ is given by:

$$\bar{c}(\alpha_1, \Delta) = \frac{\iota - \left[\tilde{t}_1(\alpha_1, \Delta) - t_2\right] A}{2\iota} = \frac{\iota - \Delta \left[1 - \frac{\gamma(\alpha_1,\Delta)}{2}\right] A}{2\iota} \tag{7}$$

Firms with $c \leq \bar{c}(\alpha_1, \Delta)$ establish their headquarters in Country 1, while firms with $c > \bar{c}(\cdot)$ choose Country 2. When $\Delta = 0$, we have that $\bar{c}(\alpha_1, \Delta) = 0.5$ and more firms locate in Country 1. To ensure that $\bar{c} \in]0, C[$ whatever Δ, we assume that $A \leq \frac{2\iota}{g(1)}$. [6] As the difference between the effective tax rate, net of profit shifting costs in the high-tax country and the nominal tax rate in the low-tax country increase, more firms choose to locate in the low-tax jurisdiction.

A reduction in Δ favors Country 1, as does a reduction in controling α_1. We can define Country 1's capital tax base semi-elasticity with respect to Δ as $\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right) = \frac{-1}{\frac{\bar{c}}{C}} \frac{\partial \frac{\bar{c}}{C}}{\partial \Delta}$, where:

$$\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right) = \frac{\left[1 - \frac{\gamma_1(\alpha_1,\Delta)}{2}\right] A}{\iota - \Delta\left[1 - \frac{\gamma(\alpha_1,\Delta)}{2}\right] A}. \tag{8}$$

Country 1's capital tax base elasticity is simply $t_1 \varepsilon(\bar{c}|t_1)$. An increase in α_1 makes firms' location decision more sensitive to tax rates, as stated in Lemma 2 below. When profit shifting is more difficult, firms give more weight to tax rates differential when choosing where to locate. Estimates of capital mobility or FDI semi-elasticities with respect to tax rates vary widely. A meta-analysis by De Mooij and Ederveen (2008) proposes an extensive investment semi-elasticity of 0.65 and an intensive one of 0.4.

[6] Some firms always locate in Country 2 because of the tax differential while some firms locate in Country 1 as long as $\iota > \left[\tilde{t}_1(\alpha_1, \Delta) - t_2\right] A = \Delta\left[1 - \frac{\gamma(\cdot)}{2}\right] A$. Note that $\Delta\left[1 - \frac{\gamma(\cdot)}{2}\right]$, is maximized when $\Delta = g(\alpha_1)$. Consequently, if $A \leq \frac{2\iota}{g(1)}$, some firms are willing to locate in Country 1 for any possible tax advantage.

Lemma 2 *Capital tax base semi-elasticity with respect to tax differential* $\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)$ *is increasing with Country 1's control intensity* α_1*.*

We can also define the the semi-elasticity of firms' location decisions with respect to profit shifting control as $\varepsilon\left(\frac{\bar{c}}{C}|\alpha_1\right) = \frac{-1}{\frac{\bar{c}}{C}}\frac{\partial\frac{\bar{c}}{C}}{\partial\alpha_1}$, where

$$\varepsilon\left(\frac{\bar{c}}{C}|\alpha_1\right) = \frac{g'(\alpha_1)\frac{\gamma_1(\alpha_1,\Delta)^2}{2}A}{\iota - \Delta\left[1-\frac{\gamma(\alpha_1,\Delta)}{2}\right]A}. \tag{9}$$

3 Tax Competition

Governments value tax revenue, but also care about the number of headquarters located in their country each of which generates benefits $R \geq 0$ for the local government. For expositional purposes, we concentrate here on the case where $\Delta > 0$, but the analysis for $\Delta < 0$ is summarized in the appendix. Whenever $\Delta > 0$, Country 1's objective function is given by

$$\Omega_1(\Delta) = \int_0^{\bar{c}} \frac{t_1[1-\gamma_1(\alpha_1,\Delta)]A + R}{C}dc = t_1\frac{\bar{c}}{C}\Big[1-\gamma_1(\alpha_1,\Delta)\Big]A + \frac{\bar{c}}{C}R. \tag{10}$$

Similarly, Country 2's objective function is given by:

$$\Omega_2(\Delta) = t_2\left[1-\frac{\bar{c}}{C}\right]A + t_2\frac{\bar{c}}{C}\gamma_1(\alpha_1,\Delta)A + \left[1-\frac{\bar{c}}{C}\right]R. \tag{11}$$

We now look at a country's optimal tax rate for a given foreign tax rate and a set of monitoring efforts. The effect of a change in tax rate t_1 on Country 1's welfare is given by

$$\begin{aligned}\frac{\partial\Omega_1(\Delta)}{\partial t_1} = & \underbrace{\frac{\bar{c}}{C}\Big[1-\gamma_1(\alpha_1,\Delta)\Big]A}_{\text{Revenue Gains from Higher Tax Rate}} + \underbrace{t_1\frac{\bar{c}}{C}\frac{\partial\big[1-\gamma_1(\alpha_1,\Delta)\big]}{\partial\Delta}A}_{\text{Revenue Losses from Profit Shifting}} \\ & + \underbrace{t_1\Big[1-\gamma_1(\alpha_1,\Delta)\Big]\frac{\partial\frac{\bar{c}}{C}}{\partial\Delta}A}_{\text{Revenue Losses from Firms Movement}} + \underbrace{\frac{\partial\frac{\bar{c}}{C}}{\partial\Delta}R}_{\text{Losses in Location Benefit}}.\end{aligned} \tag{12}$$

There are two sources of tax base erosion. As Country 1 increases its tax rate, more profits are shifted toward Country 2. At the same time, more firms choose to locate in Country 2. Define $r = \frac{R}{[1-\gamma_1(\alpha_1,\Delta)]A}$ as the size of the location benefits relative to the per-firm tax base. According to (12), we can express the first-order condition of Country 1 as the following:

$$t_1 \; \varepsilon(1-\gamma_1|\Delta) + [t_1 + r] \; \varepsilon\left(\frac{\bar{c}}{C}|\Delta\right) = 1. \tag{13}$$

Define $t_1^+(t_2)$ as Country 1's reaction function for Δ positive, which is implicitly defined by (13) above.[7] In a classic setting, a tax revenue maximizing government would like the elasticity of the tax base to equal one. In our case, there are two elasticities to consider: i) the per-firm retained profit elasticity and ii) the firms' location elasticity, commonly called capital movement elasticity. When the government increases its tax rate, some firms move away and the remaining ones shift more profits. In addition, each firm leaving generates location benefits loss R, which is worth r in relative terms.

We now look at the fiscal decision in low-tax Country 2. We have that:

$$\begin{aligned}\frac{\partial \Omega_2(\Delta)}{\partial t_2} = & \underbrace{\left[1-\frac{\bar{c}}{C}\right]A + \frac{\bar{c}}{C}\gamma_1(\alpha_1,\Delta)A}_{\text{Revenue Gains from Higher Tax Rate}} + \underbrace{t_2\frac{\bar{c}}{C}\frac{\partial\left[1-\gamma_1(\alpha_1,\Delta)\right]}{\partial\Delta}A}_{\text{Revenue Losses from Less Profit Shifting}} \\ & + \underbrace{t_2\left[1-\gamma_1(\alpha_1,\Delta)\right]\frac{\partial\frac{\bar{c}}{C}}{\partial\Delta}A}_{\text{Revenue Losses from Firms Movements}} + \underbrace{\frac{\partial\frac{\bar{c}}{C}}{\partial\Delta}R}_{\text{Losses in Location Benefit}}.\end{aligned} \tag{14}$$

The first-order condition for Country 2 can be re-written as:

$$t_2 \; \varepsilon(1-\gamma_1|\Delta) + [t_2 + r] \; \varepsilon\left(\frac{\bar{c}}{C}|\Delta\right) = \frac{1-\left[1-\gamma_1(\cdot)\right]\frac{\bar{c}(\cdot)}{C}}{\left[1-\gamma_1(\cdot)\right]\frac{\bar{c}(\cdot)}{C}}. \tag{15}$$

Define $t_2^+(t_1)$ as Country 2's reaction function for Δ positive, which is implicitly defined by (15) above. Again, the first-order condition must take into account the two tax base elasticities and the benefits from firms' location. For the low tax country, local and foreign firms contribute differently. Taxing a firm residing in Country 2 provides one unit of tax revenue.The firm's departure only reduces tax revenue by $[1-\gamma_1(\cdot)]$, because some profits are repatriated via profit shifting. The terms on the left hand side of (15) take into account the fact that fewer firms have cost advantages in Country 2. A smaller tax base prompts this jurisdiction to set a lower tax rate.

[7] See the appendix for the second-order conditions, which are satisfied.

We now look at how one country's tax rate affects the other country's best response. Because of profit shifting and size differences, reaction functions may be discontinuous. Except for these potential breaks, reaction functions feature strategic complementarity. Lemma 3 states that the slopes of the best response functions are positive.

Lemma 3 *Reaction functions are such that* $\frac{\partial t_i^+(t_j)}{\partial t_j} > 0$ *for* $i \in \{1,2\}$*, and* $j \in \{1,2\}$ *and* $i \neq j$*. Furthermore, both slopes are less then one and* $t_i^+(t_j = 0) > 0$*. The same applies to both reaction functions when* $\Delta < 0$*.*

Due to profit shifting, reaction functions can be discontinuous around the 45° line. To understand the causes of discontinuity, let us first consider Country 1's best response under complete symmetry: $\alpha_1 = \alpha_2$ and $C = 1$. As Country 1 increases t_1, the losses or the gains from profit shifted and firms' location decisions are the same whether Δ is positive or negative. Therefore, the best-response function $t_1(t_2)$ is continuous around the 45° line. Now suppose that Country 1 monitors profit shifting more aggressively, so $\alpha_1 > \alpha_2$. When $\Delta > 0$, profits flow from Country 1 to Country 2 and so monitoring α_1 is the relevant policy. When $\Delta < 0$, the flow of profits is reversed and α_2 becomes the relevant policy. Therefore, the first-order conditions are most likely discontinuous around $\Delta = 0$ when α_1 and α_2 are different. This is similar to Keen and Konrad (2013).

Differences in countries' sizes adds another source of discontinuity. Whenever $C < 1$, more firms can shift profits away from Country 1 relative to what can be shifted from Country 2. Country size differences create a similar situation to that studied in Kanbur & Keen (1993) and Mongrain & Wilson (2018). Consequently, at $\Delta = 0$ the first-order conditions may make a discrete jump for these two different reasons. The two effects can work in the same or opposite directions. Both effects can even perfectly cancel each other out. When $g(\alpha_2) = [2C - 1]g(\alpha_1)$, there is no longer a discontinuity. The following lemma summarizes the combined effects.

Lemma 4 *Whenever $g(\alpha_2) > [2C-1]g(\alpha_1)$, the best response function of Country 2 is discontinuous, while Country 1 partially follows the 45^o line. When $g(\alpha_2) < [2C-1]g(\alpha_1)$ the opposite prevails and when $g(\alpha_2) = [2C-1]g(\alpha_1)$ both reaction functions are continuous.*

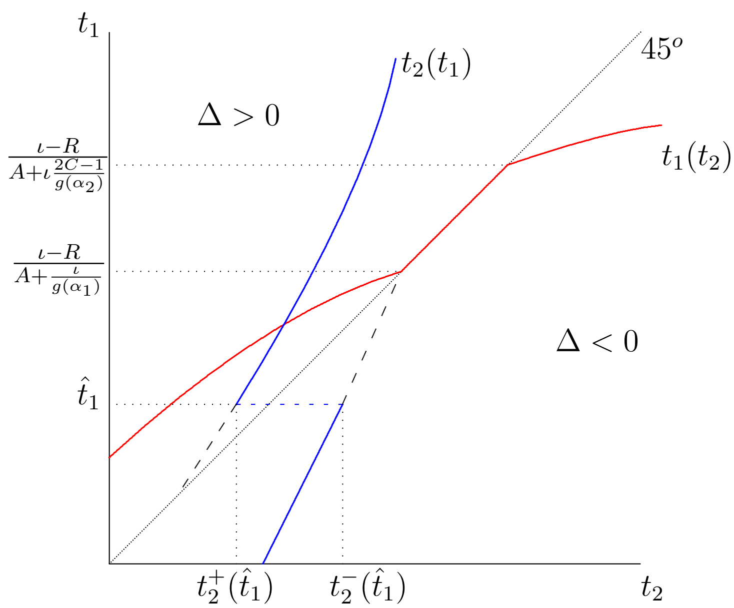

Figure 2: Best response tax rates when $g(\alpha_2) > [2C-1]g(\alpha_1)$

Figure 2 depicts reaction functions for both countries when $g(\alpha_2) > [2C-1]g(\alpha_1)$. The behavior of these best-response functions differs according to whether a country applies the higher or the lower tax rate. We start with Country 1. When t_2 is sufficiently low ($t_2 = 0$ for example), Country 1 prefers to set a higher tax rate. As t_2 increases, Country 1 increases its own tax rate. Lemma 3 states that the slope of all reaction functions is less than one, so at some point, $t_1^+(t_2)$ approaches the 45^o line. Since the first-order conditions are different on either side of the 45^o line, reaction functions are discontinuous at that point. When approaching from the left-hand side, reaction function $t_1^+(t_2)$, with a slope lower than one, would simply dip below the 45^o line. When Country 1 becomes the low-tax country however, the first-order condition suggests that the best response tax rate $t_1^-(t_2)$ should be higher than $t_1^+(t_2)$ for two reasons. With greater regulatory enforcement $\alpha_2 > \alpha_1$, profit shifted from Country 2 is less elastic than profit shifted from Country 1. Second, with $C < 1$, attracting profits from Country 2 has a smaller marginal impact on tax revenue than preventing profits from leaving Country 1. If Country 1 is the high-tax country by a small margin, it tries to prevent $\frac{1}{2C} > 0.5$ firms from shifting profits away. Contrastingly, if Country 1 is the low-tax country by a small margin, it competes to attract profits from $1 - \frac{1}{2C} < 0.5$ firms alone. For a combination of these two reasons, Country 1 prefers to set a higher tax rate. The only way to satisfy the first-order conditions on the right and the left-hand side of the 45^o is for Country 1 to simply match Country 2's tax rate. Consequently, the best response function for Country 1 must follow the 45^o line. As both tax rates t_1 and t_2 increase, the benefit of attracting profits and firms increases. At some point, Country 1 chooses to undercut Country 2 and $t_1^-(t_2)$ passes below the 45^o.

For Country 2, forces operate inversely. If Country 1 sets a high tax rate, then the smaller country prefers to be the low tax country and $t_2^+(t_1) > t_1$. As Country 1 lowers its tax rate, so does Country 2, but by a factor less than one for one. Before-tax rates even converge, Country 2 will prefer to become the high tax country, creating a jump in the reaction function. At $\hat{t}_1$, Country 2 is indifferent between setting a low tax rate $t_2^+(t_1)$ and a high tax rate $t_2^-(t_1)$. The key factor generating this discontinuity is that Country 2 strongly enforces profit shifting control. When Country 2 becomes the high-tax country, it can retain a larger portion of firms' taxable profit. The point of discontinuity $\hat{t}_1$ is implicitly defined by $\Omega_2^+\big(t_2^+(\hat{t}_1), \hat{t}_1\big) = \Omega_2^-\big(t_2^-(\hat{t}_1), \hat{t}_1\big)$.

The discontinuities are reversed for the case when $g(\alpha_2) < [2C-1]g(\alpha_1)$, as depicted in Figure 3. Note that when $g(\alpha_2) = [2C-1]g(\alpha_1)$ both reaction functions become continuous, which marks the transition from Figure 2 to Figure 3.

We now characterize equilibrium tax rates taking profit shifting control policies as given. Depending on parameters, there may be a pure strategy or a mixed strategy equilibrium. Figure 4 represents the pure strategy Nash equilibrium, where both reaction functions cross above the point of discontinuity. Country 2 is more aggressive and sets a lower tax rate. The more Country 2 is at disadvantage (smaller C), the

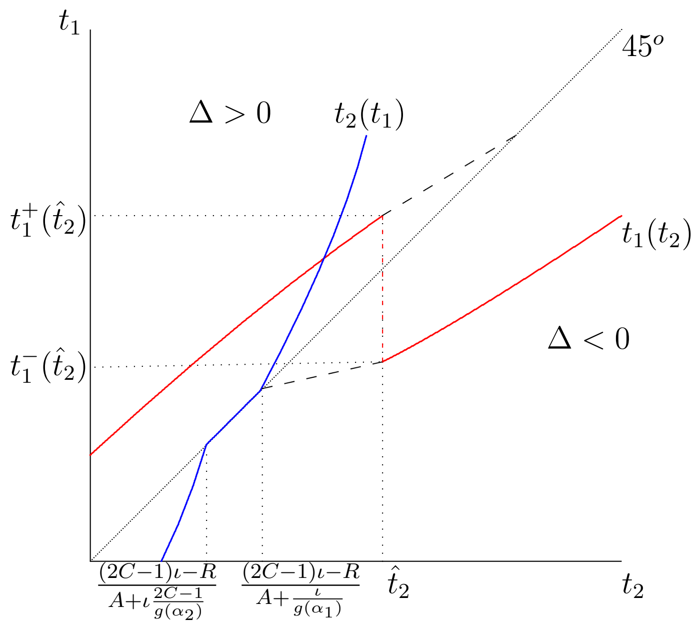

Figure 3: Best response tax rates when $g(\alpha_2) < [2C-1]g(\alpha_1)$

lower both tax rates are. Facing an adversary who sets a lower tax rate forces Country 1 to do the same.

Figure 5 represents the mixed strategy equilibrium. Reaction functions $t_1^+(t_2)$ and $t_2^+(t_1)$ now cross below the point of discontinuity $\hat{t}_1$. As a consequence, there is no pure strategy equilibrium. The mixed strategy equilibrium features Country 1 setting $\hat{t}_1$ and Country 2 being indifferent between setting $t_2^+(\hat{t}_1)$ and $t_2^-(\hat{t}_1)$. Country 2 picks each tax rate with probability q and $(1-q)$. The probability q is set so that $\hat{t}_1$ is the best response for Country 1 on average. The average best response weights two mismatched reactions: an overly high tax rate in response to $t_2^+(\hat{t}_1)$ and an overly low tax rate in response to $t_2^-(\hat{t}_1)$. As a consequence, Country 1 is sometimes the high-tax country and sometimes a low-tax country.

When $g(\alpha_2) < [2C-1]g(\alpha_1)$, the pure strategy equilibrium with tax rate t_1^* and t_2^* is still a possibility. However, there is the possibility of a mixed strategy equilibrium where Country 1 randomizes instead. Country 2 plays a pure strategy $\hat{t}_1$ and Country 1 mixes between $t_1^-(\hat{t}_2)$ and $t_1^+(\hat{t}_2)$. Figure 3 represents the pure strategy equilibrium and Figure 6 the mixed strategy equilibrium.

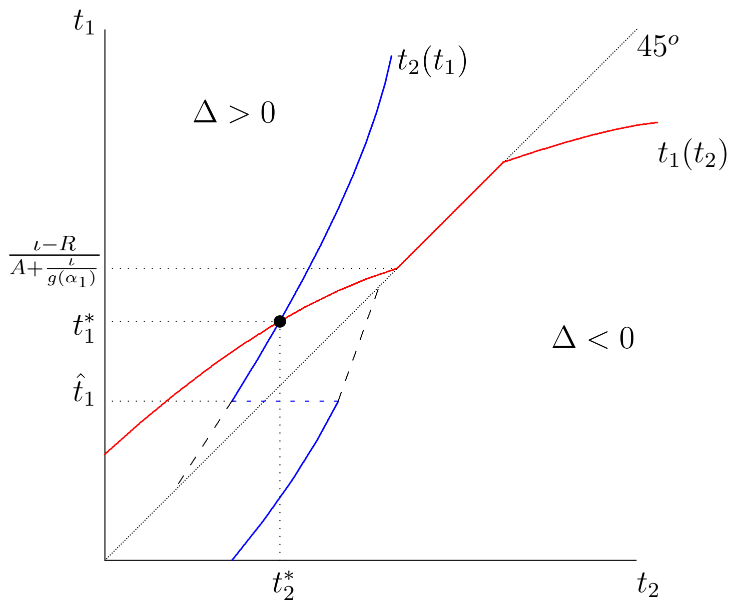

Figure 4: Pure strategy equilibrium when $g(\alpha_2) > [2C-1]g(\alpha_1)$

Proposition 1 summarizes the set of all possible equilibria where t_1^* and t_2^* solve $t_1^* = t_1^+(t_2^*)$ and $t_2^* = t_2^+(t_1^*)$ respectively.

Proposition 1

1. *If $g(\alpha_2) = [2C-1]g(\alpha_1)$, there exist a unique pure strategy Nash equilibrium t_1^* and t_2^*, where $t_1^* > t_2^*$.*

2. *If $g(\alpha_2) > [2C-1]g(\alpha_1)$ and $t_1^* \geq \hat{t}_1$, then t_1^* and t_2^* is the unique Nash equilibrium. When $t_1^* < \hat{t}_1$, there exists a mixed strategy Nash equilibrium in which Country 1 plays $\hat{t}_1$, while Country 2 mixes between $t_2^+(\hat{t}_1) < \hat{t}_1$ and $t_2^-(\hat{t}_1) > \hat{t}_1$ with probability q and $(1-q)$ as described by condition (29) found in the appendix.*

3. *If $g(\alpha_2) < [2C-1]g(\alpha_1)$ and $\hat{t}_2 \geq t_2^*$, then t_1^* and t_2^* is the unique Nash equilibrium. When $t_2^* < \hat{t}_2$, there exist a mixed strategy equilibrium where Country 2 plays $\hat{t}_2$, while Country 1 mixes between $t_1^+(\hat{t}_2)$ and $t_1^-(\hat{t}_2)$ with probability p and $(1-p)$ similar to that described above.*

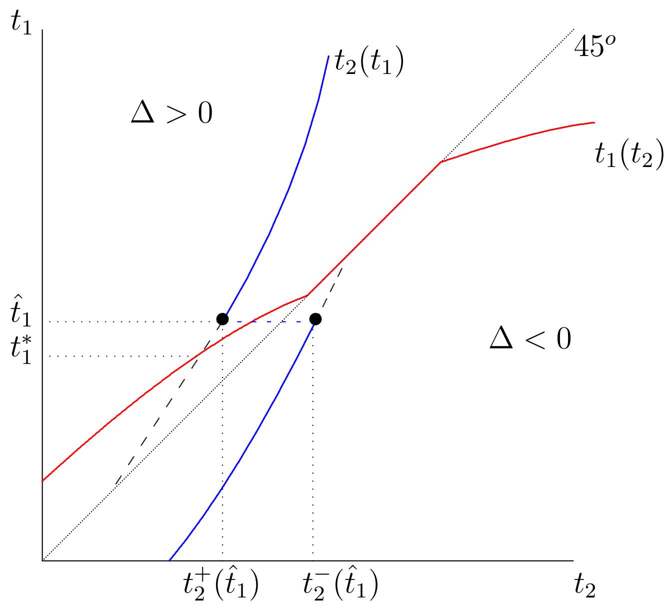

Figure 5: Mixed strategy equilibrium when $g(\alpha_2) > [2C - 1]g(\alpha_1)$

4 Profit Shifting Control

When choosing its profit shifting control strategy, each country takes into account the impact of their choice on the subsequent tax competition game. Depending on α_i' chosen, different types of equilibria may arise. Together, Lemmas 5 and 6 imply that any sets of profit shifting control efforts leading to a mixed strategy equilibrium in the tax competition game cannot be an equilibrium strategy.

Lemma 5 *Any profit shifting control strategy α_1 and α_2 such that $g(\alpha_2) < [2C - 1]g(\alpha_1)$ and where $t_2^* > \hat{t}_2$ is not a best response for Country 2.*

As stated in Proposition 1, when $g(\alpha_2) < [2C - 1]g(\alpha_1)$, there may be a tax competition mixed strategy equilibrium. In that equilibrium Country 2 plays a pure strategy $\hat{t}_1$ and Country 1 mixes between $t_1^-(\hat{t}_2)$ and $t_1^+(\hat{t}_2)$, as depicted in Figure 6. In such a case, Country 1 always sets a tax rate below t_1^*. Lower tax rate by Country 1 harms Country 2's tax revenue collection. However, Country 2 can avoid this mixed strategy equilibrium by choosing a sufficiently high level of profit shifting control. For

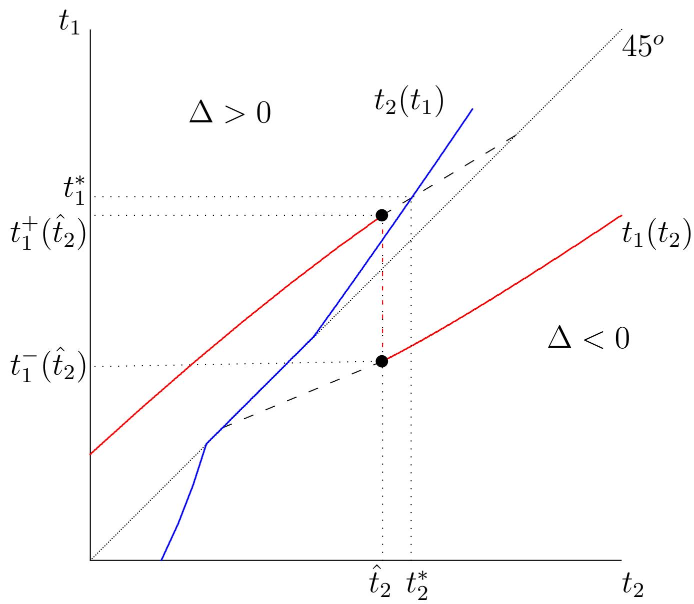

Figure 6: Mixed equilibrium when $g(\alpha_2) < [2C - 1]g(\alpha_1)$

example, when $\alpha_2 = 1$, the pure strategy equilibrium $\{t_1^*, t_2^*\}$ is unique and preferred to the mixed strategy equilibrium by Country 2.

Lemma 6 *Any profit shifting control strategy α_1 and α_2 such that $g(\alpha_2) > [2C - 1]g(\alpha_1)$ and where $t_1^* < \hat{t}_1$is not a best response for Country 1.*

When $g(\alpha_2) > [2C - 1]g(\alpha_1)$, Proposition 1 states that there may be a mixed strategy equilibrium where Country 1 plays a pure strategy and Country 2 mixes strategies as depicted in Figure 5. We show in the appendix that Country 1 gains by choosing $\tilde{\alpha}_1$ instead, where $\tilde{\alpha}_1$ is such that $g(\alpha_2) = (2C - 1)g(\tilde{\alpha}_1)$. The reason is similar to that in the previous case since tax rates are also lower under the mixed strategy equilibrium.[8]

Lemmas 5 and 6 imply that equilibrium profit shifting control strategies are such that there is always a pure strategy equilibrium in the subsequent tax competition game. Under this equilibrium, profit shifting control by Country 2 has no impact on profit shifted, on firms' location decisions, or on the equilibrium tax rates. As a

[8]A more technical explanation is presented at the end of the proof of Lemma 6.

consequence, we concentrate solely on Country 1's choice of profit shifting control. When assessing the impact of profit shifting control on Country 1's welfare, the effect α_1 has on Country 2's equilibrium tax rate plays an important role. Lemma 7 describes the interaction between profit shifting control by Country 1 and Country 2's equilibrium tax rate. As we can see in Figure 4, if an increase in α_1 leads to a downward shift of both countries' best response functions, t_2^* clearly decreases. If only one of the two reaction functions shifts down, t_2^* may still decrease depending on the relative movements of the two reaction functions. The following lemma states the condition under which an increase in α_1 induces a decrease in Country 2's equilibrium tax rate.

Lemma 7 *Under a pure strategy equilibrium in the tax competition game, an increase in α_1 leads to a decrease in equilibrium tax rate t_2^* if and only if*

$$[t_2^* + \lambda t_1^*]\left[\frac{\partial\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)}{\partial\alpha_1} + \frac{\partial\varepsilon(1-\gamma_1|\Delta)}{\partial\alpha_1}\right] + \frac{\varepsilon(1-\gamma_1|\Delta) - \varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)}{[1-\gamma_1(\cdot)]\frac{\bar{c}(\cdot)}{C}} + [1+\lambda]r\left[\frac{\partial\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)}{\partial\alpha_1} - \varepsilon(r|\alpha_1)\right] > 0, \tag{16}$$

where $\lambda = \frac{[-SOC1]}{[-SOC2-\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)-\varepsilon(1-\gamma_1|\Delta)]}$ *and where* $\varepsilon(r|\alpha_1) = -\frac{1}{r}\frac{\partial r}{\partial\alpha_1}$.

To better understand the condition above, first set R to zero. Then, only the first two terms of (16) matter. An increase in α_1 changes both firms' location decision and retained profit elasticities with respect to tax rates. When the cost of shifting profit increases, firms' location decisions become more sensitive to changes in tax rates. At the same time, profit shifting behaviours becomes less sensitive to changes in tax rates. When governments face a more elastic tax bases, tax rates tend to be lower. When the first effect dominates, higher α_1 may encourage countries to set lower tax rates. In such a case, Country 1's reaction function shifts down permanently and Country 2's reaction function is more likely to follow suit.

Profit shifting control also has a direct impact on the actual size of the total tax base Country 2 enjoys. An increase in α_1 shaves the amount of profit shifted. As a consequence, Country 2 has less to lose by lowering its own tax rate. At the same time, more firms locate in Country 2. As it becomes larger, Country 2 cares less about lowering its tax rate to attract more firms and more profits. If Country 2's overall tax base decreases, the government prefers to set a lower tax rate. In short, a smaller total tax base makes Country 2 more aggressive.

Ignoring the location benefits, when the sum of the first two terms in condition (16) is positive, an increase in α_1 causes Country 2's equilibrium tax rate to fall. If both terms are actually positive, then both reaction functions shift down. If one of the terms is negative, one of the reaction functions may shift up. The term λ, pins down the net effect by weighting the movement of both reaction functions appropriately. A similar condition with a different weight determines whether Country 1's tax rate also increases. Obviously, if both reaction functions shift down, both tax rates decrease.

When R is positive, two additional effects are present. First, instead of only gaining taxable income, Country 2 also gains the location benefit R when attracting a new firm. The impact of a change in firms' location decision elasticity is then amplified. Since an increase in α_1 makes firms more sensitive to difference in tax rates, it puts a downward pressure on both reaction functions. This effect is greater when revenue R is high relative to per-firm retained profits $[1 - \gamma_1(\cdot)]A$. At the same time, the value of this revenue relative to the per-firm retained profits, represented by r decreases when α_1 increases because the per-firm retained profits increase. The net effect is represented by the last term of (16).

In addition to influencing equilibrium tax rates, profit shifting control has a direct impact on Country 1's welfare by altering the amount of profit shifted and the allocation of capital. The following proposition states the condition under which decreasing profit shifting control is beneficial for Country 1.

Proposition 2 *Under the pure strategy equilibrium in the tax competition game, an* *increase in α_1 reduces Country 1's welfare if and only if*

$$t_1^* \varepsilon(1-\gamma_1|\alpha_1) - [t_1^* + r]\varepsilon(\bar{c}|\alpha_1) + \frac{dt_2^*}{d\alpha_1} < 0. \tag{17}$$

The first two components in (17) are the direct effects of α_1 on Country 1's welfare. On the one hand, an increase in α_1 boosts tax revenue due to the reduction in profit shifting. On the other hand, it chases firms away. Fewer firms means less tax revenue, but also smaller total benefits from firms' location. Finally, an increase in α_1, can lead Country 2 to set a higher or a lower tax rate, as stated in Lemma 7. An increase in Country 2's tax rate raisses Country 1's tax revenue. Country 1 compares these gains to the loss created by increased profit shifting.

Whenever the condition stated in Proposition 2 is satisfied at $\alpha_1 = 1$, it is beneficial for Country 1 not to take all possible actions against profit shifting, even when such actions are costless. To illustrate such a situation, we use the following simulation where we set $A = 3$, $C = 0.75$, $\iota = 2$, $R = 1.3$ and $g(\alpha) = 1 + \frac{\alpha}{10}$. We verify that the tax competition game delivers a pure strategy equilibrium. Figure 7 shows the effect of α_1 on Country 1's welfare, which is maximized for a relatively low level of profit shifting control. In the environment explored, equilibrium tax rates are strictly decreasing with α_1, which helps to make strict profit shifting control unattractive.

5 Conclusion

The first contribution of our paper was to identify two important tax bases: capital and per-firm retained profit tax bases. More stringent profit shifting control makes the per-firm retained profit tax base less elastic, but makes the mobile capital tax base more elastic. Stricter profit shifting control may put downward pressure on tax rates and may be undesirable when the second effect dominates. At the same time, profit shifting control changes the total tax base of a low-tax country. More profit shifting control reduces exports of profit, it can also chase firms away. This has an

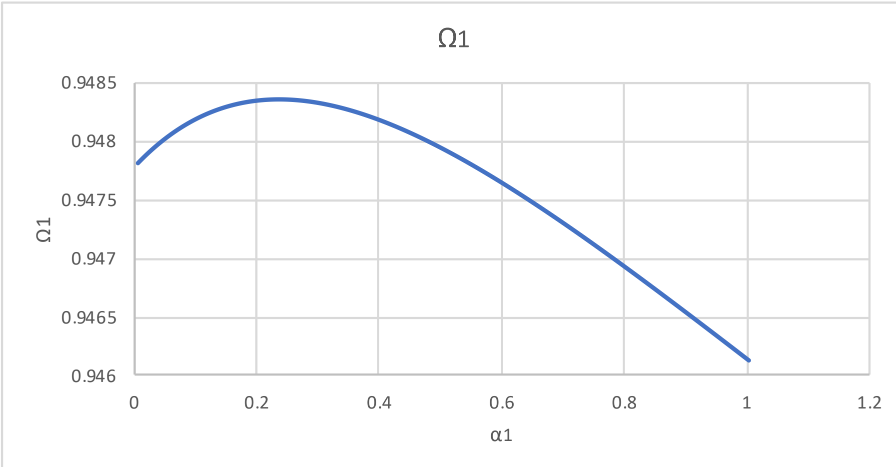

Figure 7: Effect of profit shifting control on Country 1's welfare

ambiguous effect on a high tax country's well-being. If the country loses a large number of firms, it can have a direct negative impact on tax revenue. However, if the high tax country's total tax base decreases, the low-tax country's total tax base increases which can put upward pressure on tax rates.

We purposely ignored those factors proposed in the literature as working against profit shifting control, like the presence of local and international firms allowing for fiscal discrimination or endogenous size of capital. Instead we focus on more basic mechanisms which allowed us to have endogenous tax rates for both countries in a general setting. Many papers before us introduced a parasitic tax haven posting an exogenous zero tax rate. It would be interesting to see what our model can tell about those tax havens. The case where $C = 0.5$ adequately describes a parasitic tax haven. For equal tax rates, all firms would like to set up their main production facility in Country 1. When posting a lower tax rate, Country 2 is able to attract a few headquarter, but more importantly, is able to import profits. This fits well with the idea of a parasitic tax haven. The question is, would such a country like to post a zero tax rate? If we refer back to the first-order condition (15) for Country 2, we can see that when $R = 0$, the answer is no. As soon as Country 2 posts a lower tax rate, it earns a positive tax base. Profits and firms are not fully mobile, even when $C = 0.5$.

Firms are willing to tradeoff shifting and location costs to earn some fiscal relief. As a consequence, the country will post a lower, but non-zero tax rate. When there is a positive localization rent, however, the country may like to have not tax on capital. Because of the rent, the country may prefer attracting firms than maximizing tax revenue. If R is sufficiently high, first-order condition (15) allows for the possibility of a corner solution where Country 2 prefers a zero tax rate. This implies that the localization rent is a necessary condition for the existence of parasitic tax havens.

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7 Appendix

Proof of Lemma 1: We can show that $\frac{\partial \varepsilon(1-\gamma_1|\Delta)}{\partial \alpha_1} = \frac{-g'(\alpha_1)}{[g(\alpha_1)-\Delta]^2} < 0$. ■

Proof of Lemma 2: The effect of α_1 on $\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)$ is given by:

$$\frac{\partial \varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)}{\partial \alpha_1} = \frac{-\frac{\iota}{2}\frac{\partial \gamma_1(\alpha,\Delta)}{\partial \alpha_1}A}{\left[\iota - \Delta\left[1-\frac{\gamma(\alpha_1,\Delta)}{2}\right]A\right]^2} = \frac{\frac{\iota g'(\alpha_1)A}{2\Delta g(\alpha_1)^2}}{\left[\iota - \Delta\left[1-\frac{\gamma(\alpha_1,\Delta)}{2}\right]A\right]^2}, \tag{18}$$

which is positive. ■

Second-order conditions: The second-order condition for Country 1 is given by:

$$SOC1 = -\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right) - \varepsilon(1-\gamma_1|\Delta) - t_1\frac{\partial \varepsilon(1-\gamma_1|\Delta)}{\partial t_1} - t_1\left[1+\frac{R}{t_1[1-\gamma_1(\cdot)]A}\right]\frac{\partial \varepsilon\left(\frac{\bar{c}}{C}|t_1\right)}{\partial t_1}$$
$$-\frac{R}{[1-\gamma_1(\cdot)]A}\varepsilon(1-\gamma_1|\Delta)\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right). \tag{19}$$

We can show that $\frac{\partial \varepsilon(1-\gamma_1|\Delta)}{\partial t_1} = \varepsilon(1-\gamma_1|\Delta)^2$ and that $\frac{\partial \varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)}{\partial t_1} = \varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)\left[\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right) - \varepsilon(1-\gamma_1|\Delta)\right]$, so the expression above becomes

$$SOC1 = -\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right) - \varepsilon(1-\gamma_1|\Delta) - t_1\left[\varepsilon(1-\gamma_1|\Delta) - \varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)\right]^2$$
$$-t_1\left[1+\frac{R}{t_1[1-\gamma_1(\cdot)]A}\right]\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)^2, \tag{20}$$

which is negative. Similarly, Country 2's second-order condition is given by:

$$SOC2 = -\left[\varepsilon(1-\gamma_1|\Delta) + \varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)\right]\left[1+\frac{1}{[1-\gamma_1(\cdot)]\frac{\bar{c}}{C}}\right] \tag{21}$$
$$-t_2\frac{\partial \varepsilon(1-\gamma_1|\Delta)}{\partial t_2} - t_2\left[1+\frac{R}{t_2[1-\gamma_1(\cdot)]A}\right]\frac{\partial \varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)}{\partial t_2} + \frac{R}{[1-\gamma_1(\cdot)]^2A}\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)\frac{\partial[1-\gamma_1(\cdot)]}{\partial t_2}.$$

We can rewrite the expression above as

$$SOC2 = -\frac{\varepsilon(1-\gamma_1|\Delta)+\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)}{[1-\gamma_1(\cdot)]\frac{\bar{c}}{C}} - \varepsilon(1-\gamma_1|\Delta)\left[1-t_2\varepsilon(1-\gamma_1|\Delta)\right]$$
$$-\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)\left(1-t_2\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)\left[1+\frac{R}{t_2[1-\gamma_1(\cdot)]A}\right]\right) - t_2\varepsilon(1-\gamma_1|\Delta)\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right) < 0. \tag{22}$$

The second-order condition is satisfied for low tax Country 2 as well. ■

Low tax Country 1 and High tax Country 2 – The case where $\Delta < 0$ can be summarized as below:

$$\frac{\partial \Omega_1}{\partial t_1} = \left[1-\gamma_2(\alpha_2,\Delta)\right]t_1\frac{\partial \bar{c}}{\partial t_1}\frac{A}{C} + \frac{\partial \bar{c}}{\partial t_1}R + \left[1-\frac{\bar{c}}{C}\right]t_1\frac{\partial \gamma_2(\cdot)}{\partial t_1}A$$
$$+\frac{\bar{c}}{C}A + \left[1-\frac{\bar{c}}{C}\right]\gamma_2(\alpha_2,\Delta)A. \tag{23}$$

Let $t_1^-(t_2)$ implicitly solve the equation above. High tax Country 2's first-order condition is given by:

$$\frac{\partial \Omega_2}{\partial t_2} = -\left[1-\gamma_2(\alpha_2,\Delta)\right] t_2 \tfrac{\partial \bar{c}}{\partial t_2}\tfrac{A}{C} - \tfrac{\partial \bar{c}}{\partial t_2} R - \left[1-\tfrac{\bar{c}}{C}\right] t_2 \tfrac{\partial \gamma_2(\cdot)}{\partial t_2} A + \left[1-\tfrac{\bar{c}}{C}\right]\left[1-\gamma_2(\alpha_2,\Delta)\right] A. \tag{24}$$

Let $t_2^-(t_1)$ implicitly solve the equation above. Those two equations are simply the mirror of equations (13) and (15). ■

Proof of Lemma 3: To start, evaluating (13) and (15) at $t_1 = t_2 = 0$ reveals they are both positive, so $t_1^+(t_2 = 0) > 0$ and $t_2^+(t_1 = 0) > 0$. The slope of reaction function $t_1^+(t_2)$ is given by

$$\frac{\partial t_1^+(t_2)}{\partial t_2} = \frac{-SOC1 - \left[\varepsilon(1-\gamma_1|\Delta) + \varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)\right]}{-SOC1}. \tag{25}$$

Given the second-order conditions are satisfied, $\frac{\partial t_1^+(t_2)}{\partial t_2} > 0$ and smaller than one. Similarly, we have that $\frac{\partial t_2^+(t_1)}{\partial t_1} = \frac{-SOC2 - \left[\varepsilon(1-\gamma_1|\Delta)+\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)\right]}{-SOC2}$. ■

Proof of Lemma 4: Evaluating each Country's first-order condition around $\Delta = 0$, yields :

$$\left.\frac{\partial \Omega_1^+(\cdot)}{\partial t_1}\right|_{\Delta\to 0} = \frac{A}{2C}\left[1 - t_1\left(\frac{A}{\iota}\left[1+\frac{R}{t_1 A}\right] + \frac{1}{g(\alpha_1)}\right)\right],$$

$$\left.\frac{\partial \Omega_1^-(\cdot)}{\partial t_1}\right|_{\Delta\to 0} = \frac{A}{2C}\left[1 - t_1\left(\frac{A}{\iota}\left[1+\frac{R}{t_1 A}\right] + (2C-1)\frac{1}{g(\alpha_2)}\right)\right].$$

Similarly for Country 2, we have that

$$\left.\frac{\partial \Omega_2^-(\cdot)}{\partial t_2}\right|_{\Delta\to 0} = \frac{A}{2C}\left[2C - 1 - t_2\left(\frac{A}{\iota}\left[1+\frac{R}{t_2 A}\right] + \frac{1}{g(\alpha_1)}\right)\right],$$

$$\left.\frac{\partial \Omega_2^+(\cdot)}{\partial t_2}\right|_{\Delta\to 0} = \frac{A}{2C}\left[2C - 1 - t_2\left(\frac{A}{\iota}\left[1+\frac{R}{t_2 A}\right] + (2C-1)\frac{1}{g(\alpha_2)}\right)\right].$$

When $g(\alpha_2) = [2C-1]g(\alpha_1)$, both reaction functions are continuous and the reaction functions have the same slope just above and below $\Delta = 0$.

When $g(\alpha_2) > [2C-1]g(\alpha_1)$, Country 1's first-order condition is such that $\frac{\partial \Omega_1^+(\cdot)}{\partial t_1}\Big|_{\Delta\to 0} < 0$ and $\frac{\partial \Omega_1^-(\cdot)}{\partial t_1}\Big|_{\Delta\to 0} > 0$. Therefore, Country 1's best response to $t_2 \in [\frac{\iota-R}{A+\frac{\iota}{g(\alpha_1)}}, \frac{\iota-R}{A+(2C-1)\frac{\iota}{g(\alpha_2)}}]$ is simply t_2 (i.e. the 45^o line).

Similarly, $\frac{\partial \Omega_2^+(\cdot)}{\partial t_2}\Big|_{\Delta\to 0} > 0$ and $\frac{\partial \Omega_2^-(\cdot)}{\partial t_2}\Big|_{\Delta\to 0} < 0$ for all $t_1 \in [\frac{(2C-1)\iota-R}{A+\frac{\iota}{g(\alpha_1)}}, \frac{(2C-1)\iota-R}{A+(2C-1)\frac{\iota}{g(\alpha_2)}}]$. Around $\Delta = 0$, Country 2 gains by either increasing or decreasing its tax rate. Consequently, Country 2 faces two maxima, $\Omega_2^+ = \Omega_2^+\big(t_2^+(t_1), t_1\big)$ and $\Omega_2^- = \Omega_2^-\big(t_2^-(t_1), t_1\big)$. The best response of Country 2 is the tax rate generating the highest maximum:

$$t_2(t_1) = \begin{cases} t_2^+(t_1) \text{ if } \Omega_2^+ > \Omega_2^- \\ t_2^-(t_1) \text{ if } \Omega_2^+ < \Omega_2^- \end{cases}$$

Note that at $t_1 = \frac{(2C-1)\iota-R}{A+\frac{\iota}{g(\alpha_1)}}$, we have that $\Omega_2^+ > \Omega_2^-$ as $\frac{\partial \Omega_2^+(\cdot)}{\partial t_2} = 0$ and $\frac{\partial \Omega_2^-(\cdot)}{\partial t_2} > 0$. At the opposite, when $t_1 = \frac{(2C-1)\iota-R}{A+(2C-1)\frac{\iota}{g(\alpha_2)}}$, then $\Omega_2^+ < \Omega_2^-$ as $\frac{\partial \Omega_2^+(\cdot)}{\partial t_2} > 0$ and $\frac{\partial \Omega_2^-(\cdot)}{\partial t_2} = 0$. Moreover, as $t_2^+(t_1) < t_2^-(t_1)$, a necessary condition for $\Omega_2^+ \geq \Omega_2^-$ is that

$$(1-\gamma_1^+)\bar{c}^+ > \gamma_2^-\bar{c}^- \tag{26}$$

with γ_1^+, γ_2^-, $\bar{c}^+$ and $\bar{c}^-$ being the functions $\gamma_i(\cdot)$ and $\bar{c}$ evaluated respectively at $\Delta = t_1 - t_2^+(t_1)$ and $\Delta^- = t_1 - t_2^-(t_1)$. Finally, note that

$$\frac{d(\Omega_2^+ - \Omega_2^-)}{dt_1} = \frac{(1-\gamma_1^+)\bar{c}^+ - \gamma_2^-\bar{c}^-}{C} \tag{27}$$

Equation (27) and (26) imply that when $\Omega_2^+ \geq \Omega_2^-$, then $\frac{d(\Omega_2^+ - \Omega_2^-)}{dt_1} > 0$. Therefore, there always exists $\hat{t}_1 \in [\frac{(2C-1)\iota-R}{A+\frac{\iota}{g(\alpha_1)}}, \frac{(2C-1)\iota-R}{A+(2C-1)\frac{\iota}{g(\alpha_2)}}]$ such that

$$t_2(t_1) = \begin{cases} t_2^+(t_1) \text{ if } t_1 \geq \hat{t}_1 \\ t_2^-(t_1) \text{ if } t_1 \leq \hat{t}_1 \end{cases}$$

Similarly, we can characterize reaction functions when $g(\alpha_2) < [2C-1]g(\alpha_1)$. It is now $t_2(t_1)$ who follow the 45^o line as $\frac{\partial \Omega_2^+(\cdot)}{\partial t_2}\Big|_{\Delta\to 0} < 0$ and $\frac{\partial \Omega_2^-(\cdot)}{\partial t_2}\Big|_{\Delta\to 0} > 0$ between $t_1 \in [\frac{(2C-1)\iota-R}{A+(2C-1)\frac{\iota}{g(\alpha_2)}}, \frac{(2C-1)\iota-R}{A+\frac{\iota}{g(\alpha_1)}}]$.

We also have that $\frac{\partial \Omega_1^+(\cdot)}{\partial t_1} > 0$ and $\frac{\partial \Omega_1^-(\cdot)}{\partial t_1} < 0$. For any $t_2 \in [\frac{\iota-R}{A+(2C-1)\frac{\iota}{g(\alpha_2)}}, \frac{\iota-R}{A+\frac{\iota}{g(\alpha_1)}}]$ around $\Delta = 0$, Country 1 gains by either increasing and decreasing its tax rate.

This means that for those values of t_2, the payoff of Country 1 has two maxima, $\Omega_1^+ = \Omega_1^+\big(t_1^+(t_2), t_2\big)$ and $\Omega_1^- = \Omega_1^-\big(t_1^-(t_2), t_2\big)$. The best response of Country 1 is tax rate generating the highest maximum:

$$t_1(t_2) = \begin{cases} t_1^+(t_2) \text{ if } \Omega_1^+ > \Omega_1^- \\ t_1^-(t_2) \text{ if } \Omega_1^+ < \Omega_1^- \end{cases}$$

Exactly as in the previous case, we can show that $\hat{t}_2$ exist. ■

Proof of Proposition 1: Let's start with a case where $g(\alpha_2) = [2C-1]g(\alpha_1)$. Since both $t_1^+(t_2)$ and $t_2^+(t_1)$ are increasing in their argument with a slope smaller than one, then $t_1^+(t_2)$ and $t_2^+(t_1)$ cross at t_1^* and t_2^*, where $t_1^* > t_2^*$.

Now imagine that $g(\alpha_2) > [2C-1]g(\alpha_1)$. Given (12) and (14), we know that $t_1^+(t_2)$ is above the 45^o line for $t_2 < \frac{\iota - R}{A + \frac{\iota}{g(\alpha_1)}}$ and that $t_2^+(t_1)$ is above the 45^o line for $t_1 > \frac{(2C-1)\iota - R}{A + \frac{\iota}{g(\alpha_1)}}$. Consequently, $t_1^+(t_2)$ and $t_2^+(t_1)$ cross at t_1^* and t_2^*, where $t_1^* > t_2^*$. On the contrary, $t_1^-(t_2)$ is below the 45^o line for $t_2 > \frac{\iota - R}{A + (2C-1)\frac{\iota}{g(\alpha_2)}}$ and $t_2^-(t_1)$ is below the 45^o line for $t_1 < \frac{(2C-1)\iota - R}{A + (2C-1)\frac{\iota}{g(\alpha_2)}}$. Consequently, $t_1^-(t_2)$ and $t_2^-(t_1)$ cannot cross.

This leaves us with only t_1^* and t_2^* as a candidate for a pure strategy Nash equilibrium. If $\hat{t}_1 < t_1^*$, then t_1^* and t_2^* is the only Nash equilibrium. Otherwise, there exist a mixed strategy Nash equilibrium.

Since $\hat{t}_1$ is defined by $\Omega_2^+\big(t_2^+(\hat{t}_1), \hat{t}_1\big) = \Omega_2^-\big(t_2^-(\hat{t}_1), \hat{t}_1\big)$, Country 2 is indifferent between $t_2^+(\hat{t}_1)$ and $t_2^-(\hat{t}_1)$. It is then willing to play the former with probability q and the latter with probability $1-q$. The probability q must be set so that $\hat{t}_1$ is the expected best response for Country 1 and so

$$q\frac{\partial \Omega_1^+\big(\hat{t}_1, t_2^+(\hat{t}_1)\big)}{\partial t_1} + (1-q)\frac{\partial \Omega_1^-\big(\hat{t}_1, t_2^-(\hat{t}_1)\big)}{\partial t_1} = 0. \tag{28}$$

If we define $\Delta^+ = \hat{t}_1 - t_2^+(\hat{t}_1)$ and $\Delta^- = t_2^-(\hat{t}_1) - \hat{t}_1$, the q is given by

$$q\frac{\bar{c}(\Delta^+)}{C}\Big[1-\gamma_1(\alpha_1,\Delta^+)\Big]\Bigg[1-\hat{t}_1\varepsilon\left(\frac{\bar{c}}{C}|\Delta^+\right)\left(1+\frac{R}{\hat{t}_1[1-\gamma_1(\alpha_1,\Delta^+)]A}\right)$$
$$-\hat{t}_1\varepsilon(1-\gamma_1|\Delta^+)\Bigg]+(1-q)\frac{\bar{c}(\Delta^-)}{C}\Big[1-\gamma_2(\alpha_2,\Delta^-)\Big]\Bigg[1+\frac{\gamma_2(\alpha_2,\Delta^-)}{1-\gamma_2(\alpha_2,\Delta^-)}\frac{C}{\bar{c}(\Delta^-)}$$
$$-\hat{t}_1\varepsilon\left(\frac{\bar{c}}{C}|\Delta^-\right)\left(1+\frac{R}{\hat{t}_1[1-\gamma_1(\cdot)]A}\right)-\hat{t}_1\frac{C-\bar{c}(\Delta^-)}{\bar{c}(\Delta^-)}\varepsilon(1-\gamma_2|\Delta^-)\Bigg].\quad(29)$$

Finally, we look at the case where $g(\alpha_2) < [2C-1]g(\alpha_1)$. Now it is $t_1(t_2)$ who is discontinuous and $t_2(t_1)$ who partly follow the 45^o line. Define When $\hat{t}_2 \geq t_2^*$, then t_1^* and t_2^* is a pure strategy Nash equilibrium. When $\hat{t}_2 < t_2^*$, then there exist a mixed strategy equilibrium where Country 2 plays $\hat{t}_2$ and one mixed between $t_1^+(\hat{t}_2)$ and $t_1^-(\hat{t}_2)$ with probability p and $1-p$ given by conditions similar to the ones stated above. ■

Proof of Lemma 5: Given the envelop theorem, we know that $\Omega_2^+(t_2^+(t_1), t_1)$ is increasing in t_1. Under the pure strategy Nash equilibrium, Country 2's payoff is independent of α_2. If $g(\alpha_2) < [2C-1]g(\alpha_1)$ and $\hat{t}_2 < t_2^+$, then the mixed strategy equilibrium feature Country 2 playing $\hat{t}_2$ and Country 1 mixing between $t_1^+(\hat{t}_2)$ and $t_1^-(\hat{t}_2)$. Under this mixed strategy equilibrium, Country 2's payoff is lower than under the pure strategy equilibrium. Consequently, any strategy α_2 such that $g(\alpha_2) \geq [2C-1]g(\alpha_1)$ leads to payoff for Country 2 that is better as any strategy where $g(\alpha_2) \geq [2C-1]g(\alpha_1)$ and $\hat{t}_2 < t_2^+$. Consequently, any strategy α_2 such that $g(\alpha_2) < [2C-1]g(\alpha_1)$ and $\hat{t}_2 < t_2^+$, is not a best response to α_2. ■

Proof of Lemma 6: Let's write $\tilde{t}_2^-(\alpha_2)$ such that $\tilde{t}_2^-(\alpha_2) = t_2^-(\tilde{t}_2^-(\alpha_2))\big|_{\alpha_2}$ and $\tilde{t}_2^+(\alpha_1)$ such that $\tilde{t}_2^+(\alpha_1) = t_2^+(\tilde{t}_2^+(\alpha_1))\big|_{\alpha_1}$. Those are the crossing points of respectively $t_2^-(t_1)$ and $t_2^+(t_1)$ with the 45° line

We now show that a deviation by Country 1 to $\tilde{\alpha}_1$ such that $g(\alpha_2) = (2C-1)g(\tilde{\alpha}_1)$ is profitable under the mixed strategy equilibrium. According to Proposition 1, at $\tilde{\alpha}_1$ there exists a pure strategy equilibrium (t_1^{**}, t_2^{**}) with $t_1^{**} \geq t_2^{**}$. We first show that

$t_2^+(\hat{t}_1) < \hat{t}_1 < t_2^{**}$. The first inequality comes from the definition of $t_2^+(\hat{t}_1)$. For the second one, note that given the definition of $\hat{t}_1$, we have that $\hat{t}_1 \in \left]\tilde{t}_2^+(\alpha_1), \tilde{t}_2^-(\alpha_2)\right[$. Moreover, $\tilde{t}_2^+(\tilde{\alpha}_1) = \tilde{t}_2^-(\alpha_2)$ when $g(\alpha_2) = (2C-1)g(\tilde{\alpha}_1)$. As a consequence, the best reply for Country 2 is continuous and crosses the 45° line at $\tilde{t}_2^+(\tilde{\alpha}_1)$. Since $t_1^{**} \geq t_2^{**}$, it implies that $t_1^{**} > \tilde{t}_2^+(\tilde{\alpha}_1)$ and as $\partial t_2(t_1)/\partial t_1 > 0$, it must be the case that $t_2^{**} > \tilde{t}_2^+(\tilde{\alpha}_1) > \hat{t}_1$.

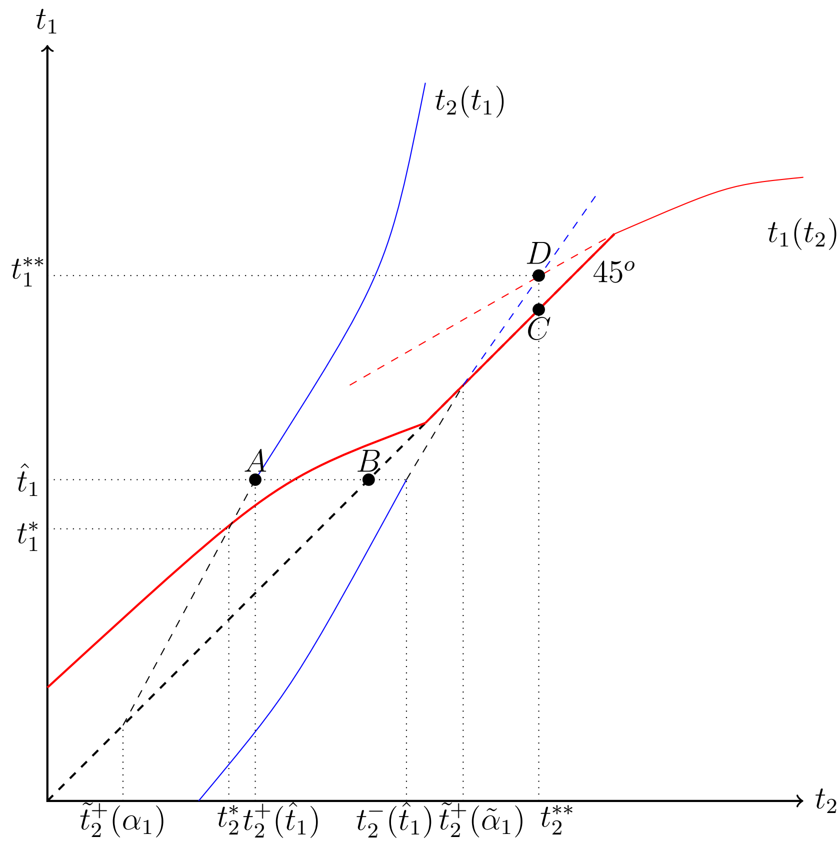
Define $\Omega_i(\alpha_1, \alpha_2, t_1, t_2)$ the payment of Country i, as a function of the profit shifting enforcement and tax rates, where

$$\Omega_1(\alpha_1, \alpha_2, \hat{t}_1, t_2^+(\hat{t}_1)) < \Omega_1(\alpha_1, \alpha_2, \hat{t}_1, \hat{t}_1) = \Omega_1(\tilde{\alpha}_1, \alpha_2, \hat{t}_1, \hat{t}_1)$$
$$< \Omega_1(\tilde{\alpha}_1, \alpha_2, t_2^{**}, t_2^{**}) \leq \Omega_1(\tilde{\alpha}_1, \alpha_2, t_1(t_2^{**}), t_2^{**}).$$

The first inequality comes from the fact that $\partial \Omega_i / \partial t_j > 0$ and $t_2^+(\hat{t}_1) < \hat{t}_1$. The second one is explained as follow when $t_1 = t_2$, there is no profit shifting and therefore payoffs are unaffected by the monitoring strategy. To understand the third one, we have to note that, because of the global inelastic supply of capital, a coordinated increase in harmonized taxes increases welfare for both countries. The last inequality is obvious as by definition of the best reply, country one has a higher payment at $t_1(t_2^{**})$ than at any other tax rate.

With $\tilde{\alpha}_1$, reaction functions become continuous and are represented above the 45^o line by the dotted curves on the figure below . Proposition 1 tells that there is a unique pure strategy equilibrium t_1^{**}, t_2^{**} with $t_1^{**} > t_2^{**}$. We already shown that $t_2^{**} > t_2^+(\hat{t}_1)$. Following the figure below , it easy to see that Country 1 prefers B to A for α_1 as t_2 is larger in B. Note also that when $t_1 = t_2$, Country 1 is not affected by a change in α, this means that country 1 prefers C with $\tilde{\alpha}_1$ to B with α_1. Lastly, by definition of the best reaction function, we have that Country 1 prefers D to C both with $\tilde{\alpha}_1$. Therefore by transitivity announcing $\tilde{\alpha}_1$ is a profitable deviation. ■

Proof of Lemma 7: The sign of $\frac{dt_2^*}{d\alpha_1}$ can be found by using implicit derivatives and Cramer's rule on equations (13) and (15), which jointly determine t_2^*. According to Cramer's Rule, $\frac{dt_2^*}{d\alpha_1} = -\frac{|h|}{|H|}$, where $|H|$ is the determinant of the matrix composed of


t_1
$t_2(t_1)$
t_1^{**}
D
45^o
$t_1(t_2)$
C
A
B
$\hat{t}_1$
t_1^*
t_2
$\tilde{t}_2^+(\alpha_1)$
t_2^*
$t_2^+(\hat{t}_1)$
$t_2^-(\hat{t}_1)$
$\tilde{t}_2^+(\tilde{\alpha}_1)$
t_2^{**}

the derivatives of equations (13) and (15) with respect with t_1 and t_2, while $|h|$ is the determinant of the similar matrix, but with respect with t_1 and γ_1 instead. From the proof of Lemma 3, we know that $|H| = [-SOC1][-SOC2] - [-SOC1 - \varepsilon\left(\frac{\bar{c}}{C}|\Delta\right) - \varepsilon(1-\gamma_1|\Delta)][-SOC2 - \varepsilon\left(\frac{\bar{c}}{C}|\Delta\right) - \varepsilon(1-\gamma_1|\Delta)]$, which is positive. Similarly we can show that $|H| = [-SOC1][-\frac{dFOC2}{d\alpha_1}] + [-SOC2 - \varepsilon\left(\frac{\bar{c}}{C}|\Delta\right) - \varepsilon(1-\gamma_1|\Delta)][-\frac{dFOC1}{d\alpha_1}]$, where

$$-\frac{dFOC1}{d\alpha_1} = t_1\frac{\partial\varepsilon(1-\gamma_1|\Delta)}{\partial\alpha_1} + (t_1+r)\frac{\partial\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)}{\partial\alpha_1} + \varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)\frac{\partial r}{\partial\alpha_1}; \tag{30}$$

$$\begin{aligned}-\frac{dFOC2}{d\alpha_1} = t_2\tfrac{\partial\varepsilon(1-\gamma_1|\Delta)}{\partial\alpha_1} + (t_2+r)\tfrac{\partial\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)}{\partial\alpha_1} + \varepsilon\left(\tfrac{\bar{c}}{C}|\Delta\right)\tfrac{\partial r}{\partial\alpha_1}\\ +\tfrac{\varepsilon(1-\gamma_1|\Delta)-\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)}{[1-\gamma_1(\cdot)]\frac{\bar{c}(\cdot)}{C}}.\end{aligned} \tag{31}$$

We can then show that $\frac{dt_2^*}{d\alpha_1}$ is negative if and only if

$$\begin{aligned}[\lambda t_1^* + t_2^*]\left[\frac{\partial\varepsilon(1-\gamma_1|\Delta)}{\partial\alpha_1} + \frac{\partial\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)}{\partial\alpha_1}\right] + \frac{\varepsilon(1-\gamma_1|\Delta) - \varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)}{[1-\gamma_1(\cdot)]\frac{\bar{c}(\cdot)}{C}}\\ +[1+\lambda]r\left[\frac{\partial\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)}{\partial\alpha_1} - \varepsilon(r|\alpha_1)\right] > 0,\end{aligned} \tag{32}$$

where $\lambda = \frac{[-SOC1]}{[-SOC2-\varepsilon\left(\frac{\bar{c}}{C}|\Delta\right)-\varepsilon(1-\gamma_1|\Delta)]}$ and where $\varepsilon(r|\alpha_1) - \frac{1}{r}\frac{\partial r}{\partial\alpha_1}$. ■

Proof of Proposition 2: The effect of a change in α_1 on Country 1's welfare is given by

$$\frac{d\Omega_1(t_1^*, t_2^*, \alpha_1)}{d\alpha_1} = \frac{\partial\Omega_1(\cdot)}{\partial t_1}\frac{dt_1^*(\cdot)}{d\alpha_1} + \frac{\partial\Omega_1(\cdot)}{\partial t_2}\frac{dt_2^*(\cdot)}{d\alpha_1} + \frac{\partial\Omega_1(\cdot)}{\partial\gamma_1}\frac{d\gamma_1(\cdot)}{d\alpha_1} + \frac{\partial\Omega_1(\cdot)}{\partial\bar{c}(\Delta)}\frac{d\bar{c}(\Delta)}{d\alpha_1}. \tag{33}$$

Given the envelop theorem, $\frac{\partial\Omega_1(\cdot)}{\partial t_1} = 0$. Using the first-order conditions for Country 1's $\frac{\partial\Omega_1(\cdot)}{\partial t_2} = \left[1-\gamma_1(\cdot)\right]\frac{\bar{c}(\Delta)}{C}A + \frac{\bar{c}(\Delta)}{C}R$. We can also show that $\frac{\partial\Omega_1(\cdot)}{\partial\gamma_1} = -t_1^*\frac{\bar{c}(\Delta)}{C}A < 0$ and $\frac{\partial\Omega_1(\cdot)}{\partial\bar{c}} = t_1^*\left[1-\gamma(\cdot)\right]\frac{A}{C} > 0$. Using these, the equation above can be rewritten as:

$$\frac{d\Omega_1(\cdot)}{d\alpha_1} = \left[1-\gamma_1(\cdot)\right]\frac{\bar{c}(\Delta)}{C}\left[t_1^*\varepsilon(1-\gamma_1|\alpha_1) - [t_1^*+r]\varepsilon\left(\frac{\bar{c}}{C}|\alpha_1\right) + \frac{dt_2^*}{d\alpha_1}\right]A. \tag{34}$$

Welfare is decreasing in α_1 whenever $t_1^*\varepsilon(1-\gamma_1|\alpha_1) - [t_1^*+r]\varepsilon\left(\frac{\bar{c}}{C}|\alpha_1\right) + \frac{dt_2^*}{d\alpha_1} < 0$. ■