The costs of taxation in the presence of inequality

Katinka Holtsmark* and Åsmund Sunde Valseth†‡

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Abstract

This paper proposes an adjustment to the traditional theoretical definition of the marginal cost of public funds (MCF). The adjusted definition more precisely accounts for the distributional aspects of taxation than the standard MCF used in the current literature. Using the adjusted definition results in a higher MCF than using the standard definition in all allocations with income inequality. Moreover, due to its regressive distributional consequences, we show that the MCF of a uniform lump-sum tax is always greater than one when not combined with distortive taxes. With an optimal combination of a uniform lump-sum tax and a linear income tax, the MCF can also be greater than one. These findings are in contrast to the previous literature on the MCF using the standar definition which does not fully capture the distributional effects of taxation.

Keywords: Marginal cost of public funds, lump-sum taxes, public goods.

JEL-codes: H20, H40, H50.

*Corresponding author. Department of Economics, University of Oslo. k.k.holtsmark@econ.uio.no, +47 98640788.

†asmundsv@gmail.com.

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1 Introduction

Knowledge about the real costs of taxation is central in designing policy, including optimal taxation and public goods provision. The main contribution of this paper is to propose an adjustment to the standard way of theoretically defining the marginal cost of public funds (MCF). With the proposed adjustment, the MCF will precisely account for the distributional effects of taxation, in contrast to the standard definition of the MCF used in the current literature.

The MCF is the ratio between the marginal value of funds in the public sector and the marginal value of income in the private sector. Raising public revenue by taxation may affect aggregate welfare not only through reducing private income, but also by creating distortions in the economy and by changing the distribution of income in the population. If the MCF is different from one, the marginal value of funds in the public sector is either larger or smaller than the value of income in the private sector due to these effects of taxation – either distortionary of distributional. The MCF is a key variable in many different policy decisions. First of all, the costs of taxation are relevant in determining the optimal size of the public sector. More specifically, the MCF is in many countries explicitly included in public sector cost-benefit analysis. As an example, the MCF is set to 1.2 in Norwegian public sector cost-benefit analysis (Norwegian Ministry of Finance,, 2014).

We use a stylized model of taxation and public goods provision in line with for example Sandmo (1998) and Jacobs (2018) to illustrate the effects of taxation. In line with Mirrlees (1971), we introduce inequality by letting individuals be heterogeneous in their unobserved productivity leves, reflected in their wage rates. The unobserved heterogeneity in productivity across individuals creates the well-known trade-off for the government between efficiency and equity. Because poorer individuals have a higher marginal utility from income, raising public revenue by taxing labor income has favourable distributional effects compared to using only a uniform lump-sum tax. However, while the uniform lump-sum tax alone does not create distortions in the economy, an income tax distorts the labor supply and is thus costly in terms of efficiency. The MCF is intended to measure the marginal cost of raising public revenue including both distortionary and distributional effects of taxation.

In this framework, the standard theoretical definition of the MCF measures the shadow value of public funds in the second-best allocation relative to the *average marginal utility of income* in the population, see for example (Atkinson & Stiglitz, 1980; Sandmo, 1998; Gahvari, 2006; Kleven & Kreiner, 2006; Kreiner & Verdelin, 2012; Jacobs, 2018).[1] In other words, the standard definition of the MCF measures the total marginal cost of raising public revenue relative to the marginal cost of raising public revenue by use of a uniform lump-sum tax. This definition precisely captures the distortionary effects of taxation that has been well-known since Pigou (1947), since a uniform lump-sum tax alone does not have distortionary effects. However, the distributional effects of taxation in the second-best allocation are captured only to the extent that they are different from those of a uniform lump-sum tax. And since a uniform lump-sum tax is regressive – the poor bear a higher burden than the rich both in terms of budget share and in terms of utility – this means that this definition of the MCF will *underestimate* the distributional costs (or overestimate the distributional gain) of taxation on the margin.

To fully and precisely capture not only the distortionary effects of taxation, but also the distributional effects, the shadow value of public funds must be measured against the marginal cost of raising revenue using a hypothetical collection scheme that does not create distortions, and also does not affect the income distribution. We show that if the shadow value of public funds in the second-best allocation is measured relative to the *harmonic mean of the marginal utility of income* across the population, the MCF will indeed measure both the distortionary and the distributional effects precisely. That is because the harmonic mean of the marginal utility of income represents the marginal cost of raising public revenue using an individualized collection scheme (not dependent on observing productivity levels) that gives the same marginal welfare loss from all individuals in the economy.

We show that using the proposed adjusted definition of the MCF results in a measure of the MCF that is always higher than the standard definition, when there is income inequality. This finding is in line with the difference between using

[1] Jacobs (2018) proposes an adjustment to this measure not directly related to the adjustment proposed in this paper. A similar normalization has also been proposed by Håkonsen (1998), based on Diamond (1975).

the uniform lump sum tax – which is a regressive tax scheme – as a benchmark, and using the individualized tax scheme that we propose – which is distribution neutral – as the benchmark.

We also consider the MCF given our adjusted definition in the specific setting where the government is constrained to use only a combination of a uniform lump-sum tax and a linear income tax. We show that the MCF can be either larger than one, equal to one or smaller than one in the second-best allocation in this case. This findings are in contrast to the current literature, where the standard definition of the MCF has led to the conclusion that the MCF cannot be larger than one if the government has access to a uniform lump-sum tax (see for example Sandmo (1998)). The findings are also in contrast to the conclusion of Jacobs (2018) that the MCF must be equal to one when there is access to lump-sum taxation.

Furthermore, we show that the MCF will approach infinity if the total tax burden is such that the income of the poorest individual approaches zero. This result is completely in line with the intuition, we can in reality not use lump-taxation to a very large extent exactly for this reason. However, the result is in contrast to the conclusion using the standard measure of the MCF that it cannot go above 1 if a uniform lump-sum tax can be used.

Finally, we also consider optimal provision of a public good. The distributional characteristics of public goods must also be included in the analysis, see for example Atkinson and Stiglitz (1980), Sandmo (1998) or Wilson (1991). However, in parallel to how the distributional effects of taxation have not been precisely accounted for in the standard measure of the MCF, the distributional properties of public goods have also been imprecisely measured in the literature. We propose a new and precise measure of the distributional characteristics of a public good. Furthermore, we propose a way of expressing the adjusted Samuelson rule for optimal provision of a public good that distinguishes clearly between the distributional effects of taxation and the distributional consequences of providing the public good.

The second-best solution to a government's problem of public goods provision and redistribution depends on whether distribution of income and welfare across individuals matters or not. If distribution is irrelevant, and a distortive income tax is the only source for public funding, and the labor supply curve is upward sloping, a number greater than one should be applied as a corrective factor to see the real social costs of providing a public good (Atkinson & Stern, 1974; Stiglitz & Dasgupta, 1971; Ballard & Fullerton, 1992).

When distribution matters, the second-best is less straightforward because the distributional aspects of both public goods and the taxes used to fund them should be taken into account (Atkinson & Stiglitz, 1980; Wilson, 1991; Sandmo, 1998).

With regards to the public good itself, a progressive public good from which the poor derives greater marginal utility than the rich increases welfare more than a regressive good for which it is the other way around (when the aggregate marginal willingness to pay is the same). It follows that with concerns for distribution, an additional argument is raised against the conclusion that provision of public goods should always be lower in the second-best than in the first-best (Pigou, 1947).

To tackle these issues in a transparent and consistent manner, both in the evaluation of public projects and in the design of tax policy, one should use methods that distinguish precisely between the distributional properties of public goods themselves and of the taxes used for funding (Slemrod & Yitzhaki, 2001; Kaplow, 1996). We show that the standard MCF measure attributes parts of the distributional effects of taxation to the public good. This leads to both an underestimation of the funding costs and a corresponding misrepresentation of the distributional effects of public projects. The measures for the funding costs and for the distributional characteristics of public goods developed in this paper distinguish clearly between the two sources of distributional consequences of providing a tax-funded public good, and therefore solve this problem.

The paper is organized as follows. The model is presented in Section 2.1, which is followed by an illustration of the first-best allocation in the model economy as a benchmark in Section 2.2. In Section 2.3, we present our proposed adjusted measure of the MCF and illustrate how our measure compares to the standard measure of the MCF in the literature for a few specific cases of constraints facing the government. We further illustrate the potential consequences of using our adjusted measure of the MCF by use of numerical simulations in Section 3, and, finally, we conclude in Section 4.

2 A general measure of the MCF

Sandmo (1998, p. 366) defines the MCF as follows:

> [...] *the multiplier to be applied to the direct resource cost in order to arrive at the socially relevant shadow price of resources to be used in the public sector.*

We believe this definition is in line with the common understanding of the concept, and will therefore base our analysis on this definition.

Pigou (1947) first touched upon the need for a multiplier of this kind because of the potential distortionary effects of taxation. Among others, Sandmo (1998) points out, however, that we cannot disregard distributional concerns when we consider the effects of taxation. After all, many of the challenges we face when considering optimal taxation – and economic policy more generally – are closely related to inequality and redistribution. Sandmo (p. 366) writes:

> *Presumably the main reason why we have distortionary taxes is precisely the distributional problem; if issues of equity and justice could be disregarded altogether, the design of an efficient tax system would be a much less challenging task.*

Therefore, we must include the costs and benefits from taxation related to changes in the distribution of income, when calculating the MCF.

In the following, we will illustrate how the MCF is currently measured mathematically in the economics literature and discuss how this measure should be adjusted in order to fully and precisely account for the distributional effects of taxation. In order to do this, we must consider the problem facing a government that needs to raise public revenue. We attempt to illustrate our main point in a setting as simple as possible, in order to be able to focus on the key mechanisms.

2.1 The model

Let there be a total of n individuals, with subscript $i \in \{1, \ldots, n\}$ denoting individual i. The labor supply of individual i is given by h_i and is used in the production of both a private consumer good and a pure public good G. Consumption of the private good by individual i is given by c_i. Following Mirrlees (1971), the labor productivities of individuals, given by w_i for individual i (in units of the private consumer good), vary across individuals. The variation in labor productivity is the (only) source of inequality across individuals in the model.

Let q be the unit cost of producing the public good in terms of the private good. The private consumer good is the numeraire and the resource constraint of the economy is given by:

$$\sum_i c_i + qG = \sum_i w_i h_i. \tag{1}$$

Preferences are symmetric across individuals and represented by the following utility function:

$$u_i = u(c_i, h_i, G). \tag{2}$$

Let u_{ik} denote the derivative of the utlity function with respect to argument k for individual i. We assume that $u_{ic} > 0$, $-u_{ih} > 0$, $u_{iG} > 0$, and that the utility function is strictly quasi-concave. For our notation to be in line with previous literature, let $\lambda_i \equiv u_{ic}$ denote the marginal utility of consumption. With the private consumer good as numeraire, λ_i also represents the marginal utility of income (net of taxes). Define also the arithmetic average $\bar{\lambda} \equiv \frac{1}{n}\sum_i \lambda_i$.

2.2 The first-best allocation

We define the first-best allocation as the maximum of a utilitarian welfare function $W = \sum_i u_i$.[2] The first-best is given by the following $2n$ first-order conditions:

$$\sum_j u_{jG} = u_{ic}q \quad \forall i, \tag{3}$$

$$\sum_j u_{jG} = -\frac{u_{ih}}{w_i}q \quad \forall i. \tag{4}$$

[2]Using a simple utilitarian welfare function allows us to focus on our main point regarding the measure of the MCF under income inequality. The modeling choice is in line with for example Sandmo (1998) and Jacobs (2018) . For discussion of more general functional forms, i.e. $W = \sum_i W(u_i(\cdot))$, see for example Kaplow (2008, p. 42)

It follows from Equation (3) that the marginal utility of income must be equal for all individuals in the first-best. Furthermore, reorganizing Equation (3) shows that the first-best provision of the public good is defined by the original Samuelson rule:

$$\sum_i \frac{u_{iG}}{u_{ic}} = q. \tag{5}$$

The equation states that the individuals' total marginal willingness to pay must equal the unit cost of producing the public good (Samuelson, 1954).

Equation (4) ensures welfare-maximizing division of labor supply across individuals: The marginal utility of leisure must equal the marginal value of labor supply for all individuals.

2.3 The second-best allocation and the MCF

We assume that the government want to maximize $W = \sum_i u_i$. Both redistribution and provision of the public good requires collection of public revenue. The government cannot observe the productivity of each individual in the economy. Therefore, taxation must either be pure lump-sum taxation, or it must be based on taxation of wage income. The inability of the government to use an individual lump-sum tax based on the productivity level ω_i for individual i is the reason the costs of taxation arise.

Throughout this section, we will consider a relatively simple linear tax scheme where the government can tax wage income with the rate $t \in [0, 1]$ in addition to applying a uniform lump-sum tax (or transfer) given by b. $b > 0$ indicates a lump-sum tax, while $b < 0$ indicate a lump-sum transfer.[3]

We assume throughout that the wage rate for an individual is equal to the

[3] Under optimal taxation in the tradition of Mirrlees (1971), the actual costs of taxation in the second-best are lower than under taxation using only a linear income tax. However, the question of how to mathematically measure the MCF – which is the main issue investigated in this paper – is of course not dependent of the specific tax instruments available to the government. To keep the model as simple and transparent as possible, we therefore use this simple tax scheme when we illustrate the government's problem and when exemplifying the effects of our proposed adjustment to the measure of the MCF.

labor productivity w_i. Individual i solves the problem:

$$\max_{c_i,h_i} u(c_i, h_i, G) \quad \text{s.t.} \quad c_i = (1-t)w_i h_i - b.$$

The following first-order condition solves the problem and defines, together with the budget constraint and the time constraint, the demand functions for the consumption good and the labor supply functions:

$$-\frac{u_{ih}}{u_{ic}} = (1-t)w_i \tag{6}$$

$$\Rightarrow \quad c_i(t,b,G), \qquad h_i(t,b,G).$$

Let $v_i(t,b,G)$ be the indirect utility of individual i. It follows that:

$$\frac{\partial v_i}{\partial b} = -\lambda_i, \tag{7}$$

$$\frac{\partial v_i}{\partial t} = -\lambda_i w_i h_i, \tag{8}$$

$$\frac{\partial v_i}{\partial G} = u_{iG}. \tag{9}$$

The government's problem is given by:

$$\max_{t,b,G} \sum_i v_i(t,b,G) \quad \text{s.t.} \quad t\sum_i w_i h_i(t,b,G) + nb = qG, \tag{10}$$

where equation (10) represents the public budget constraint. The Lagrangian corresponding to the government's problem is:

$$L_g = \sum_i v_i(t,b,G) + \mu\left(t\sum_i w_i h_i(t,b,G) + nb - qG\right), \tag{11}$$

where μ represents the shadow value of public revenue. In the second-best allocation, this shadow value of public revenue must reflect the marginal cost of collecting public revenue.[4] Therefore, the multiplier μ in the government's Lagrangian is the

[4]This is true independently of the exact constraints facing the government. Thus, the multi-

natural starting point for measuring the MCF.

Towards the end of this section, we will solve the government's problem and discuss the size of the MCF for the linear income tax scheme available to the government. Before that, however, we will provide a general mathematical measure of the MCF that fully and precisely captures both the distortionary and distributional marginal costs (or gains) from taxation.

The shadow value μ represents the total marginal welfare cost of collecting public revenue, including both the distortionary and the distributional effects of taxation in addition to the direct marginal cost of lower private consumption. To arrive at the relevant multiplier in line with Sandmo's definition, this total marginal cost must be measured against the pure direct marginal welfare cost of lower private consumption. The result will be a multiplier that is 1 if there are no additional costs of taxation on the margin – the direct loss from reduced private consumption is the only cost of raising public revenue. Furthermore, the multiplier is below one if there are gains from taxation on the margin – either in terms of lower distortions or a more favorable income distribution. Finally, the multiplier is larger than one if there are costs to taxation on the margin either in terms of distortions or distribution.

The key to the difficulty of measuring the MCF when there is inequality is that there is no obvious measure of this direct cost of lower private consumption. In a model with identical consumers, consumption is typically the same across all consumers in the second-best and the marginal resource cost can be measured by $\lambda_i = \lambda$ for all i. When the marginal utility of income is the same across all individuals, this λ represents the direct resource cost of collecting one dollar, irrespective of which individual it is collected from. The MCF can then be measured by μ/λ, which is indeed the measure of MCF in literature with identical individuals when the approach of Atkinson and Stern (1974) and Stiglitz and Dasgupta (1971) is adopted.

With heterogeneous labor productivity across consumers, the marginal utility of income, λ_i, will typically *not* be identical across individuals in the second-best, because redistribution is costly. It is then not clear how to represent the cost of

plier μ reflects the marginal cost of collecting public revenue also in the case where the government is allowed to choose from either a wider or a more narrow set of tax schemes.

reduced private consumption, because the cost depends on which individual that consumes less.

The currently standard way of mathematically defining the MCF in the literature is by using the arithmetic mean, $\bar{\lambda}$, as a measure of the marginal welfare cost of reduced private consumption (Sandmo, 1998; Gahvari, 2006; Kleven & Kreiner, 2006; Kreiner & Verdelin, 2012; Jacobs, 2018). Correspondingly, the MCF has been measured by the ratio $\mu/\bar{\lambda}$. However, as we will show in the following, this measure of the MCF does not precisely measure the true marginal costs of taxation.

The reason for the misrepresentation of the true marginal costs of taxation that arise when using the standard measure of the MCF is that the arithmetic mean, $\bar{\lambda}$, reflects the cost of collecting tax revenue in a manner that changes the income distribution. The arithmetic mean, $\bar{\lambda}$, reflects the marginal cost of collecting tax revenue by reducing consumption for each individual by the same amount, irrespective of their marginal utility of consumption, λ_i. The problem is that this collection scheme *increases* relative after-tax income inequality because it reduces income relatively more for low-income individuals than for high-income individuals. Or in other words: It increases inequality because it reduces utility more for the poor than for the rich. The costs measured by $\bar{\lambda}$ thus include not only the direct resource costs of taxation, but also the welfare costs resulting from this increased inequality. When the total marginal cost of revenue collection, μ, is measured relative to $\bar{\lambda}$, the resulting measure of the MCF does therefore not capture the true distributional costs of taxation.

For the MCF to precisely capture not only the distortionary effects of taxation, but also the distributional effects, *one must measure μ relative to the cost of collecting one dollar from the private sector in a way that does neither decrease nor increase inequality.* In other words, one must measure μ relative to the welfare cost of collecting one dollar using a hypothetical non-distortionary lump-sum tax affecting the utility of all individuals in the economy identically. This property of a tax scheme is what we define as distribution-neutrality, as formalized in the following definition:

Definition 1 (**Taxation and distribution**). *A tax scheme is distribution-neutral if the tax payments (or transfers) are allocated so that all individuals experience* *the same loss (or gain), measured in utility. A tax scheme is progressive if the tax payments (or transfers) are allocated so that poor individuals experience a lower loss (or a higher gain) than the rich, measured in utility. A tax scheme is regressive if the tax payments (or transfers) are allocated so that the poor experience a higher loss (or a lower gain) than the rich individuals, measured in utility.*

We believe this definition of distribution neutrality is in line with the common understanding of how to evaluate changes in the income distribution. Given this definition, our first proposition provides a measure for the MCF that fully captures both distortionary and distributional effects of taxation.

Proposition 1 (**The marginal cost of public funds**). *The marginal cost of taxation – including both distortions and changes in the income distribution – are precisely captured if and only if the marginal cost of public funds (MCF) is measured as follows:*

$$MCF = \frac{\mu}{\theta} \quad \textit{where} \quad \theta \equiv \frac{n}{\sum_i \frac{1}{\lambda_i}}. \tag{12}$$

Proof. For the measure of the MCF to be in line with the definition of Sandmo (1998), it must measure μ relative to the marginal welfare cost of a distribution-neutral tax scheme. λ_i represents the marginal welfare cost for individual i of a reduced income. $1/\lambda_i$ thus represents the income reduction resulting in a welfare loss of one unit. Correspondingly, $\frac{1}{n}\sum_i(1/\lambda_i)$ represents the average income reduction corresponding to a one unit reduction in welfare. Finally, θ, which is the harmonic mean of λ_i across the population, then represents the welfare loss following the collection of one dollar by use of an individual lump-sum tax set such that the utility loss is the same for all individuals.[5][6] □

The standard measure of the MCF in the literature, $\mu/\bar{\lambda}$, measures the welfare costs of taxation against the cost of collecting one dollar through a lump-sum

[5]For other applications of the harmonic mean, see for example Komic (2011) and Coggeshall (1886).

[6]With a social welfare function of the type mentioned in footnote 2, the corresponding approach would imply to define θ as the harmonic mean of β.

tax collecting $1/n$ from each individual in the economy (a ULS tax). Measured in utility, this tax puts a higher burden on poor individuals compared to the rich, and increases inequality. It follows that this measure of the MCF will misrepresent the true costs of taxation because the distributional effects are misrepresented. The easiest way to see this is perhaps to think of the MCF of the ULS tax itself. With $MCF = \mu/\bar{\lambda}$ the use of a ULS tax (without other tax instruments used at the same time) would result in an MCF measured to 1, even though a ULS tax in reality clearly represents a *regressive* tax scheme that increases inequality.

Moreover, it is intuitive that the distributional cost of a ULS tax depends on the initial income distribution. To illustrate this, consider a change in the initial distribution that increases income inequality by *increasing* income of an individual i with $\lambda_i < \bar{\lambda}$ with Δ, while *decreasing* income of an individual j with $\lambda_j > \bar{\lambda}$ with the same amount, Δ. Recall that the marginal utility of income for each individual is decreasing in income itself. Thus, it is clear that the welfare costs of collecting one dollar through a ULS tax is higher after this redistribution toward increased inequality than in the economy with the initial income distribution. An MCF defined as μ/θ will reflect this finding. In contrast, the increased welfare costs of a ULS tax after the described reallocation of income will *not* be reflected in the MCF if it is measured by $\mu/\bar{\lambda}$, which will be equal to one in both cases.

In the next section, we will illustrate the difference between the currenty used standard MCF measure and the measure that we propose in Proposition 1 by investigating the marginal cost of taxation in the second-best allocation following from the solution of the government's problem presented earlier in this section (see Equation 11).

2.3.1 The MCF with uniform lump-sum tax and linear income tax

The government solves the problem represented by the Lagrange function in Equation 11.

The three first-order conditions of the government become:

$$\sum_i \lambda_i = \mu \left(t \sum_i w_i \frac{\partial h_i}{\partial b} + n \right), \tag{13}$$

$$\sum_i \lambda_i w_i h_i = \mu \left(\sum_i w_i h_i + t \sum_i w_i \frac{\partial h_i}{\partial t} \right), \tag{14}$$

$$\sum_i u_{iG} = \mu \left(q - t \sum_i w_i \frac{\partial h_i}{\partial G} \right). \tag{15}$$

Together with the public budget constraint, these three first-order conditions give the second-best levels of the ULS tax, b, the linear income tax, t and public consumption, G, in addition to the shadow value of public revenue, μ.

Some reorganizing of the first-order conditions is useful to clarify their interpretation. First, define the distributional characteristics of a public good, in line with those of a tax scheme from Definition 1:

Definition 2 (**Public goods and distribution**). *A public good is distribution neutral if increased provision of the good gives the same increase in utility for all individuals. A public good is progressive (regressive) if increased provision of the good gives a higher (lower) increase in utility for the poor individuals than for the rich individuals. This definition corresponds to:*

$$Cov\left(\frac{1}{\lambda_i}, u_{iG}\right) \begin{cases} < 0 & \Rightarrow \quad G \textit{ is progressive} \\ = 0 & \Rightarrow \quad G \textit{ is distribution neutral} \\ > 0 & \Rightarrow \quad G \textit{ is regressive} \end{cases}$$

The covariance between $1/\lambda_i$ and u_{iG} represents the distributional characteristics of the public good and has a straight forward interpretation: $1/\lambda_i$ represents the income increase necessary for individual i to get an increase in utility of one unit – it will be higher for a rich (highly productive) individual than for a poor (less productive) individual in our model. Hence, the covariance represents the strength of the (positive or negative) association between individuals' income (net of taxes) and their marginal utility from the public good.

This definition of the distributional characteristics of a public good will be helpful in interpreting the conditions determining the second-best provision of the public good. Note that by the definition of covariance, we have that $\frac{1}{n}\sum_i u_{iG}\sum_i \frac{1}{\lambda_i} = \sum_i \frac{u_{iG}}{\lambda_i} - n\ Cov(u_{iG}, \frac{1}{\lambda_i})$. Dividing Equation (15) by θ and reorganizing, we get:

$$\sum_i \frac{u_{iG}}{\lambda_i} - n\ Cov\left(u_{iG}, \frac{1}{\lambda_i}\right) = \frac{\mu}{\theta}\left(q - t\sum_i w_i \frac{\partial h_i}{\partial G}\right). \tag{16}$$

The second-best provision of the public good depends on three factors in addition to the MCF$\equiv \mu/\theta$:

- The direct resource cost of providing the good, q.
- The effect of provision on labor supply (and hence public revenue): If the income tax, t, is positive, the equilibrium labor supply is inefficiently low. In this case, a positive (negative) effect of increased provision of the public good on labor supply increases (decreases) efficiency in the economy and therefore also the optimal second-best provision.
- The distributional characteristic of the public good: If the good is progressive (regressive) according to Definition 2, the second-best provision is larger (smaller) than compared to the case where the good is distribution-neutral.

Both the direct resource cost and the increase (or decrease) in public revenue resulting from increased provision is multiplied by the MCF.

Here, it is useful to to make a comparison to the size the MCF would take in the second-best allocation following from the government's first-order conditions provided by equations (13) – (13) if one were to use the standard measure of the MCF, $\mu/\bar{\lambda}$. As already discussed, the standard measure misrepresents the distributional costs (or gains) of collecting public revenue. In a similar manner (and as a mechanical consequence of this misrepresentation) the standard measure also misrepresents the welfare effects of provision of the public good through underestimating the distributional gains (or costs) of the public goods providsion. The standard measure measures the distributional characteristics of the good by the covariance between the marginal utility of income, λ_i and the marginal *willingness* *to pay* for the good (see Sandmo (1998)). Although higher willingness to pay of course will be a result of higher marginal utility – all else equal – this is not a precise measure of the distributional characteristics. The reason is that higher willingness to pay will also be a result of higher income: Because of this relationship between willingness to pay and income, the covariance implicitly used as a measure of the distributional effects of public goods provision when the standard measure of the MCF is used, will measure any public good to be less progressive (or more regressive) than what it is according to our Definition 2.

An example illustrating the misrepresentation of the distributional effects of public good provision, is the case where $Cov(\frac{u_{iG}}{\lambda_i}, \lambda_i) = 0$, representing a distribution-neutral public good with the standard setup. The pure mechanical relationship between λ_i and the willingness to pay, $\frac{u_{iG}}{\lambda_i}$ ensures that $Cov(\frac{u_{iG}}{\lambda_i}, \lambda_i) = 0$ (only) if the poor have higher marginal utility from provision of the good than the rich: The good is in fact *progressive*. Or the other way around: A public good that is distribution neutral according to our Definition 2 will result in $Cov(\frac{u_{iG}}{\lambda_i}, \lambda_i) < 0$ because the willingness to pay for the rich will be higher than that for the poor.

To summarize: When using the standard measure of the MCF the distributional characteristics both of the public good and of taxation will be measured in a way that is not consistent with the welfare effects of distributional changes. The result is that the costs of collecting public revenue are measured to be lower than they are in reality. And correspondingly, the benefits of public good provision are underestimated. The resulting allocation of resources to provision of the public good is of course independent of how the first-order condition is reorganized and interpreted. However, the standard measure of the MCF used in the current literature cannot be used in any other situation without making an adjustment for the misrepresentation of the distributional effects of taxation.

Now, we can use Equation (13) and (14) to find two expressions for the MCF. First, dividing Equation (13) by θ and reorganizing, we get:

$$\frac{\mu}{\theta} = \frac{1 - Cov\left(\frac{1}{\lambda_i}, \lambda_i\right)}{1 + \frac{1}{n} t \sum_i w_i \frac{\partial h_i}{\partial b}}, \tag{17}$$

representing the MCF when the ULS tax is used to increase public revenue.

To interpret this expression, notice first that $Cov(\frac{1}{\lambda_i}, \lambda_i)$ (which is ≤ 0) represents the distributional effects of an increase in the ULS. If $\lambda_i = \bar{\lambda}$ for all i – there is no inequality – the covariance will be 0 and the ULS tax will be distribution neutral according to Definition 1. As inequality increases, the absolute value of the covariance will increase: The ULS tax becomes more costly because low-income individuals take a higher loss. In Equation (17), we see that the MCF is indeed increasing in the absolute value of the covariance.

Next, the term $\frac{1}{n}t\sum_i w_i \frac{\partial h_i}{\partial b}$ represents the average effect on public revenue of an increase in the ULS tax. This is the pure income effect of increased taxation – which is efficiency enhancing if $t > 0$ and the labor supply is therefore inefficiently low.

To summarize, it is clear from Equation (17) that the MCF for the ULS tax can be smaller than, equal to or larger than one, depending on the two factors:

- The income distribution: Larger inequality increases the (distributional) costs of ULS taxation and therefore the MCF.
- The income effect on labor supply: The higher the marginal income tax, t, and the stronger the effect of the ULS tax on labor supply, the higher is the efficiency gain from ULS taxation and therefore the lower is the MCF.

Here, it may be useful to note that Equation (17) is similar, but not identical, to Equation (17) in Sandmo (1998), which describes the MCF for a ULS tax with MCF= $\mu/\bar{\lambda}$. The main difference between the two expressions is – not surprisingly – that the MCF in Sandmo (1998) does not include the distributional costs of ULS taxation. This is a direct consequence of measuring the MCF by the shadow value of public funds relative to the marginal cost of ULS taxation ($\bar{\lambda}$). By definition, this standard measure of the MCF excludes the distributional costs of the a ULS tax.

Finally, we can use Equation (14) (divide by θ and reorganize) to represent the

MCF when the linear income tax is used on the margin:

$$\frac{\mu}{\theta} = \frac{\frac{1}{n}\sum_i w_i h_i - Cov\left(\frac{1}{\lambda_i}, \lambda_i w_i h_i\right)}{\frac{1}{n}\sum_i w_i h_i + \frac{1}{n} t \sum_i w_i \frac{\partial h_i}{\partial t}}, \tag{18}$$

$Cov(\frac{1}{\lambda_i}, \lambda_i w_i h_i)$ measures the marginal welfare effects of the distributional changes from the tax, sine $\lambda_i w_i h_i$ is the marginal increase in tax payment by individual i for the income tax t. Moreover, the term $\frac{1}{n} t \sum_i w_i \frac{\partial h_i}{\partial t}$ measures the distortionary effects of the income tax, depending on the effect of the tax on labor supply.

Again, we see that the MCF may be smaller than, equal to or larger than one, and we can summarize the determinants of its size as follows:

- The income distribution: The more strongly the marginal income tax affects the income distribution, the lower are the costs of taxation by use of this tax, and thus the MCF.

- The (substitution and income) effects on labor supply: The stronger the distortionary effect on labor supply, the higher the costs of taxation, and thus the MCF.

Again, comparing with Sandmo (1998), we see that Equation (18) is similar to Sandmo's Equation (19). Qualitatively, this measure of the MCF in Sandmo (1998) includes the same two determinants. However, the distributional effects of the tax are not measured precisely: Sandmo's measure includes the distributional effects relative to those of the ULS tax. The result is that his measure of the MCF overestimates the progressivity of the income tax. The result is – again – that the MCF is underestimated.

Together with the public budget constraint, the three first-order conditions of the government determine the second-best income tax rate, lump-sum tax and provision of the public good. Moreover, given these choices, the MCF is determined. In the second-best the MCF is the same irrespective of which tax instrument is used, so we must have:

$$\frac{\mu}{\theta} = \frac{1 - Cov\left(\frac{1}{\lambda_i}, \lambda_i\right)}{1 + \frac{1}{n} t \sum_i w_i \frac{\partial h_i}{\partial b}} = \frac{\frac{1}{n}\sum_i w_i h_i - Cov\left(\frac{1}{\lambda_i}, \lambda_i w_i h_i\right)}{\frac{1}{n}\sum_i w_i h_i + \frac{1}{n} t \sum_i w_i \frac{\partial h_i}{\partial t}}.$$

Our next proposition summarizes our findings on the size of the MCF when the government can use a linear income tax (in combination with a ULS).

Proposition 2 (**MCF with optimal taxation**). *With an optimal combination of a linear income tax, t, and a uniform lump-sum tax, b,*

- *The MCF can be smaller than, equal to or larger than 1 in the second-best allocation: MCF* $\lesseqqgtr 1$.
- *The MCF approaches infinity when the ULS is approaching the income of the poorest individual in the economy: MCF$\rightarrow \infty$ as $tw_jT + b \rightarrow w_jT$ for j such that $w_j = \min(w_1, \ldots, w_n)$.*

Proof. MCF $\lesseqqgtr 1$ follows directly from the three first-order conditions.
As $b + tw_jT \rightarrow w_jT$ for j such that $w_j = \min(w_1, \ldots, w_n)$ we get $\lambda_j \rightarrow \infty$.
As $\lim_{\lambda_j \rightarrow \infty} Cov(\frac{1}{\lambda_i}, \lambda_i w_i h_i) = -\infty$ it follows from Equation (17) and (18) that MCF$\rightarrow \infty$ in this case. □

In Section **??** we provide a few numerical examples illustrating the size of the MCF under optimal taxation. In a few special cases, it is possible to be more specific also on the theoretical size of the MCF. Our next result concerns the case where there is no income tax and therefore no distortions in the economy:

Proposition 3 (**MCF with ULS tax**). *When a uniform lump-sum tax is the only tax implemented, and at least two of the n individuals have different productivities, then*

- *The MCF is always larger than one in the second-best allocation: $MCF > 1$.*
- *The MCF approaches infinity ad the size of the ULS approaches the income of the poorest individual in the economy: $MCF \rightarrow \infty$ as $b \rightarrow w_jT$ for j such that $w_j = \min(w_1, ..., w_n)$.*

Proof. $MCF > 1$ follows directly from Equation (17) when $t = 0$ and there exist $\lambda_i \neq \bar{\lambda}$. As $b \to w_j T$ for j such that $w_j = \min(w_1, \ldots, w_n)$ we get $\lambda_j \to \infty$. As $\lim_{\lambda_j \to \infty} Cov(\frac{1}{\lambda_i}, \lambda_i) = -\infty$ it follows from Equation (17) that MCF$\to \infty$ in this case. □

With the measure of the standard measure of the MCF, the MCF$= 1$ when $t = 0$ and MCF< 1 when $t > 0$.

However, as propositions 2 and 3 illustrates, the "fact" that the MCF≤ 1 – and equal to 1 if only lump-sum taxes are used – is based on a measure of the MCF that is not in line with the common understanding of the concept. More specifically, these conclusions are based on a measure of the MCF that underestimates the negative – or overestimates the positive – effects of taxation on inequality.

Our final result concerns provision of a distribution-neutral public good in the special case with no income tax:

Proposition 4 (**Optimal provision of a distribution neutral public good**). *When the public good on the margin is distribution neutral and a uniform lump-sum tax is the only tax implemented, provision of the public good should be lower than prescribed by the Samuelson rule (which corresponds to the first-best provision).*

Proof. When $t = 0$, MCF$= \mu/\theta > 1$ according to Proposition 3. From Equation (16) it follows that

$$\sum_i \frac{u_{iG}}{\lambda_i} = \frac{\mu}{\theta} q$$

when $Cov(u_{iG}, 1/\lambda_i) = t = 0$. When the total willingness to pay for the public good is higher than according to the Samuelson rule, it means that provision is lower. □

3 Numerical illustrations

To illustrate some of the results of the present paper, simulations of a numerical model with 10 individuals (for examples representing the 10 deciles of the income distribution) were carried out. I should be emphasized that the numerical

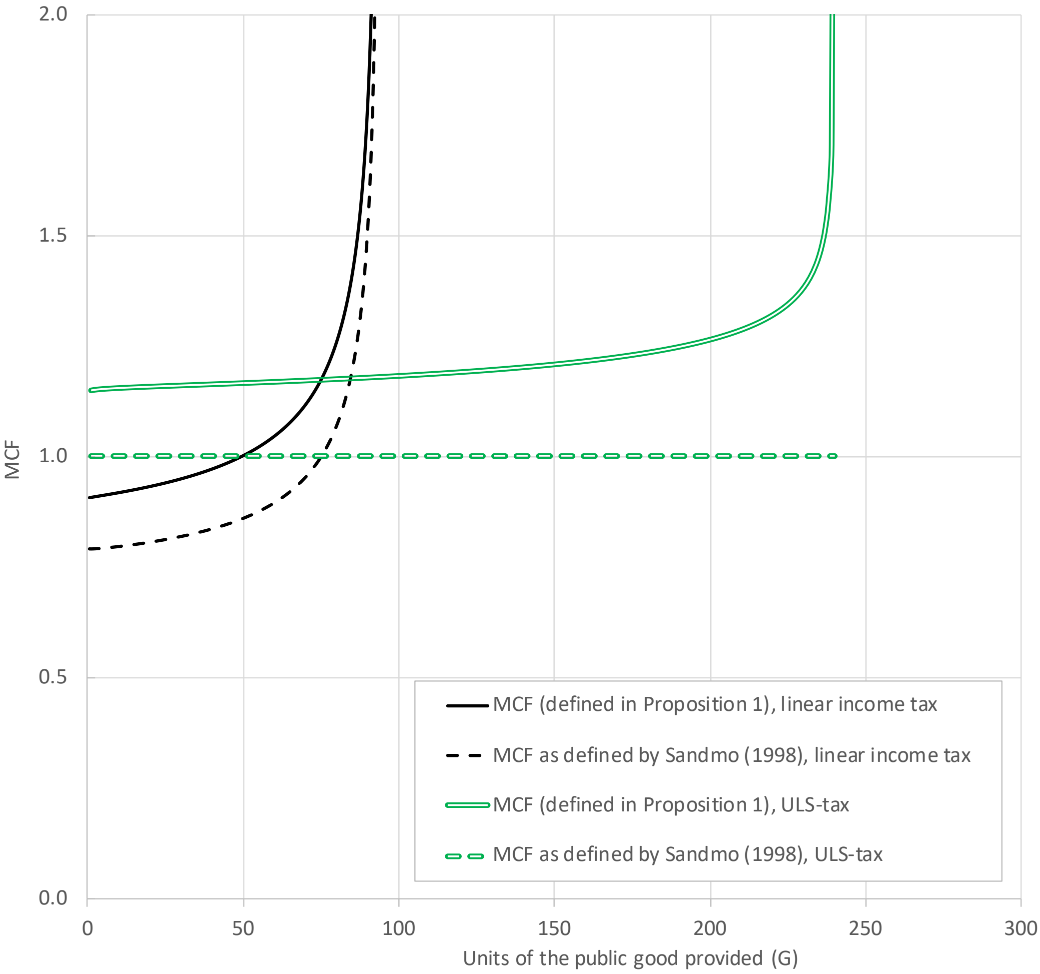

Figure 1: The MCF for different levels of public good provision funded with a lump-sum tax or a linear income tax, respectively. The green solid double-line and the solid black line show the MCF based on our measure presented in Proposition 1 for the case with a linear income tax and a uniform lump-sum tax, respectively. The green broken double-line and the broken black line show the MCF based on the measure proposed by Sandmo (1998) and adopted in the literature, for a linear income tax and a uniform lump-sum tax, respectively.

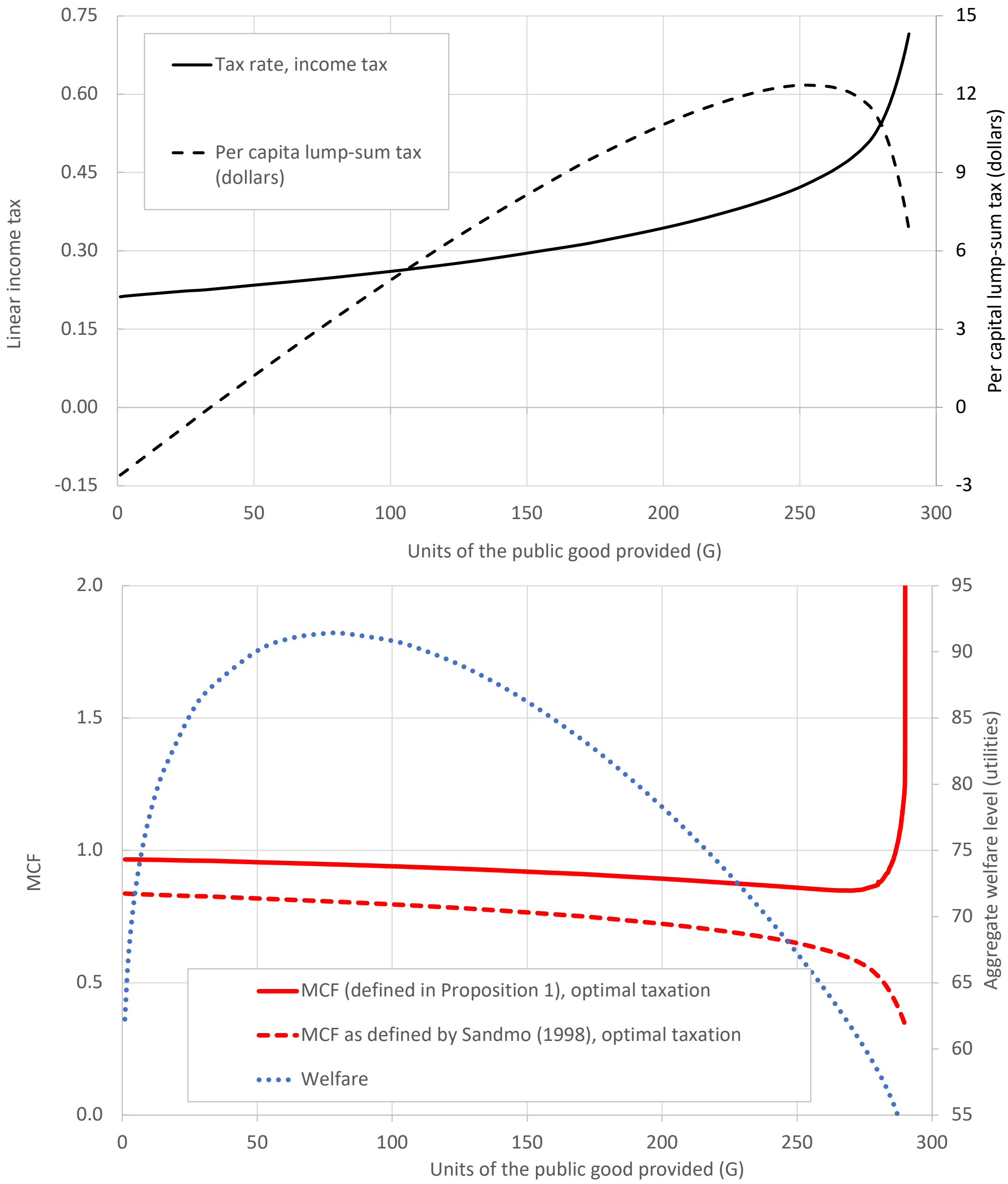

Figure 2: A numerical example with different levels of public good provision funded with an optimal combination of a uniform lump-sum tax and a linear income tax. The upper diagram shows the tax levels and the lower shows the MCF levels as well as the welfare. Maximum welfare is reached when $t = 0.25$ and $b = 3.30$.

simulations are made for illustrative purposes to provide intuition to the results. Because the parameters of the numerical model and the functional form of the utility functions lack empirical basis, the results of the simulations could not provide information on the likely level of MCF. For detailed description of the applied model and all numerical assumptions, see Appendix A.

The utility functions given in equation (2) were specified as CES-functions. The elasticity of substitution between leisure and private consumption, se to 1.25, was calibrated to give labor supply elasticities in the interval 0.15-0.20, as this corresponds to results from the empirical literature. The public budget constraint is binding, which means that in all simulations the entire net public revenue is used for production of the public good or transfers (negative lump-sum taxes).

Figure 1 shows two numerical examples. In one public good provision is funded with a ULS tax, in the other with a linear income tax. The horizontal axis measures provision of the public good. The vertical axis measures MCF. The green solid double-line shows the MCF of a uniform lump-sum tax when the ULS tax is the source of public funding and the MCF is measured according to Proposition 1. Because a ULS tax is regressive, and in accordance with Proposition 3, the MCF with ULS is found to be greater 1 at all tax levels and converges towards infinity when the ULS tax becomes close to 24, because the individual with lowest productivity has a time endowment $T = 24$ and a productivity $w_1 = 1$. This is consistent with the second bullet point of Proposition 3.

For comparison, the MCF of a ULS tax as measured by Sandmo (1998) is given by the green, broken double-line. As already discussed, Sandmo's measure will give MCF$= 1$ in the case where the ULS is the only tax implemented, see for example Sandmo (1998, p. 372).

The black solid line shows the MCF of a linear income tax when the this tax is the only source of public funding and the MCF is measured according to Proposition 1. For tax rates below 0.34, the MCF of the income tax is less than 1. For comparison, the black broken line shows MCF measured as by Sandmo (1998).

Figure 2 shows a numerical example where different levels of public good provision are funded with an optimal combination of a ULS tax and a linear income tax. For zero and low levels of public good provision the lump-sum tax is negative. In accordance with Proposition 2, the lower diagram shows that the MCF with our definition and with optimal taxation could be both below, equal to, and higher than 1. Consistent with the second bullet point in Proposition 2, the MCF converges toward infinity as the total tax bill becomes close to the maximum possible income level of the poorest individual in the economy. However, because the ULS tax now is combined with an income tax, this happens at a higher level of public good provision.

The broken line of the lower diagram (Figure 2) shows that the MCF with measured as by Sandmo (1998) is always below 1 with optimal taxation, in accordance with findings i several papers, see for example Sandmo (1998, p. 376), Jacobs (2018) and Håkonsen (1998).

4 Conclusion

Decisions about the provision of public goods should be based on a clear understanding of both the benefits of public goods and the costs of funding them with taxes. If there is inequality and the benefits of public goods are measured in monetary terms, attention must be paid to the fact that money has greater value to the poor than to the rich. When redistribution is costly, a corrective factor has to be applied to the willingness to pay for a public good in order to provide more of the goods that benefit the poor and less of the goods that benefit the rich. For the funding, both the efficiency loss from distortive taxation and the distributional properties of taxes have to be taken into account. The MCF should be a corrective factor that captures both these aspects of taxation, to be applied to the direct cost of providing a public good to arrive at the actual cost to society of providing it.

The main contribution of this paper is to establish a theoretical definition for the MCF that more correctly accounts for the distributional aspects of taxation than the standard theoretical measured used in the current literature. Correspondingly, the paper also proposes a corrective factor for the willingness to pay for public goods that accurately captures the distributional characteristics of a public good.

Sandmo (1998, p. 366) defined the MCF as "the multiplier to be applied to the direct resource cost in order to derive the socially relevant shadow price of resources to be used in the public sector" . This is in accordance with most work within this field and this definition has been our starting point. However, we show in this paper that the MCF measure of the previous literature attributes parts of the distributional effects of taxation to the measure for the distributional characteristics of the public good. The reason for this failure to account precisely for the distributional effects of taxation in the way the MCF is mathematically defined is that the literature uses the regressive ULS tax as the benchmark against which the distributional effects of other taxes are measured.

Appendices

A Description of the model used in the numerical example

In the numerical model used for illustrative purposes in section 4, there are $n = 10$ individuals with the following utility function:

$$u_i = x\left(\alpha^{1-\rho}c_i^{\rho} + \beta^{1-\rho}l_i^{\rho} + \gamma^{1-\rho}G^{\rho}\right)^{\frac{1}{\rho}} \tag{A.1}$$

where x, α, β, γ, and ρ are parameters. Define

$$s = \frac{1}{1-\rho}. \tag{A.2}$$

s is the elasticity of substitution between consumption and leisure and was set to 1.25. The parameters α and β where calibrated such that $\alpha^{1-\rho} = 0.3$ and $\beta^{1-\rho} = 0.7$.

The wage rates (for example \$/hour) were:

$$\begin{aligned} w_1 &= 1 \\ w_i &= (w_{i-1})^g \qquad \text{if} \quad i > 1, \end{aligned}$$

were g is a parameter which was calibrated to a value that gave a Gini-index of gross income distribution of 27.5 in second-best. Both the individual wage rates and individual before and after tax incomes in second-best are shown in Figure 2.

The complete list of applied parameter values follows:

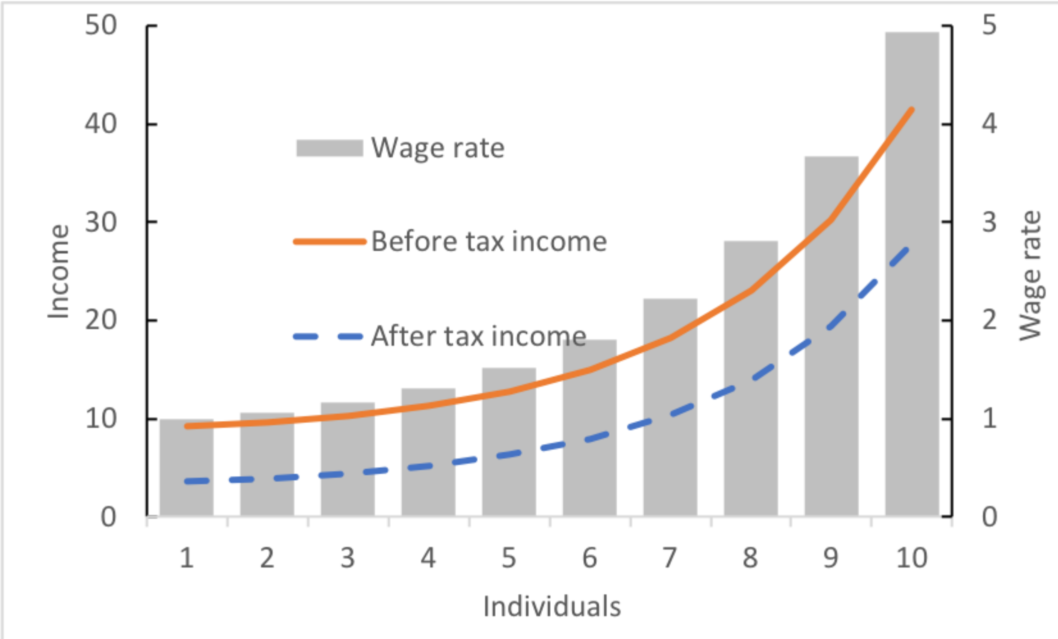

Figure 3: Wage rates w_i and gross and net incomes in second-best, for $i = 1, ..., 10$.

Parameter	Value	Parameter	Value
x	0.25	γ	0.10
α	0.22	β	0.64
ρ	0.20	s	1.25
T (hours available)	24.00	g	1.03

second-best was achieved with a tax rate $t = 0.25$ combined with an effective lump-sum tax $b =$ \$3.30. Individuals spent between 8.4 and 9.2 time units working of total time endowments of $T = 24$ time units. A tax $t = 0.41$ gave welfare maximum if the lump-sum transfer was set to zero. endappendices

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