Direct vs. Indirect Taxation – A Revisit

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Abstract

We show in this paper that the marginal cost of public funds (MCF) does not depend on whether public revenue is collected by taxation of consumer goods or by taxation of income from factors supplied on the market by households. Atkinson and Stern (1974) concluded in their seminal paper that "[...] whether the Conventional Rule provides an under- or over-estimate depends on the choice of taxed good [....]." Although this conclusion can be interpreted in different ways, it has created confusion in the literature on the MCF and has been the basis for literature arguing that the standard measure of the MCF has weaknesses and should be replaced by alternative measures (Jacobs, 2018; Jacobs & de Mooij, 2015; Håkonsen, 1998). We show that the choice between direct and indirect taxation does not affect the MCF.

Keywords: marginal cost of public funds, taxation, Samuelson rule
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1 Introduction

Atkinson and Stern (1974) made an important contribution to the literature on public finance. However, in their discussion, they state that "[...] whether the Conventional Rule provides an under- or over-estimate depends on the choice of taxed good [....]." This statement has been given the interpretation that the marginal distortionary cost of taxation depends on the choice between direct taxation – taxation of factor income

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– and indirect taxation – taxation of consumer goods. This interpretation has caused confusion in the literature on the MCF. In Håkonsen (1998), Jacobs and de Mooij (2015), and Jacobs (2018), the claimed sensitivity of the marginal cost of taxation with respect to the choice between direct and indirect taxation is emphasized as a serious weakness of the traditional MCF concept. This alleged weakness of the concept is used as an argument in favor of using alternative measures of the cost of public funds. The contribution of this paper is to disentangle the misunderstanding concerning the effect on the costs of taxation of the choice of taxed goods and show that the marginal cost of public funds (MCF) is not influenced by the choice between direct and indirect taxation.

The size of the MCF has consequences in a large range of policy areas. Importantly, the MCF is an important factor in determining optimal public spending. For example, the MCF is typically used as a parameter in cost-benefit analyses for public projects, and the size of this parameter will typically strongly affect the perceived costs of such projects. Moreover, the determinants of the MCF are central to the design of a good tax scheme. Correct, precise, and reliable knowledge regarding the determinants of the MCF is therefore important to policy makers. The confusion in the literature is therefore important to disentangle.

We use the same stylized model as is used in by Atkinson and Stern (1974). The model allows investigation of second-best taxation and provision of public goods in a simple, yet fairly general, setting with a set of identical households optimizing their consumption and supply of labor and other production factors on the market. We show how the costs of taxation affect second-best provision of the public good, and that these costs – and their consequences for public goods provision – are independent of the choice between direct and indirect taxation.

The intuition behind our result is simple. A consumption tax has the same effect on economic behavior as an income tax, *as long as the relative prices are the same.* Taxation affects the value for households of supplying factors – for example labor – in the market. More precisely, the distortionary costs of taxation arise as a result of the tax wedges between relative consumer prices and relative producer prices. However, the tax wedge, and thus the distortions, are exactly the same independently of whether taxation lowers factor income or increases consumer prices, as long as the changes in the relative prices are the same. Therefore, the distortionary costs of taxation, and the MCF, are not affected by the choice between direct and indirect taxation.

An important starting point for Atkinson and Stern (1974) was the discussion in Pigou (1947) on the optimal size of the public sector when taxes cause "damage" to economic efficiency. According to Pigou (1947, p. 34) such "damage" means that

> "[...] expenditure ought not to be carried so far as to make the real yield of the last unit of resources expended by the government equal to the real yield of the last unit left in the hands of the representative citizen."

The original Samuelson rule for provision of public goods states that first-best public expenditure is such that the sum of the marginal rates of substitution between public goods and private consumption equals the marginal rate of transformation (Samuelson, 1954). A reasonable interpretation of Pigou's statement is that the Samuelson rule represents inefficiently high public expenditure when taxation is distortionary. Stiglitz and Dasgupta (1971) showed that his conclusion must be nuanced. They found that (p. 159)

> "[...] if there is only one consumption good and one factor, labor, whether the Conventional Rule represents an under or over supply depends simply on whether the supply curve of labor is backward bending or upward sloping."[1]

In contrast, Atkinson and Stern (1974, p. 126) stated that

> "[...] Stiglitz and Dasgupta failed to point out that whether the Conventional Rule provides an under- or over-estimate depends on the choice of taxed goods [....]."

When the government solves the problem of optimal taxation, taxes on all goods and factors will not be determined, one can always normalize the tax on one good without affecting the resulting second-best allocation. The interpretation that has been given to the above statement in Atkinson and Stern (1974) by later literature (Håkonsen (1998), Jacobs and de Mooij (2015), Jacobs (2018)) is that the distortionary costs of taxation depend on this normalization. More precisely, that these costs depend on whether consumer goods or factor supplied in the market are taxed. In other words, it has been interpreted as a claim that – all else equal – the cost of an incremental increase in the provision of a public good is lower if the good is financed by a labor income tax than if a tax on consumption is used (in an economy with only normal goods).

We show in this paper that this conclusion is not valid. With the model applied by Atkinson and Stern (1974), the distortionary effects of a consumption tax are exactly the same as the distortionary effects of an income tax. The two tax instruments also lead to the same MCF and to the same modification of the Samuelson rule.

[1] Atkinson and Stern (1974) and Stiglitz and Dasgupta (1971) used the term *Conventional Rule* for what we today usually label the *Samuelson rule* attributable to Samuelson (1954).

To our knowledge, the confusion regarding the sensitivity of the MCF to the choice of taxed goods has not been disentangled in the literature. On the contrary, a wrongful conclusion concerning this sensitivity has been adopted in later literature. Boadway and Keen (1993, p. 473) write that

> "different choices of untaxed numeraire may give rules for the optimal provision of the public good that diverge from the Samuelson Rule in opposite directions."

Furthermore, Håkonsen (1998), Jacobs and de Mooij (2015), and Jacobs (2018) conclude that the alleged sensitivity with respect to the choice between direct and indirect taxation reveals a serious weakness in the traditional measure of the MCF. Based on this conclusion, they discuss and propose alternative measures. Jacobs (2018) states that

> "[...] the most regularly used definition [of the MCF], e.g., in Atkinson and Stern (1974), Ballard and Fullerton (1992), and Sandmo (1998), [is] highly sensitive to the choice of the untaxed numeraire good"

and uses this claimed weakness of the traditional MCF concept as one of three arguments in favor of an alternative concept.[2] Based on this alternative MCF concept, Jacobs (2018) further concludes that the MCF is equal to one in the second-best allocation and that no correction of the Samuelson rule is necessary. Jacobs and de Mooij (2015) build on the same arguments when they argue that Pigouvian taxes should not be adjusted to take the MCF into account.

The paper is organized as follows. In Section 2, we present a basic model of provision of public goods and taxation with the same set up as used by Atkinson and Stern (1974). In Section 2.1, we consider first-best provision of public goods as a benchmark, while in Section 2.2, we examine the second-best solution with taxation and present the modified Samuelson rule as well as the main result. We discuss the division between the substitution and income effects in the two-good case in Section 2.3. Finally, we conclude in Section 3.

[2]The two other concerns of Jacobs (2018) and Jacobs and de Mooij (2015) related to the conventional concept are that the MCF of lump-sum taxes then is not equal to one with optimal taxation and that the MCF of distortionary taxes is not directly related to the excess burden of taxation in the absence of distributional concerns. The discussion in our paper is unrelated to these other two concerns.

2 Model and results

The model presented in the following is the same as was used by Atkinson and Stern (1974), because we want our analysis to be as easily comparable with theirs as possible.

The economy considered is assumed to have the production constraint

$$G(\mathbf{X}, e) = 0, \tag{1}$$

where $\mathbf{X} = (X_1, ..., X_n)$ is a vector of the n private goods consumed and factors supplied by households on the market, while e is provision of a public good. Assume throughout, without loss of generality, that if good i is a consumption good, then $X_i \geq 0$, and if good i is a supplied factor, then $X_i \leq 0$. Let $G_i > 0$ denote the first derivative of the function $G(\cdot)$ with respect to argument $i = 1, ..., n, e$. Let $p_i \geq 0$ be the prevailing equilibrium producer price of good $i = 1, \ldots, n$, and define good 1 as numeraire so that $p_1 = 1$. Moreover, assume, as a normalization, that $G_1 = 1$. All firms in the market are competitive, and there are constant returns to scale in production. Together these assumptions mean that we have $G_i = p_i$ for $i = 1, .., n$ in the market equilibrium.

Let there be h identical households and let $\mathbf{x} = (x_1, ..., x_n)$ be the vector of goods consumed and factors supplied by each individual household, with $h\mathbf{x} = \mathbf{X}$. $\mathbf{q} = (q_1, ..., q_n)$ is the vector of consumer prices. Each household maximizes utility, $u(\mathbf{x}, e)$, subject to the budget constraint $\mathbf{q} \cdot \mathbf{x} = 0$. The utility function is increasing and concave in all arguments. Let $u_i > 0$ denote the marginal utility of good or factor i for $i = 1, ..., n$ and let $u_e > 0$ denote the marginal utility of the public good.

To produce the public good, e, the government must reallocate resources. This is done by raising a total tax revenue of $\mathbf{t} \cdot \mathbf{X}$, where $\mathbf{t} = (t_1, ..., t_n)$ is the vector of taxes such that $q_i = p_i + t_i$, $i = 1, .., n$.[3] If good i is a consumption good, then $t_i > 0$ if good i is taxed, and if good i is a factor, then $t_i < 0$ if the factor is taxed. The taxes will take the opposite signs if goods or factors are subsidized.

[3] In more recent literature, it is more common to model with ad valorem taxes, with $q_i = (1 + t_i)p_i$; see, for example, Dahlby (1998), Christiansen (2007), Sandmo (1998) and Jacobs (2018). We adopted the approach applied by Atkinson and Stern (1974) when modeling with specific (quantity) taxes. Our conclusions are not affected by this choice.

2.1 The first-best allocation

The first-best allocation of resources in the economy is given by the solution to the following problem:

$$\max_{\mathbf{x},e} \; h \; u(\mathbf{x}, e) \quad \text{s.t.} \quad G(h\mathbf{x}, e) = 0.$$

This problem is solved by the n first-order conditions:

$$h\frac{u_e}{u_i} = \frac{G_e}{G_i}, \;\; \forall i \in \{1, ..., n\}, \tag{2}$$

which, together with the resource constraint, determine optimal consumption and supply of all private goods and optimal provision of the public good. Equation (2) states that the marginal cost of producing the good (the marginal rate of transformation) should equal the aggregate marginal value of access to the good for all households (the marginal rate of substitution). Equation (2) corresponds to Equation (2) in Samuelson (1954). When Equation (2) is expressed in terms of income (i.e., the numeraire good), it is what Atkinson and Stern (1974), with reference to Aaron and McGuire (1970), define as the Samuelson rule.

2.2 The second-best allocation and the MCF

To simplify the notation and exposition in this section, we define the set of relative consumer prices

$$m_j(q_1, q_j) \equiv \frac{q_j}{q_1}, \;\; \forall j \in \{1, ..., n\}. \tag{3}$$

and the corresponding vector $\mathbf{m} \equiv (m_1, ..., m_n)$. m_j represents the consumer price of good j measured in units of the numeraire good *after taxes*. The household's budget constraint can be written $\mathbf{m} \cdot \mathbf{x} = 0$. The household's maximization problem is the following:

$$\max_{\mathbf{x}} \; u(\mathbf{x}, e) \quad \text{s.t.} \quad \mathbf{m} \cdot \mathbf{x} = 0.$$

The corresponding Lagrange function can be written as follows:

$$L^H = u(\mathbf{x}, e) - \alpha \mathbf{m} \cdot \mathbf{x},$$

where α is the Lagrange multiplier. The n first-order conditions become:

$$u_j = \alpha m_j, \ \forall j \in \{1, \ldots, n\}.$$

They determine equilibrium demand and supply of all private goods as functions of the vector of relative prices, $\mathbf{m}$, and public expenditure, e, given by $\mathbf{X}(\mathbf{m}, e) = h\mathbf{x}(\mathbf{m}, e)$. The taxes determine the consumer prices and hence also the relative consumer prices. However, for any vector of relative prices $\mathbf{m}$, there is an infinite number of vectors $\mathbf{t}$, all with corresponding vectors $\mathbf{q}$, that would give the same vector of relative consumer prices. Specifically, different tax vectors $\mathbf{t}$ with different weights on direct versus indirect taxation may give the same relative price vector $\mathbf{m}$.

The first-order conditions also determine the equilibrium Lagrange multiplier $\alpha(\mathbf{m}, e) = u_1$. The Lagrange multiplier thus represents the marginal utility of income (net of taxes), as $p_1 = 1$.

The representative household's indirect utility function is denoted by $v(\mathbf{m}, e)$. Let v_{m_i} and v_e denote the derivatives with respect to m_i and e, respectively. The derivatives are given by:

$$v_e = u_e \text{ and } \ v_{m_i} = -\alpha x_i, \ \forall i \in \{1, ..., n\}. \tag{4}$$

Given the households' behavior, the government sets the supply of the public good, e, and the taxes, $\mathbf{t}$, which determine the consumer prices, $\mathbf{q}$. The government solves the following maximization problem:

$$\max_{\mathbf{q}, e} h \ v(\mathbf{m}, e) \quad \text{s.t.} \quad G(\mathbf{X}(\mathbf{m}, e), e) = 0.$$

The corresponding Lagrange function can be written:

$$L^G = h \ v(\mathbf{m}, e) - \mu \ G(\mathbf{X}(\mathbf{m}, e), e),$$

where μ is the Lagrange multiplier. The $n+1$ first-order conditions are given by:

$$\frac{dL^G}{de} = hv_e - \mu \left[\sum_{i=1}^{n} G_i \frac{\partial X_i}{\partial e} + G_e \right] = 0, \tag{5}$$

$$\frac{dL^G}{dq_1} = h \sum_{j=1}^{n} v_{m_j} \frac{\partial m_j}{\partial q_1} - \mu \sum_{i=1}^{n} G_i \sum_{j=1}^{n} \frac{\partial X_i}{\partial m_j} \frac{\partial m_j}{\partial q_1} = 0, \tag{6}$$

$$\frac{dL^G}{dq_j} = hv_{m_j} \frac{\partial m_j}{\partial q_j} - \mu \sum_{i=1}^{n} G_i \frac{\partial X_i}{\partial m_j} \frac{\partial m_j}{\partial q_j} = 0, \ \forall j \in \{2, ..., n\}. \tag{7}$$

Lemma 1. *The $n+1$ government first-order conditions given by equations* (5)–(7) *are only n independent equations and determine, together with the resource constraint, only the relative consumer prices, $\mathbf{m}$, the supply of the public good, e, and the Lagrange multiplier, μ, not the full set of taxes, $\mathbf{t}$, and not the full set of consumer prices, $\mathbf{q}$.*

Proof. Use $v_{m_j} = -\alpha x_j$ for all $j \in \{1, \dots, n\}$, $\partial m_j / \partial q_1 = -m_j / q_1$ for all $j \in \{2, \dots, n\}$, $\partial m_1 / \partial q_1 = 0$, and reorganize Equation (6) to get:

$$\alpha X_1(\mathbf{m}, e) = \mu \sum_{i=1}^{n} \sum_{j=2}^{n} G_i m_j \frac{\partial X_i(\mathbf{m}, e)}{\partial m_j}. \tag{8}$$

Correspondingly, use $v_{m_j} = -\alpha x_j$ for all $j \in \{1, \dots, n\}$, $\partial m_j / \partial q_j = 1/q_1$ for all $j \in \{2, \dots, n\}$, and reorganize (7), to get:

$$-\alpha X_j(\mathbf{m}, e) = \mu \sum_{i=1}^{n} G_i \frac{\partial X_i(\mathbf{m}, e)}{\partial m_j}, \quad \forall j \in \{2, ..., n\}. \tag{9}$$

Multiplying each side of Equation (9) by m_j, summing over all $j \in \{2, ..., n\}$ and using $\sum_{j=2}^{n} X_j m_j = -X_1$ give Equation (8). Together with the resource constraint, the first-order conditions are thus a system of $n+1$ independent equations that determine $n+1$ variables: $\{m_2, \dots, m_n\}$, e and μ. □

Lemma 1 means for example – as is well-known in the literature on optimal taxation – that any one tax, t_j, can be set to zero without that affecting the equilibrium allocation.

Equation (5) can be reorganized by dividing by α, using $G_i = p_i$ and using the fact that

$$-\mathbf{p}\cdot\frac{\partial \mathbf{X}}{\partial e}=\mathbf{t}\cdot\frac{\partial \mathbf{X}}{\partial e},$$

which follows from the household's budget constraint $(\mathbf{p}+\mathbf{t})\cdot\mathbf{x}=0$. This reorganization results in a modified Samuelson rule stating that the sum of the marginal willingness to pay for the public good across all households, adjusted by the multiplier α/μ, must equal the unit cost in production of the public good minus the change in public revenue from a marginal increase in the public good supply:

$$\frac{\alpha}{\mu}h\frac{u_e}{u_1}=G_e-\mathbf{t}\cdot\frac{\partial \mathbf{X}}{\partial e}. \tag{10}$$

To what extent, and in what direction, the Samuelson rule should be modified to achieve the second-best solution depends of the size of the multiplier α/μ.[4]

The following proposition presents the main contribution of this paper:

Proposition 1. *For any set of second-best equilibrium producer prices,* $\mathbf{p}$*, and relative consumer prices,* $\mathbf{m}$*, as defined by Equation* (3)*, the marginal cost of taxation and, correspondingly, the multiplier* α/μ *to be used to modify the Samuelson rule, are determined. Given the second-best relative prices, the multiplier is not influenced by the choice between direct and indirect taxation. The multiplier is given by:*

$$\frac{\alpha}{\mu}=-\frac{1}{X_j(\mathbf{m},e)}\sum_{i=1}^{n}p_i\frac{\partial X_i(\mathbf{m},e)}{\partial m_j},\qquad \forall j\in\{2,...,n\}. \tag{11}$$

Proof. Reorganizing Equation (7) gives Equation (11). It follows from this expression that when public expenditure, e, the producer prices, $\mathbf{p}$, and the relative consumer prices, $\mathbf{m}$ are determined (giving the second-best allocation), the multiplier α/μ. is unambiguously determined, independent of the choice by the government between direct and indirect taxation. □

Proposition 1 states that for a given set of second-best relative consumer prices, the distortionary costs of taxation are the same and independent of the choice between direct and indirect taxation. The intuition behind the result is not complicated: The distortionary costs of taxation arise when the value of supplying factors on the market, relative to the cost of acquiring consumer goods, is changed by taxation. The key to

[4]In more recent literature, the inverse of the multiplier, i.e., μ/α, is the standard measure of the MCF and is usually included as a factor on the right-hand side of the equation (Sandmo, 1998; Gahvari, 2006; Kleven & Kreiner, 2006; Kreiner & Verdelin, 2012; Jacobs, 2018).

this result is that the tax wedges, defined as the difference between the relative consumer prices and the relative producer prices, are the same independently of whether the supplied factors or the consumer goods are taxed.

Technically, it is clear that the tax burden can be shifted from factors to consumer goods, or the opposite, without that changing the relative consumer prices $\mathbf{m}$. As an example, start with a set of taxes $\mathbf{t} = (t_1, ..., t_n)$, which gives the set of consumer prices $\mathbf{q} = (q_1, ..., q_n)$ and the set of relative consumer prices $\mathbf{m} = (m_1, ..., m_n)$. Then, change the tax on good or factor k from t_k to $\tilde{t}_k \neq t_k$. To keep relative consumer prices unchanged, a new set of taxes $\tilde{\mathbf{t}} = (\tilde{t}_1, ..., \tilde{t}_n)$ can be implemented, such that:

$$\frac{p_k + \tilde{t}_k}{p_z + \tilde{t}_z} = \frac{p_k + t_k}{p_z + t_z}, \quad \Leftrightarrow \quad \tilde{t}_z = \frac{q_z}{q_k}(p_k + \tilde{t}_k) - p_z, \quad \forall z \in \{1, ..., n\}. \tag{12}$$

Given these changes, the set of relative prices is still $\mathbf{m} = (m_1, ..., m_n)$ and the total tax revenue is not changed, i.e., $\tilde{\mathbf{t}} \cdot \mathbf{X}(\mathbf{m}, e) = \mathbf{t} \cdot \mathbf{X}(\mathbf{m}, e)$. According to Proposition 1 the multiplier α/μ is the same for the two tax schemes $\mathbf{t} = (t_1, ..., t_n)$ and $\tilde{\mathbf{t}} = (\tilde{t}_1, ..., \tilde{t}_n)$.

Equation (11) corresponds to Equation (4) in Atkinson and Stern (1974), which they used as the basis for their discussion of the size of the multiplier μ/α. However, there is an important difference between the two equations. While Equation (11) in this paper applies also when $t_1 \neq 0$, Equation (4) in Atkinson and Stern (1974) applies only when $t_1 = 0$. To see this, reorganize Equation (7), using $v_{m_j} = -\alpha x_j$, $\partial m_j/\partial t_j = 1/(1 + t_1)$, $G_j = p_j$, for all $j \in \{1, ..., n\}$, and the household's budget constraint $(\mathbf{p} + \mathbf{t}) \cdot \mathbf{X} = 0$, to get

$$\frac{\alpha}{\mu} = (1 + t_1) \frac{\frac{d}{dt_j}(\mathbf{t} \cdot \mathbf{X})}{X_j}, \ \in \{1, ..., n\}, \tag{13}$$

which corresponds directly to Equation (4) in Atkinson and Stern (1974). However, their Equation (4) does not include the first factor on the right-hand side of (13), $(1 + t_1)$. This is of course unproblematic, since one can – as mentioned in the above discussion – normalize by setting one tax equal to zero without changing the second-best allocation. However, Atkinson and Stern (1974) go on to discuss the size of the multiplier, μ/α, based on their Equation (4) using a two-good example in two specific cases: The first case is the one corresponding to the normalization that has already been made, where the tax on the numeraire good, good 1, is set to zero. The second case, however, is the case where the tax on *the other good*, good 2 is set to zero while the tax on good 1 is positive. In this second case, however, a problem arises because Equation (4) does not hold when $t_1 \neq 0$.[5]

2.3 Income and substitution effects

This section follows up the discussion by Atkinson and Stern (1974) related to the division between the substitution effect and the income effect in the two-good case. On page 123 they explained their findings regarding the effect of the choice of taxed good on the multiplierr with the following statement:

> "The reason for the dependence of the sign on the choice of the taxed good is fairly clear: the 'income' effect of taxation reduces the revenue from a consumption tax given normality but increases the revenue from a factor tax given normality of leisure."

In the following, it will be demonstrated that this consideration is too partial. In the end, the income effect of taxation is not influenced by the choice between direct and indirect taxation.[6]

Let good 1 be the consumer good and good 2 be the labor supply. For simplicity, assume first that there is a tax on labor income ($t_2 < 0$), while $t_1 = 0$. Then, from Equation (13) we have:

$$\frac{\alpha}{\mu} = 1 + \frac{t_2}{X_2}\frac{dX_2}{dt_2}. \tag{14}$$

Applying the Slutsky equation gives:

$$\frac{\alpha}{\mu} = \underbrace{1 \underbrace{-t_2\frac{dX_2}{dI}}_{>0} + \underbrace{\frac{t_2}{X_2}\frac{dX_2^c}{dt_2}}_{<0}}_{\lesseqgtr 1}, \tag{15}$$

[5]The analysis of Atkinson and Stern (1974) may also be interpreted with a different definition of α in mind. If we ignore the implicit definition of α from their Equation (3), their starting point may have been a Lagrange function and Lagrange multiplier of the households' maximization given by $L'^{H} = u(\boldsymbol{x}, e) - \alpha' \, \boldsymbol{q} \cdot \boldsymbol{x}$. In that case $\alpha' \equiv u_1/(1+t_1)$ and Equation (4) in Atkinson and Stern (1974) holds for all normalizations (i.e. also in the case with $t_1 \neq 0$). However, with this definition of α', the multiplier of the modified Samuelson rule is equal to $(1+t_1)\alpha'/\mu$. Thus, also in this case, a discussion of the size of the multiplier in the case where $t_1 \neq 0$) based on the right-hand side of Equation (4) in Atkinson and Stern (1974) representing the multiplier is problematic.

[6]For other studies considering the two-good case in relation to the MCF, see, for example, Browning, Gronberg, and Liu (2000), Sandmo (1998), Kreiner and Verdelin (2012), Kleven and Kreiner (2006), Kaplow (1996), Jacobs (2018), or Håkonsen (1998).

where dX_2/dI gives the marginal change in X_2 following a change in an exogenously given income I, and the index c indicates that we are considering the compensated demand. Thus dX_2^c/dt_2 represents the substitution effect of the tax change.

If leisure is a normal good, then $dX_2/dI > 0$ (because $X_2 < 0$, less factor supply means that X_2 increases). Thus, the second term on the right-hand side of Equation (15) is positive. Furthermore, the substitution effect means that $dX_2^c/dt_2 < 0$ (because $t_2 < 0$, increasing t_2 means less taxation of the factor, which gives substitution toward more factor supply, i.e., that X_2 is reduced). Thus, if leisure is a normal good, then the substitution and income effects pull in different directions and we cannot conclude whether the expression on the right-hand side of Equation (15) is less than or greater than 1. If the income effect is stronger than the substitution effect, then the right-hand side of Equation (15) is greater than 1. This is in line with Stiglitz and Dasgupta (1971, p. 159) stating that

> "[...] whether the conventional rule represents an under or over supply depends simply on whether the supply curve of labor is backward bending or upward sloping."[7]

However, Atkinson and Stern (1974) stated that this result applies only if labor is taxed, not if consumption is taxed. Let us therefore also consider the case with a consumption tax $t_1 > 0$ while $t_2 = 0$.

Using Slutsky, Equation (13) can be rewritten to give:

$$\underbrace{\frac{\alpha}{\mu} = (1+t_1)\underbrace{\left[1 \underbrace{-t_1\frac{dX_1}{dI}}_{<0} + \underbrace{\frac{t_1}{X_1}\frac{dX_1^c}{dt_1}}_{<0}\right]}_{<1}}_{\lesseqgtr 1}. \tag{16}$$

The square bracket in (16) is less than one. Because Atkinson and Stern (1974) did not include the factor $(1+t_1)$ on the right-hand side of their Equation (4), they concluded that with a consumption tax, the multiplier is always less than one. However, when the

[7] Atkinson and Stern (1974, p. 120) pointed to the unclear meaning of Stiglitz and Dasgupta's use of the expression *under or over supply* in this context, because different interpretations are possible. The question of the size of the multiplier in the modified Samuelson rule, which it is reasonable to assume that Stiglitz and Dasgupta had in mind, has to be distinguished from the question of the level of provision of public goods in first-best compared to second-best. Atkinson and Stern (1974) showed that the provision of public goods can be lower in second-best than in first-best also in the case of a backward bending labor supply curve.

first factor $(1 + t_1)$ is taken into account, this cannot be guaranteed. In the case with a consumption tax, α/μ can also be greater than one.

Finally, since the size of α/μ is determined only by the relative prices, not the choice between a tax on factor income or a tax on consumption, Stiglitz and Dasgupta's conclusion referred to above applies also in the case with a tax on consumption.

3 Conclusion

The work of Atkinson and Stern (1974) stands out as an important contribution to the literature in public economics. The authors shed new light on issues related to the costs of taxation and the appropriate output level of public goods. Their findings underline the importance of distinguishing between the size of "the appropriate benefit measure for incremental output of the public good" and the question of "the appropriate output level for public goods" (Atkinson & Stern, 1974, p. 120) . Moreover, they show how the income effects of taxation might counteract the distortionary effects of taxation. As a result, we cannot rule out that the marginal costs of taxation can be less than 1, even before the distributional effects of taxation are taken into account.

However, the statements of Atkinson and Stern regarding the effect on the multiplier to be applied in the Samuelson rule of the choice between direct and indirect taxation has created confusion in the literature on the MCF. Later contributions on the costs of taxation have claimed that the findings of Atkinson and Stern (1974) reveal a serious weakness in the traditional MCF measure; see Håkonsen (1998), Jacobs and de Mooij (2015), and Jacobs (2018).

We have shown in this paper that the costs of taxation, and the MCF, are independent of whether consumption goods or supplied factors are taxed. It follows that the modified Samuelson rule does not depend on the choice between direct and indirect taxation.

The economics literature should provide knowledge and understanding of the size and interpretation of the MCF that is precise and correct. The MCF is important in a large range of policy decisions. The size of the MCF not only is of theoretical interest, but also affects practical policy. Thus, disentangling the confusion in the literature regarding the effect on the MCF of the choice between direct and indirect taxation is important.

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