Optimal Carbon Taxation and Horizontal Equity:

A welfare-theoretic approach with application to German household data

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This version: February 15, 2021

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Abstract: We develop a model of optimal carbon taxation and redistribution taking into account horizontal equity concerns. Within each income decile, households are heterogeneous in terms of how efficient an exogenous technology allows them to convert carbon-intensive energy into individual well-being. We then investigate how horizontal equity is considered in the economy's welfare maximizing tax structure by deriving first- and second-best policy rules. Further, we characterize optimal non-linear carbon taxes, which the government can use when individual households' energy efficiency is not directly observable. Subsequently, we apply our finding to empirical data on energy consumption in Germany to quantify optimal policies.

Keywords: carbon taxation, horizontal equity, redistribution.

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1 Introduction

The trade-off between equity and efficiency is one of the central topics in economics and economic policy. Analysing this central trade-off with respect to the implementation of climate policy poses an urgent challenge: On the one hand economic theory clearly suggests that Pigouvian carbon pricing should be at the heart of economically efficient and environmentally effective climate policy (Pigou 1920, Nordhaus 2019), which is broadly supported by economists worldwide (Financial Times 2019, The Economist 2021) as well as by recent empirical findings (Andersson 2019, Gugler et al. 2021). On the other hand there are many factors that impede the timely required implementation and political feasibility of carbon pricing (Levi 2021, Edenhofer et al. 2021). Among these factors distributional consequences for low-income households are a key concern (Shammin and Bullard 2009, Parry 2015, Pizer and Sexton 2019) resulting in a political debate, which is often charged with emotions and thus provides breeding ground for conflicts, turmoil and deadlock. The yellow vest movement, for example, rose in France in November 2018 to protest against fuel price increases due to CO_2 taxation. It is one example illustrating that climate policy needs to be accompanied by appropriate redistribution measures that address equity concerns.

Due to the regressive effects of carbon pricing (Wang et al. 2016, Ohlendorf et al. 2021)[1], the existing literature has mostly focused on vertical equity between different income deciles. Empirical studies, however, have highlighted the importance of horizontal equity since distributional effects show an even larger variation within income groups (Poterba 1991, Rausch et al. 2011, Cronin et al. 2019, Pizer and Sexton 2019). The impact of carbon pricing on carbon-intensive energy consumption may vary across households with similar incomes for example due to household characteristics and behavior such as the climate surrounding the household, commuting distance of its members or the energy efficiency standard of a building (Rausch et al. 2011). Indeed, in the public debate negative distributional outcomes for households that are hardship cases due to their

[1] The literature provides somewhat ambiguous results on the distributional consequences of climate policy. Regressive effects have been found to be more likely in higher income countries (Ohlendorf et al. 2021).

high carbon footprints, have been used to argue against effective and efficient carbon pricing.

Because household characteristics that determine horizontal inequality are mostly outside the scope of governmental regulation (Kaplow 1989, 1992), only very few studies have included horizontal equity concerns in a welfare-theoretic framework (Slesnick 1989, Auerbach and Hassett 2002, Fischer and Pizer 2019). Moreover, horizontal equity may be incompatible with economic efficiency (Stiglitz 1982, Elkins 2006), if individuals with equal circumstances had to pay the same taxes regardless of individual behavior in terms of labor-leisure decisions or externality generating activities (Elkins 2006). Economic efficiency, in turn, would require to differentiate between these types of behavior in order to maximize welfare at the lowest possible cost while internalizing externalities. Despite these shortcomings, policy makers should take into account both vertical and horizontal equity as the political support for a tax reform will be weakened if a majority of people in low income deciles will be worse-off after the reform (Cronin et al. 2019).

In this paper we put forward a welfare-theoretic model that captures the trade-off between economic efficiency and horizontal inequality due to heterogeneous carbon-intensive energy expenditures shares. We use the model to analyse how the additional effect on horizontal inequality due to the introduction of climate policy is considered in welfare-optimal first- and second best-policy instruments like carbon taxes and transfer payments. Subsequently, we apply our findings to empirical data on energy consumption in Germany to quantify optimal policies. We thereby contribute to the literature on optimal environmental taxation (Sandmo 1975, Klenert et al. 2018, Jacobs and van den Ploeg 2019, for example) with heterogeneous households (Cremer et al. 2003, Kaplow 2008).

Methodologically our approach builds on Cremer et al. (2003) and Kaplow (2008), who suggest non-linear (energy) taxation to take into account households' heterogeneity. In our case, however, the heterogeneity is not modelled as a 'taste' as in Cremer et al. (2003) and Kaplow (2008) but, at least in the short-run, as exogenously given by a heterogeneous technology parameter capturing how efficient households can convert energy into individual well-being. Hence, we differ by making an explicit case for considering horizontal inequality due to heterogeneous within-decile energy expenditure shares, which has been identified as a central reason for concern with respect to the political feasibility of carbon pricing (Pizer and Sexton 2019). We show how public finance can optimally target hardship cases that form a central element in the opposition against carbon pricing by balancing societal preferences for economic efficiency and horizontal equity. Further, to purely focus on how carbon taxation impacts horizontal inequality, we abstract from an explicit representation of labor-leisure choices, but explore how carbon taxation itself and the resulting revenue should be redistributed to households to take the impacts on horizontal equity into account.

We show that the government's first best solution to address horizontal inequality is to set the carbon tax equal to the Pigouvian level and recycle the carbon tax revenue through household-specific transfer payments. The numerical application to German household data reveals that, depending on the societal preferences about horizontal equity versus economic efficiency, it is welfare-optimal to redistribute a higher fraction of the tax revenue to either very efficient households with low carbon footprints or hardship cases with high carbon-intensive energy expenditure shares. For the rather likely case that the government can not perfectly observe all individual household characteristics, we also compute a non-linear household-specific carbon tax that only relies on information about the households' carbon-intensive energy consumption.

The paper is structured as follows: Section 2 discusses related literature on the economics of horizontal equity in detail. Section 3 sets up the model while section 4 introduces functional forms and applies the theoretical model to German household data. The final section 5 summarizes our results and discusses limitations of the approach taken in this paper.

2 Related Literature

The literature on the economics of horizontal equity can be divided into (i) empirical studies that quantify the magnitude of horizontal inequality due to some (environmental) policy reform, (ii) theoretical and applied modeling studies suggesting a welfare measure that disentangles vertical from horizontal equity and (iii) theoretical studies that consider horizontal equity within an optimal taxation framework. In the following we briefly summarize each of these literature strands.

The first literature strand focuses on empirical analyses and reports considerable within-decile variation in energy expenditure shares. Poterba (1991) analyses gasoline expenditures in the United States and finds considerable within-decile variability especially among low-income households. Pizer and Sexton (2019) confirm the high variation in energy expenditures shares also for other countries like Mexico and the United Kingdom. Rausch et al. (2011) shows that the impacts of carbon taxation in the United States puts indeed the highest burden on low-income deciles, while Cronin et al. (2019) also consider the capacity of existing transfer payments to address horizontal equity. They show that a uniform increase in all existing transfer payments increases horizontal inequality even further, which thus calls for a more targeted redistribution approach.

The second strand of the literature on the economics of horizontal equity deals with designing explicit welfare indices that incorporate horizontal equity. Slesnick (1989) proposes a welfare measure for horizontal equity that is consistent with social choice axioms and is calculated as the difference between welfare under a horizontally egalitarian distribution and the existing distribution of individual welfare. Using data on commodity taxation in the United States from 1947-1985 the study finds increasing horizontal inequality due to the heterogeneous effects of taxation on households' welfare. Auerbach and Hassett (2002) argue that horizontal equity should be justified within the context of the Atkinson inequality aversion index (Atkinson 1970). They differentiate between aversion to vertical and aversion to horizontal inequality by using a two parameter specification similar to the one that has been suggested by Epstein and Zin (1989) to disentangle preferences for risk from preferences for intertemporal substitution. When applying the suggested index to income tax data for 1994 in the United States, they find horizontal inequality to be the less severe the higher the standard Atkinson inequality aversion index.

The most recent study by Pizer and Sexton (2019) proposes a welfare measure that can incorporate both vertical and horizontal equity and is based on the concept of equal sacrifice relative to a status-quo (Slesnick 1989, Kahneman and Tversky 1979). Specifically, welfare for a policy induced change in net income is modelled to be the difference between the average income change due to environmental regulation and a weighted average of the within-decile deviation of individual income from a reference point that is the average burden of environmental regulation in the own income decile. Findings show that non-Pigouvian policies like tradable performance standards lead to more horizontal inequality as compared to Pigouvian policies like a cap and trade system with equal per household rebates. Compared to the literature on welfare measures that Pizer and Sexton (2019) build on, this paper specifically introduces within-decile heterogeneity in households' energy expenditure shares in the modelling structure, but otherwise applies a standard utilitarian social welfare function to evaluate policies. While in Pizer and Sexton (2019) more horizontal equity is always welfare-increasing, this paper not only considers the positive effect on welfare due to a more egalitarian within-decile income distribution, but also takes into account that society sacrifices efficiency gains when compensating hardship cases with higher transfer payments. Our approach can thus be considered more general as we seek to understand under which conditions (i.e. social preferences) a benevolent government would care about horizontal equity without prescribing a distinct welfare measure in the first place.

In this paper we mainly build on the third strand of the literature on optimal taxation (Ramsey 1928, Diamond and Mirrlees 1971), that aims at implementing a tax system that maximizes a social welfare function subject to economic constraints (Mankiw et al. 2009). Traditional utilitarian welfare theory (Bentham 1789) is based on the principle of diminishing marginal utility of income, which motivates the dominating interest in vertical equity in optimal taxation models. Horizontal equity, in turn, allows treating tax payers at equal positions equally, which is the more fundamental and widely accepted principle of fairness as an acceptable pattern of differentiation between income groups must be chosen (Musgrave 1990). Nevertheless, the literature on optimal taxation and horizontal equity is relatively scarce (Atkinson and Stiglitz 1976, Fischer and Pizer 2019).

Stiglitz (1982) shows that horizontal equity cannot be derived from a utilitarian social welfare function and can be inconsistent with Pareto optimality. This is because randomization of the tax system enables the government to differentiate between high and low ability types at lower cost by taxing individuals with equal circumstances such that the high ability type is even more productive thereby raising average productivity and economic output. Jordahl and Micheletto (2005) incorporate a horizontal equity constraint in the problem of finding an optimal utilitarian tax structure, which has already been suggested by Atkinson and Stiglitz (1976) in order to circumvent the equity-efficiency trade-off when horizontal equity is built into the measurement of social welfare itself. The horizontal equity constraint is based on the interpretation of Bossert (1995) in terms of 'equal transfers for equal circumstances' and requires that heterogeneous households with the same abilities should pay the same taxes. Kaplow (2008) reconsiders central results of optimal income and commodity taxation when preferences are heterogeneous and are either observable or not. Based on a utility function that can embody different types of heterogeneity, results reveal that preference heterogeneity can lead to both higher and lower levels of income taxation depending on the type of heterogeneity, it's strengths, and the concavity of private utility and the social welfare function. However, both Jordahl and Micheletto (2005) and Kaplow (2008) do not make any explicit connection to environmental policy and carbon taxation specifically.

Within the optimal taxation literature there is an established sub-field on optimal taxation and environmental externalties[2] that goes back to Pigou (1920). Later Sandmo (1975) contributed the seminal paper based on a model of optimal linear taxation of a commodity that generates a negative atmospheric externality. The optimal commodity tax rule that results from this modeling setup includes one additive term that corrects for the externality thereby fulfilling the so-called 'additivity property'. The resulting optimal tax system has two main objectives including (i) the correction of the environmental externality and (ii) achieving a distribution among heterogeneous individuals that is optimal according to a particular social welfare function.

[2]See Aronsson and Sjögren (2018) for a very good overview of this literature.

This literature has been extended to also consider heterogeneous households (Cremer et al. 2003) which could in principle also cover horizontal inequality as a source of heterogeneity. Cremer et al. (2003) analyses how taste heterogeneity in households' preferences is captured in different systems of optimal environmental taxation and applies the model to energy consumption in France. The authors argue that type-specific non-linear environmental taxes should be implemented when individual consumption levels are observable as in the case of electricity consumption.

3 Theoretical Model

We introduce a parsimonious model in order to convey a few basic intuitions about optimal policies. We begin by characterizing the first best optimal allocation that a social planer would implement (section 3.1.1). Then, in section 3.1.2, we compare the first best with the outcomes that a government can achieve by using different sets of policy instruments. The first best can also be achieved by a benevolent government that sets a uniform carbon tax and in addition can make household specific transfers. With a uniform transfer and individualized carbon tax rates, the government faces an equity-efficiency trade-off and the first-best cannot be achieved. In section 3.2, we discuss possible extensions of the model, that is, efficiency enhancing investments and the possibility to substitute carbon-free for fossil-based energy.

We assume that a benevolent government seeks to maximize the welfare of $j = 1, ..., n$ heterogeneous households, which derive utility u^j from a numeraire consumption good c^j and carbon-intensive energy services E^j. We assume that households have exogenously given income y^j, that they demand raw energy $\widetilde{E}^j$ and use a technology f to convert it to energy services E^j. Households are heterogeneous with respect to their capital endowments x_0^j, which implies that . For now, we will assume that they cannot make additional investments in efficiency enhancing capital. Later, we will discuss how relaxing this assumption affects our results. Thus, we have

$$
\begin{aligned}
u^j =& u(c^j, E^j) \\
E^j =& \alpha_j \widetilde{E}^j = f(x_0^j)\widetilde{E}^j
\end{aligned} \tag{1}
$$

where $x_0^j > 0$ and $f' > 0 > f''$. Their budget equation is

$$
b^j = y^j + R^j = c^j + \underbrace{(p_E + t_E^j)}_{=:q^j} \widetilde{E}^j + x^j(1-s) \tag{2}
$$

Possible policy instruments that the government could implement include carbon taxes t_E^j on CO_2-intensive energy consumption and transfers R^j – which may be uniform or household-specific.

3.1 Basic model

In the basic model, we assume that households heterogeneity is exogenous and given by the differences in the α_j, $j = 1, 2$. The Lagrangian of households is given by

$$
L^H = u(c^j, E^j) + \lambda^j(b^j - c^j - \frac{q^j E^j}{\alpha^j}), \tag{3}
$$

where we have used (1) to eliminate raw energy $\widetilde{E}^j$. The first order conditions are as follows:

$$
\frac{\partial u^j}{\partial c^j} = \lambda^j \tag{4}
$$

$$
\frac{\partial u^j}{\partial E^j}\frac{\alpha^j}{q^j} = \lambda^j \tag{5}
$$

Combining equations (4) and (5) shows that in the optimum the marginal rate of substitution between consuming an additional unit of energy and consuming an additional unit of the numeraire consumption good must be equal to the market price of energy scaled by the energy efficiency parameter α. Moreover, (5) reveals that the marginal utility of the individual household's income λ^j depends on the energy efficiency parameter α, i.e. we have $\lambda(\alpha)$ with $\lambda'(\alpha) = \frac{\partial u^j}{\partial E}\frac{1}{q^j} > 0$.

An individual household's maximization results in the conditional demand functions $c^j = c(b^j, q^j)$ and $E^j = E(b^j, q^j)$, which together determine the household's indirect utility function $v^j = v(b^j, q^j)$.

3.1.1 Social planner optimum

The first-best optimal allocation that a social planner would choose is determined by maximizing a Bergson-Samuelson social welfare function $W(u^1, ..., u^n)$ with $\frac{\partial W}{\partial u^j} \geq 0$ and $\frac{\partial^2 W}{\partial {u^j}^2} \leq 0$ for all j, subject to an exogenous aggregate environmental target $E^* = \sum_j \widetilde{E}^j$ and a resource constraint $\sum_j y^j - c^j - p\widetilde{E}^j = 0$. We abstract from an explicit representation of environmental damages to keep the analysis as simple as possible. The social planner's Lagrangian is

$$L^{SP} = W(u^1, ..., u^n) + \mu(\bar{E} - \sum_j \widetilde{E}^j) + \gamma(\sum_j y^j - c^j - p\widetilde{E}^j), \tag{6}$$

Proposition 1. *The social optimum in the basic model can be achieved under the condition that*

$$0 = W_u^j u_E^j \alpha_j - \mu - \gamma p \quad \forall j \tag{7}$$

$$0 = W_u^j u_c^j - \gamma \quad \forall j \tag{8}$$

The social planner chooses an allocation that balances households' welfare weights W_u^j, their marginal utilities u_E^j and u_c^j and their energy efficiencies α_j. Thus, normative distributional social preferences are balanced with efficiency in consumption.

From the social planner's first-order conditions it follows that

$$\frac{W_u^i}{W_u^j} = \frac{u_E^j}{u_E^i}\frac{\alpha_j}{\alpha_i}$$
$$\text{and} \quad \frac{W_u^i}{W_u^j} = \frac{u_c^j}{u_c^i} \quad \forall i, j.$$

To interpret these equations, assume that i has a higher normative welfare weight than j ($W_u^i > W_u^j$). Then, i must also have a higher level of numeraire consumption. Moreover, if $\alpha_i = \alpha_j$, household i must also have higher level of energy service consumption. However, if j is more energy efficient than i ($\alpha_i < \alpha_j$), then the difference between normative welfare weights could be offset by energy efficiency considerations. Social optimality requires the social planner to allocate relatively more energy service consumption to households that are more efficient in transforming energy services to utility and hence social welfare.

3.1.2 Optimal governmental policies for the decentralized economy

Uniform carbon tax, individual targeted transfers. The government maximizes the same social welfare function $W(u^1, ..., u^n)$, subject to the same aggregate environmental target $E^* = \sum_j \widetilde{E}^j$ and its budget constraint $\sum_j (t_E \widetilde{E}^j - R^j) = 0$. Using the indirect utility function implies that the government maximizes social welfare taking into account the individual household's optimization behavior.

We abstract from problems of self-selection to clarify the fundamental mechanisms that determine the optimal tax system.

The Lagrangian of the government's optimization problem is

$$L^G = W(v^1, ..., v^n) + \mu(E^* - \sum_j \widetilde{E}^j) + \gamma \sum_j \left(t_E \widetilde{E}^j - R^j \right) \tag{9}$$

The first order conditions for the optimal individual carbon taxes and lump-sum transfers read

$$\frac{\partial L^G}{\partial t_E} = \sum_j \frac{\partial W}{\partial v^j}\frac{\partial v^j}{\partial q} - \mu \sum_j \frac{\partial \widetilde{E}^j}{\partial q} + \gamma \left(t_E \sum_j \frac{\partial \widetilde{E}^j}{\partial q} + \sum_j \widetilde{E}^j \right) = 0 \tag{10}$$

$$\frac{\partial L^G}{\partial R^j} = \frac{\partial W}{\partial v^j}\frac{\partial v^j}{\partial I^j} - \mu \frac{\partial \widetilde{E}^j}{\partial I^j} + \gamma \left(t_E \frac{\partial \widetilde{E}^j}{\partial I^j} - 1 \right) = 0 \tag{11}$$

Proposition 2. *If the government can only use a uniform carbon tax $t_E^j = t_E \; \forall j$ but can implement targeted transfers, it can achieve the first-best optimum by setting*

$$t_E^* = \frac{\mu}{\gamma} \; \forall j, \tag{12}$$

The optimal transfers R^{j*} must ensure that the households' marginal rates of substitution, weighted by energy efficiency α_j, are equal for all households,

$$\frac{\mu}{\gamma} + p = \alpha_j \frac{{u_E^j}^*}{u_c^{j*}} = \alpha_j {MRS_{c,E}^j}^* = {MRS_{c,\widetilde{E}}^j}^* \; \forall j \tag{13}$$

and the governments budget is balanced.

$$t_E^* \bar{E} = \sum_j R^{j*} \tag{14}$$

Asterisks denote the values obtained from evaluation at the optimal allocation.

The proof is in the appendix.

Individual carbon taxes, uniform lump-sum transfer Let's assume that the government can target individual households by setting individual carbon tax rates, but must use the same lump-sum transfer for all households.

The Lagrangian of the government is

$$L^G = W(v^1, ..., v^n) + \mu(E^* - \sum_j \widetilde{E}^j) + \gamma \sum_j \left(t_E^j \widetilde{E}^j - R \right) \tag{15}$$

The first order conditions read

$$L_R^G = \sum_j W_{v^j} v_{b^j}^j - \mu \sum_j \frac{\partial \widetilde{E}^j}{\partial b^j} + \gamma \sum_j \left(t_E^j \frac{\partial \widetilde{E}^j}{\partial b^j} - 1 \right) = 0 \tag{16}$$

$$L_{t_{\widetilde{E}^j}}^G = W_{v^j} v_{q_E^j}^j - \mu \frac{\partial \widetilde{E}^j}{\partial q_E^j} + \gamma \left(t_E^j \frac{\partial \widetilde{E}^j}{\partial q_E^j} + \widetilde{E}^j \right) = 0 \tag{17}$$

Definition 1. *We define the social shadow price of achieving the environmental target E^* measured in terms of public funds, μ/γ, as the* Pigouvian *component of environmental taxation.*[3]

By using the private households' marginal utility of income $\lambda^j(\alpha)$ and Roy's identity[4] we can derive the rule for individual carbon taxes from (17) as given in Proposition 3.

Proposition 3. *When the government can perfectly observe the household specific energy efficiency α and skill level w the optimal household specific carbon tax can be written as*

$$t_E^{j\,*} = \frac{\mu}{\gamma} + \underbrace{\frac{\partial W}{\partial v^j} \frac{\lambda^j}{\gamma} \frac{\widetilde{E}^j}{\frac{\partial \widetilde{E}^j}{\partial q^j}} - \frac{\widetilde{E}^j}{\frac{\partial \widetilde{E}^j}{\partial q^j}}}_{\Phi_j}. \tag{18}$$

The optimal individual carbon tax is the sum of two components:

(i) a Pigouvian component μ/γ, which is the same for each household.

(ii) a distributional, household-specific component Φ_j, which takes into account differences in disposable income resulting from heterogeneous α^j. If energy expenditures

[3] We are aware of the fact that the Pigouvian component is typically associated with capturing the sum of marginal environmental damages. We abstract from environmental damages here and instead introduce a fixed upper bound on total energy use. However, our shadow price μ/γ plays a very similar role as the corresponding shadow price in the literature on optimal taxation and environmental externalities.

[4] Specifically we use $\frac{\partial v^j}{\partial q_E} = -\underbrace{\frac{\partial v^j}{\partial I^j}}_{\lambda^j} \widetilde{E}^j$ and $\frac{\partial v^j}{\partial b} = \underbrace{\frac{\partial v^j}{\partial I^j}}_{\lambda^j}$

only make up a negligible part of the household's total expenditure, this term is very small and the tax is close to the Pigouvian level. The greater the share of energy expenditure, the more the tax deviates from the shadow price, i.e distributional aspects have a larger impact on the optimal carbon tax rule.

Corollary 1. *In setting the optimal energy tax, the government faces an equity-efficiency trade-off. The equity motive is determined by the welfare function, while the efficiency motive is determined by the curvature of households' indirect utility function with respect to disposable income (measured by λ^j), their energy-efficiency (measured by α_j) and the marginal social value of public funds γ.*

Proof. If $\Phi_j = 0$, household j is taxed at the Pigouvian level. In general, however, Φ_j could be greater or less than zero, implying that the optimal individual carbon tax lies above or below the Pigouvian level. Without loss of generality we focus in the following on the case in which $\Phi_j < 0$ and, hence, $t_j^{E*} < \frac{\mu}{\gamma}$. The discussion of the case $\Phi_j > 0$ would be analogous. Assuming that energy is a normal good in the sense that $\frac{\partial \widetilde{E}^j}{\partial q^j} < 0$, we have

$$t_E^{j\,*} < \frac{\mu}{\gamma} \quad \Longleftrightarrow \quad \Phi_j < 0 \quad \Longleftrightarrow \quad \frac{\gamma}{\lambda^j} < \frac{\partial W}{\partial v^j}. \tag{19}$$

Above inequality (19) allows us to infer four reasons for the government to tax a household below the Pigouvian rate. We now discuss each of these four reasons assuming all else is equal. First, the optimal individual carbon tax is more likely to be set below the Pigouvian level if the government puts a relatively high marginal welfare weight on household j, i.e. $\frac{\partial W}{\partial v^j}$ is relatively large. Households whose utility contributes more to social welfare thus have to bear less of the tax burden. Second, (19) is more likely to hold, the larger the households marginal utility of income λ^j is. Since marginal utility is decreasing, the government has a motive to put less tax burden on poorer households. Third, the government wants to shift energy consumption towards the household that generates most utility from a given quantity of energy, i.e. the household with the highest α^j. It holds that $\frac{\partial \lambda^j}{\partial \alpha^j} > 0$ and thus, the higher α^j, the more likely it is that (19) holds and household j is taxed below the Pigouvian level. Fourth, the lower the social marginal value of public funds γ, the more likely (19) will hold. If, in contrast, γ is very high and, hence, additional public funds would contribute relatively strongly to social welfare, the optimal tax is less likely to be below the Pigouvian level. □

Corollary 2. *When the government is constrained to use the uniform lump-sum transfer R instead of a household specific transfer but can implement household specific carbon taxes, it cannot achieve the first-best allocation*

The proof is in the appendix.

3.2 Extensions

3.2.1 Energy efficiency enhancing investments by households

Assuming that households' energy efficiency α_j is fixed limits the model's applicability to the short run. To relax this assumption, one could think of households as being able to invest part of their income to improve their energy efficiency α_j. Final energy services consumed by household j would then be $E^j = \alpha_j \widetilde{E}^j = f(x_0^j + x^j)\widetilde{E}^j$, where $x_0^j, x^j > 0$ and $f' > 0 > f''$. In addition to the carbon tax and the lump-sum transfer, the government could implement a subsidy on efficiency-enhancing investments.

However, extending the model to allow for household investments in efficiency-enhancing capital yields very similar results to the ones obtained from the basic model. For example, Proposition 2 still remains valid, that is, the first-best can be implemented with a uniform carbon tax and individualized transfers. If the government is restricted to a uniform carbon tax and a uniform transfer, it is welfare enhancing to allow for a subsidy on efficiency-enhancing investments. For details, see Appendix, section B.

3.2.2 Decarbonization of final energy production

In the preceding analysis we have abstracted from the possibility to decarbonize energy production. The only mitigation option to achieve the environmental target E^* was to reduce energy consumption, for which the government used the energy tax t_E. The model's production side, however, can be extended to include a production function for energy that takes renewable X and non-renewable (fossil) resources Z as input factors where factor prices p_X and p_Z are fixed. Then, the environmental target consists of keeping the use of fossil resources below a certain treshold, $Z \leq Z^*$

Now, the government can use a uniform carbon tax τ on the use of Z in production to incentivize substitution of renewables X for fossil resources Z. Household specific carbon taxes as considered in the simple model above cannot be modeled under this setup. Instead, we can consider household specific subsidies ς_i for energy expenditures.

In the remainder of the paper, we assume a CES function for energy production with substitution elasticity $\sigma \in \mathbb{R}_0^+$, share parameter $a \in (0,1)$ and scaling parameter A.

$$E(X,Z) = A\left(aX^\rho + (1-a)Z^\rho\right)^{\frac{1}{\rho}}$$
$$\rho = \frac{\sigma - 1}{\sigma}$$

Then, the price of raw energy p_E is determined by the factor prices and can be derived from the energy producer's first order conditions.

$$p_E \frac{\partial E}{\partial X} = p_E A(aX^\rho + (1-a)Z^\rho)^{\frac{1-\rho}{\rho}} aX^{\rho-1} = p_X \tag{20}$$
$$p_E \frac{\partial E}{\partial Z} = p_E A(aX^\rho + (1-a)Z^\rho)^{\frac{1-\rho}{\rho}} (1-a)Z^{\rho-1} = p_Z + \tau \tag{21}$$

4 Numerical Model

4.1 Functional forms

We assume Stone-Geary type utility functions for the households of the form

$$u^j(c^j, \tilde{E}^j) = \frac{\left(c^{j\,\beta}\left(\alpha_j \tilde{E}^j - \bar{E}\right)^{1-\beta}\right)^{1-\eta}}{1-\eta} \tag{22}$$

where $\bar{E}$ denotes a subsistence level of utility-relevant energy consumption and α_j the conversion efficiency for raw energy $\tilde{E}$ energy to utility-enhancing energy-intensive services such that $\alpha_j \tilde{E}^j = E^j$. The subsistence requirement enables us to model non-constant energy-expenditure shares within the same income decile and is thus suited particularly well to capture horizontal inequality (see the following two subsections).

Following Kaplow (2010) we assume a Bergson-Samuelson social welfare function of the form

$$W(u) = \sum_j (1-\eta)^{-\gamma} \frac{u^{1-\gamma}}{1-\gamma}, \tag{23}$$

which reduces to a simple utilitarian social welfare function $W(u) = \sum_j u^j$ for $\gamma = 0$. While η is the elasticity of the marginal utility of comprehensive household consumption, i.e. reflecting the curvature of the individual household utility function u, γ measures governmental inequality aversion as reflected by the curvature of the social welfare function $W(u)$. Kaplow (2010) showed that the combined concavity parameter is then given by $\epsilon = \eta + (1-\eta)\gamma$.[5]

We use equivalent variation (EV) to monetize households' policy costs. Given household j's exogenous budget y^j (excluding transfer payments) and energy price $q = p + t_E$, EV^j is obtained by the following indifference condition:

$$v^j(y^j + EV^j, p) = v(y^j, q). \tag{24}$$

[5] Note that every ϵ can also be obtained by varying only η while holding γ constant equal to zero. We follow this strategy in the numerical optimization to obtain the results in section 4.3.

The aggregate monetized social welfare loss is then given by the sum of the individual household's EV^j weighted by the marginal utility of income $\frac{\partial v^j}{\partial b^j}$ and the household's welfare weight $\frac{\partial W}{\partial v^j}$ (see for example Fankhauser et al. (1997)).

$$\sum_j \frac{\partial W}{\partial v^j} \frac{\partial v^j}{\partial b^j} EV^j \tag{25}$$

4.1.1 Baseline model (no investments into energy efficiency)

Let $b^j = c^j + q^j \tilde{E}^j$ be total expenditures and $m_E^j = \frac{q_E^j \tilde{E}^j}{b^j}$ the energy expenditure share. With $\frac{\partial u^j}{\partial E^j} \alpha^j = q^j \frac{\partial u^j}{\partial c^j}$ from the first order conditions of the short-run household optimization problem (equations (4) - (5)), we obtain for the energy expenditure share:

$$m_E^j = 1 - \beta + \frac{\beta q \bar{E}}{\alpha_j b^j} \tag{26}$$

Hence, for a homothetic utility function (with $\bar{E} = 0$), energy expenditure shares are constant and equal $1 - \beta$. Because of subsistence energy consumption $\hat{E}$, energy expenditure shares decrease with rising total expenditure. Further, if energy conversion efficiency α_j is high, energy expenditure share is lower. Importantly, there is no horizontal heterogeneity in energy expenditure shares if preferences are homothetic and $\bar{E} = 0$.

4.1.2 Investments into energy efficiency

In the case that households can increase energy efficiency α^j according to $\alpha^j = f(x_0^j + x^j)$, we assume an iso-elastic production function for energy efficiency: $f(z) = \psi_0 z^\varepsilon$. First-order conditions become

$$m_E^j = (1 - \beta)(1 - m_X) + \frac{\beta q_e^j \hat{E}}{\alpha_j b^j} \tag{27}$$

$$m_X^j = m_E^j - \frac{(1 - s) x_0^j}{b^j} \tag{28}$$

with the adjusted budget equation $b^j = c^j + q_E^j E^j + (1-s)x^j = (m_C^j + m_E^j + m_X^j)b^j$ and m_y denoting the budget expenditure share to y.

4.2 Calibration

We determine the structural parameters of the utility function based on German household data from the EVS 2018. We split households in 10 income deciles, based on their adult-equivalent household expenditures, as expenditure are a better proxy for permanent-income than annual income, which changes strongly over the life-cycle of an individual. We then calculate for each income decile ten energy expenditure deciles, given a grid of 10×10 household types. Since, this paper focuses on horizontal inequality within income deciles the later analysis only considers 10 energy-efficiency deciles within the median income decile, i.e 10 different household types in this grid. The left panel in Fig. 1 shows this heterogeneity for German household data over different income deciles.

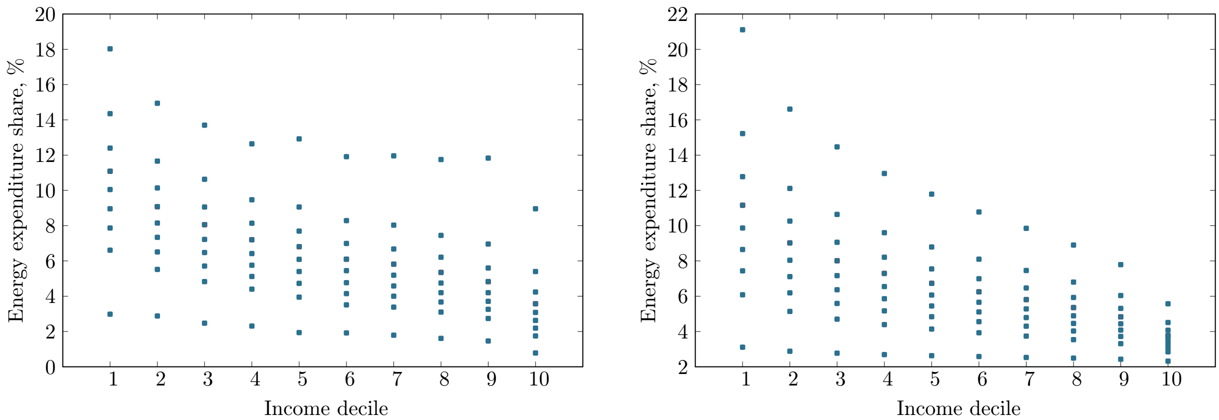

Figure 1: Hetereogeneity of households. Left: mean expenditure shares in 10x10 grid based on EVS 2018 data. Right: mean expenditure shares from fitted model.

We estimate the parameter by implementing linear regression on (26) using energy expenditure decile dummies to estimate the coefficient $\frac{\beta q_e \bar{E}}{\alpha_j}$. For calculating α^j, we further need to impose values on q^E and $\bar{E}$. Because $\bar{E}$ scales with α^j, there is an additional degree of freedom and we can set $\bar{E} = 1$ without loss of generality of the

α^j	y^j
5.007498409	9315.925
2.503749205	12213.454
1.232721058	14329.374
0.914952033	16327.830
0.745341376	18318.064
0.627822982	20446.003
0.537748476	22903.540
0.456117483	26152.103
0.371109856	31254.175
0.255868986	51494.737

Table 1: Calibrated energy efficiency levels α^j and exogenous income levels y^j for modeling household heterogeneity. The median income decile is $\tilde{y} = 18318.064$ Euro/year.

model. [6] We can therefore set $\bar{E} = 1$ and obtain α_j directly from the expenditure share data with (26). In EVS 2018, mean annual energy expenditures amount to 1752 Euro per household, implying 882.50 Euro per capita. From the environmental accounting (YEAR XXX), carbon emissions related to households energy consumption are 2.95 tCO_2 per capita. This implies a price of energy (measured in ton of carbon) of $q = 462$ Euro/tCO_2.

[6]This can be seen from the utility function (22). With $\tilde{E}^j := E^j/\hat{E}$, we get

$$u^j(c^j, E^j) = \frac{\left(c^{j\,\beta}\left(\alpha_j \tilde{E}^j \bar{E} - \hat{E}\right)^{1-\beta}\right)^{1-\eta}}{1-\eta} = \frac{\left(c^{j\,\beta}\left((\alpha_j \tilde{E}^j - 1)\bar{E}\right)^{1-\beta}\right)^{1-\eta}}{1-\eta}$$

$$= \frac{\left(c^{j\,\beta}\left(\alpha_j \tilde{E}^j - 1\right)^{1-\beta}\bar{E}^{1-\beta}\right)^{1-\eta}}{1-\eta}$$

$$= \frac{\left(c^{j\,\beta}\left(\alpha_j \tilde{E}^j - 1\right)^{1-\beta}\right)^{1-\eta}}{1-\eta}\left(\bar{E}^{1-\beta}\right)^{1-\eta} = u^j(c^j, \tilde{E}^j; \bar{E} = 1)\left(\bar{E}^{1-\beta}\right)^{1-\eta}$$

Hence, utility will be just scaled by the factor $\bar{E}^{(1-\beta)(1-\eta)}$.

4.3 Numerical Results

In the following we present the results of the numerical optimization[7] of climate policy instruments that achieve a 20% reduction in aggregate household CO_2-intensive energy consumption. In order to purely focus on horizontal inequality, we focus on the variation of energy expenditures within the median income decile.[8] Socially optimal policy instruments are then calculated for ten different efficiency deciles α^j within that median income decile. Efficiency decile 1 includes the most efficient households ($\alpha^1 = 5.01$) whereas efficiency decile 10 captures the least efficient households ($\alpha^{10} = 0.26$). In addition we vary the households' elasticity of the marginal utility of comprehensive consumption η and thereby also the combined concavity parameter ϵ. It measures the combined aversion to horizontal inequality, from 0.1 (low inequality aversion) to 2 (high inequality aversion), which allows us to capture different degrees of overall societal aversion to horizontal inequality.

As a first step we consider the first-best optimum that can be achieved with a uniform carbon tax that must equal the social cost of energy consumption (Pigouvian component), while the resulting tax revenue should be redistributed household-specifically (see Proposition 2). For that case we calculate an optimal uniform carbon tax of $t_E^* = 694$ Euro/ton CO_2 that achieves a 20% reduction in aggregate household energy consumption.[9]

Figure 2 shows how the resulting carbon tax revenue should be optimally redistributed to the ten household types in order to take horizontal inequality within the median income decile into account. How this 'should' be done depends on the overall societal aversion to horizontal inequality reflected by ϵ that is influenced by both private inequality aversion η and governmental inequality aversion γ. Panel A depicts the aggregate

[7]Results have been calculated using the AMPL programming language together with the Knitro solver (version 10.2).

[8]The median income is $\tilde{y} = 18318.064$ Euro per year.

[9]Note that in the short-term households cannot invest in more energy efficient technologies and hence, the carbon tax that is necessary to instantaneously reduce CO_2 emissions by 20% is relatively high.

transfer payments that targets both horizontal inequality without climate policy and the additional effect on horizontal inequality through the introduction of carbon taxation. Panel B isolates the transfer payments without climate policy (business as usual-BAU), while panel C shows the additional transfer targeting the pure effect of carbon taxation. In panel D these additional transfers are expressed as a percentage of household's policy costs as measured by EV.

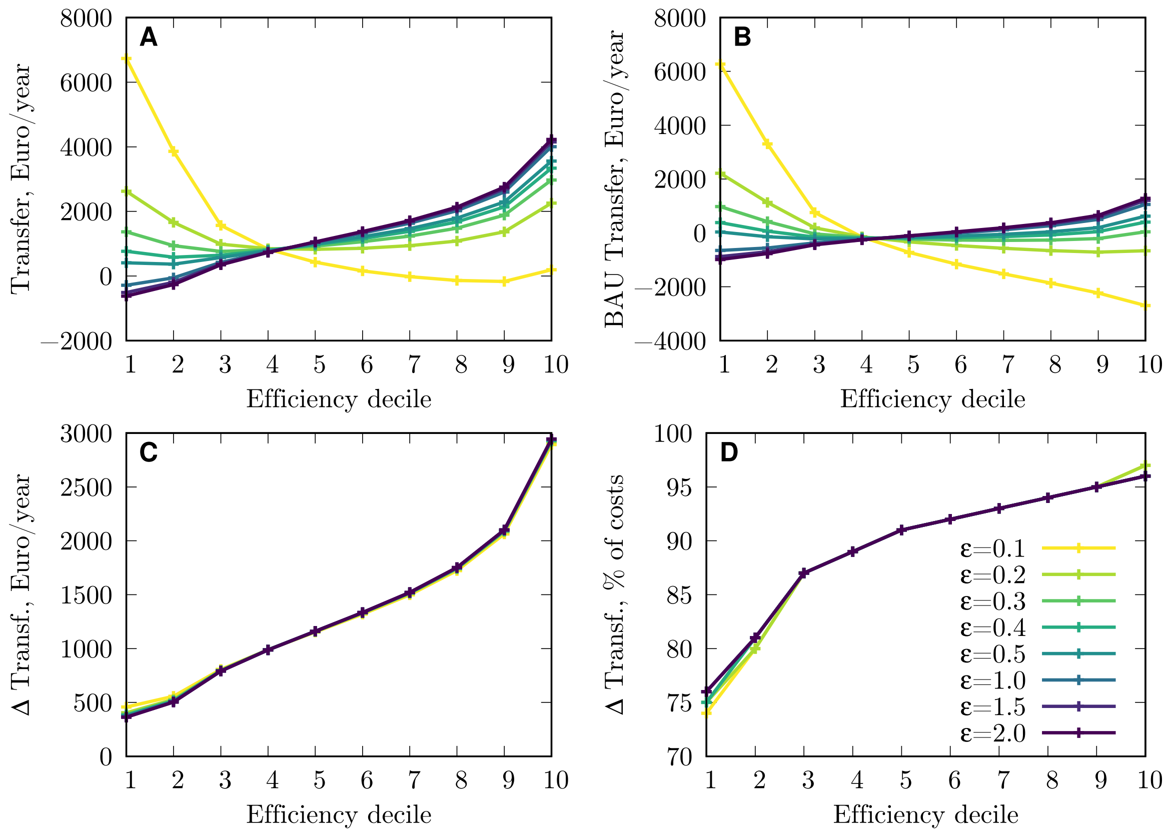

Figure 2: **Optimal differentiated transfers** for energy-efficiency deciles (1=high, 10=low) and inequality aversion $\epsilon = [0.1; 2]$. **A** shows transfers with an optimal uniform carbon tax $t_E^* = 694$ Euro/ton CO_2 set to achieve a 20% reduction in total carbon emissions, **B** depicts transfers without climate policy, i.e. $t_E = 0$, **C** shows the additional transfer due to the introduction of climate policy and **D** expresses this change as a percentage of households' policy costs.

Comparing panels A and B with panels C and D reveals that while transfer payments that target horizontal equity in the absence of climate policy vary with inequality aversion ϵ, the additional transfers due to the introduction of climate policy are almost independent of ϵ. In the first case (panels A and B) the equity-efficiency trade-off in allocating transfers to households becomes clearly visible: When ϵ is rather low, higher transfers are given to the most energy-efficient households as they can best convert the additional income into well-being. The higher ϵ, i.e. the higher the societal preference to care about horizontal equity for a given level of private household inequality aversion, the more it is socially optimal to allocate higher transfers to less energy-efficient households. In the second case (panels C and D) the additional transfers increase in the energy-efficiency decile irrespective of governmental inequality aversion. When climate policy is introduced, it is thus always socially optimal to take horizontal inequality into consideration by redistributing a higher fraction of the carbon tax revenue to less energy-efficient households. Depending on the energy-efficiency decile, panel D shows that the additional transfers cover between 74% and 96% of households' policy costs.

In a second step we now turn to the case of differentiated carbon taxes with en equal-per-household redistribution of tax revenue that amounts to between 1404 ($\epsilon = 0.1$) and 1128 Euro per year ($\epsilon = 2$). Figure 3 panel A depicts the optimal carbon tax for each efficiency decile. While for low values of ϵ the efficiency arguments dominates the socially optimal amount of the tax, i.e. households that are very energy efficient receive a preferential treatment by the government in form of a lower carbon tax, the equity argument dominates for higher ϵ implying lower taxes for less energy-efficient households.

In panel B we additionally calculate non-linear carbon taxes of the form $\hat{t}_E^j = t_{E_0} + t_{E_1} \times \tilde{E}^j$ for the case when the government cannot directly observe the households' heterogeneous energy-efficiency levels α^j. This is probably the most policy relevant case since the energy-efficiency of households will most likely only partly be observable by the government for example by using energy certificates of buildings that reveal the efficiency of households' heating devices. Other factors that influence α^j, like mobility preferences and commuting distance, are rather outside the scope of governmental regulation as already pointed out by Kaplow (1992). Thus, the suggested simple rule for differentiated carbon taxation would have the merit of being able to target horizontal inequality – and thus likely increase the political feasibility of carbon taxation – without knowing the

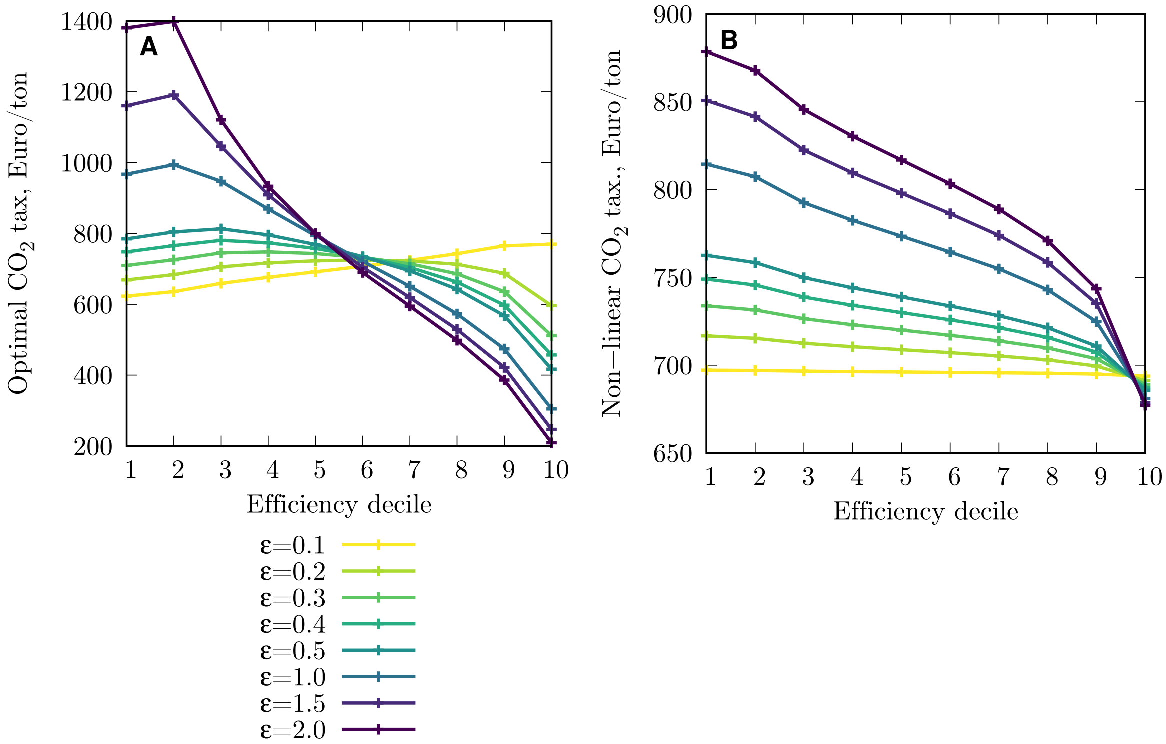

Figure 3: **Optimal differentiated carbon taxes** for 10 energy-efficiency deciles (1=high, 10=low) and inequality aversion $\epsilon = [0.1; 2]$. Panel **A** shows the optimal taxes on CO_2 emissions to achieve a 20% reduction in overall emissions and **B** depicts optimal non-linear taxes when the energy efficiency levels cannot be observed by the government.

determinants of horizontal inequality in the background. Compared to panel A, panel B shows that that the simple rule for the carbon tax captures the trade-off between efficiency and horizontal equity in a similar but less pronounced way. While the carbon tax is almost uniform across household types for low inequality aversion, it favors less-energy efficient households through lower taxes for higher inequality aversion. Table 2 provides an overview of the numerical results on the three household-specific policy instruments to target horizontal equity analysed in this paper.

In a last step we now compare the suggested policy instruments in terms of their implied impact on monetized aggregate social welfare. Figure 4 illustrates the results of Proposition 2 and Corollary 2. The first best solution to tackle horizontal inequality can be achieved by a household-specific redistribution of the revenue from uniform car-

Household-type (Efficiency decile)	**Δ Transfers [€/year] (% of costs covered)**	**Taxes [€/tCO₂]**	**Non-linear taxes [€/tCO₂]**
Low aversion to horizontal inequality $\epsilon = 0.1$			
1 = Most efficient	458 (74%)	623	697
5 = Medium efficient	1152 (91%)	692	696
10 = Least efficient	2893 (97%)	770	674
Medium aversion to horizontal inequality $\epsilon = 1$			
1 = Most efficient	367 (76%)	967	815
5 = Medium efficient	1160 (91%)	794	773
10 = Least efficient	2942 (96%)	305	681
High aversion to horizontal inequality $\epsilon = 2$			
1 = Most efficient	363 (76%)	1380	879
5 = Medium efficient	1160 (91%)	800	817
10 = Least efficient	2945 (96%)	209	677

Table 2: Household-specific policy instruments to target horizontal equity for different energy efficiency deciles.

bon taxation. In this case the social welfare impact due to the introduction of climate policy can be minimized at around -0.65% of total household income before redistribution. If in turn the government redistributes tax revenue on an equal-per-household basis it has to accept higher welfare losses. The welfare loss increases with the combined inequality aversion ϵ from 0.7% for $\epsilon = 0.1$ to 1.4% for $\epsilon = 2$. Between these two options the second-best policies that take horizontal inequality into account consists of differentiated carbon taxation and non-linear carbon taxation while both option include equal-per-household redistribution of tax revenue. We find that while differentiated carbon taxes only lead to an aggregate welfare loss that is around 0.1 percentage points higher, the differentiated non-linear carbon tax performs only marginally better compared to the case with uniform carbon taxation. However, the more society values a horizontally equitable distribution of the impacts of carbon taxation as reflected by ϵ, the better is the performance of non-linear taxes from an aggregate societal point of view. Furthermore, despite the similar performance on an aggregate level, non-linear carbon taxation will likely increase the political feasibility and acceptability of carbon taxes, since households' policy costs in low energy efficiency deciles will be partly covered as compared to uniform taxation.

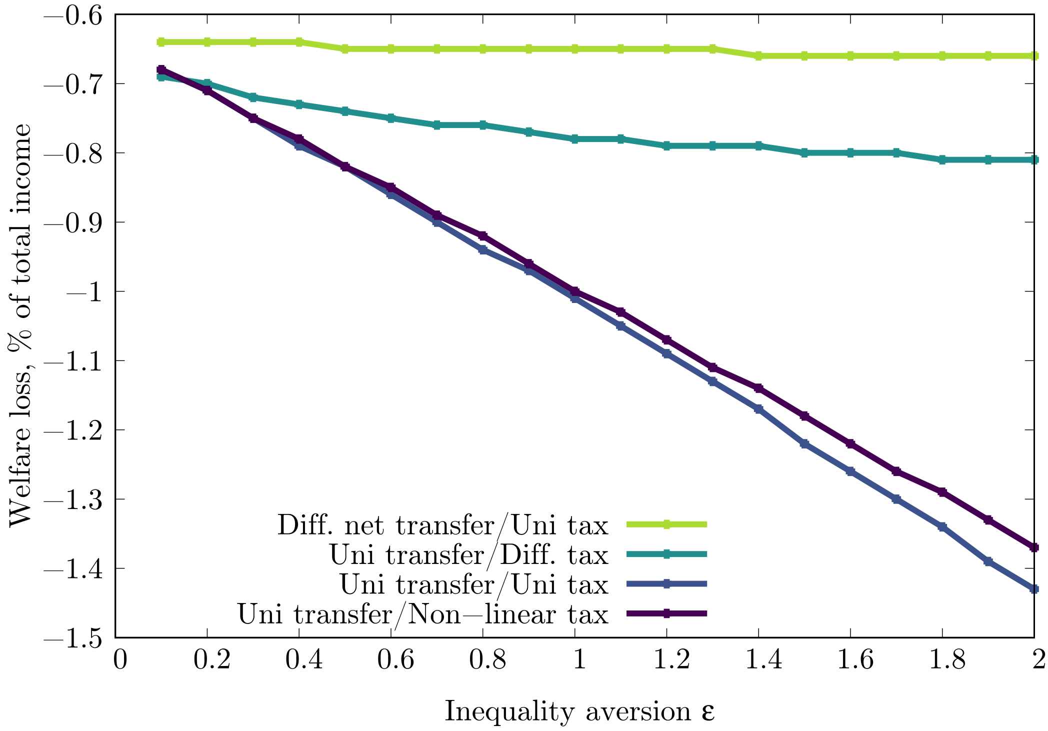

Figure 4: Social welfare changes with different policy instruments.

5 Conclusions

In this paper we have developed a welfare-theoretic model of optimal carbon taxation and redistribution to examine how horizontal inequality due to energy regulation is considered within the stylized economy's welfare maximizing tax structure. Our model captures households that are heterogeneous in terms of how efficient they can convert energy into individual well-being. We have derived explicit policy rules for first and second-best carbon taxes and transfers payments.

We show that the government's first best solution to address horizontal inequality is to set the carbon tax equal to the Pigouvian level and recycle the carbon tax revenue through household-specific transfer payments. If transfers are uniform horizontal equity can also be considered by individual carbon taxes. We show that both the household-specific carbon tax consists of two components. The first is equal to all households and reflects the marginal social cost of achieving the environmental target. The second component depends on individual household characteristics and hence also takes differences in disposable income due to heterogeneous energy efficiencies into account. If energy expenditures only make up a negligible part of the household's total expenditure, this term is very small and the tax is close to the Pigouvian level. The greater the share of energy expenditure, the more the tax deviates from the Pigouvian level, i.e distributional aspects have a larger impact on the optimal carbon tax rule.

The numerical application to German household data reveals that, depending on the societal preferences about horizontal equity versus economic efficiency, it is welfare-optimal to redistribute a higher fraction of the tax revenue to either very efficient households with low carbon footprints or hardship cases with high carbon-intensive energy expenditure shares. For the rather likely case that the government can not perfectly observe all individual household characteristics, we also compute a non-linear household-specific carbon tax that only relies on information about the households' carbon-intensive energy consumption.

References

Andersson, J. J. (2019, November). Carbon Taxes and CO2 Emissions: Sweden as a Case Study. *American Economic Journal: Economic Policy 11*(4), 1–30.

Aronsson, T. and T. Sjögren (2018). Optimal taxation, redistribution, and environmental externalities. *International Review of Environmental and Resource Economics 11*(3), 233–308.

Atkinson, A. B. (1970). On the measurement of inequality. *Journal of Economic Theory 2*(3), 244 – 263.

Atkinson, A. B. and J. E. Stiglitz (1976). The design of tax structure: Direct versus indirect taxation. *Journal of Public Economics 6*(1-2), 55–75.

Auerbach, A. J. and K. A. Hassett (2002). A new measure of horizontal equity. *The American Economic Review 92*(4), 1116–1125.

Bentham, J. (1789). *An introduction to the principles of morals and legislation.* London: T. Payne and Son.

Bossert, W. (1995). Redistribution mechanisms based on individual characteristics. *Mathematical Social Sciences 29*(1), 1–17.

Cremer, H., F. Gahvari, and N. Ladoux (2003). Environmental taxes with heterogeneous consumers: an application to energy consumption in france. *Journal of Public Economics 87*(12), 2791–2815.

Cronin, J. A., D. Fullerton, and S. Sexton (2019). Vertical and horizontal redistributions from a carbon tax and rebate. *Journal of the Association of Environmental and Resource Economists 6*(S1), S169–S208.

Diamond, P. A. and J. A. Mirrlees (1971). Optimal taxation and public production i: Production efficiency. *The American Economic Review 61*(1), 8–27.

Edenhofer, O., M. Franks, and M. Kalkuhl (2021). Pigou in the 21st century: a tribute on the occasion of the 100th anniversary of the publication of the economics of welfare. *International Tax and Public Finance*, 1–32.

Elkins, D. (2006). Horizontal equity as a principle of tax theory.

Epstein, L. G. and S. E. Zin (1989). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica 57*(4), 937–969.

Fankhauser, S., R. S. Tol, and D. W. Pearce (1997). The aggregation of climate change damages: a welfare theoretic approach. *Environmental and Resource Economics 10*, 263–91.

Financial Times (2019). Eu economists call for carbon taxes to hit earlier net zero goal. *Financial Times*. https://www.ft.com/content/137b9da8-99c4-11e9-8cfb-30c211dcd229.

Fischer, C. and W. A. Pizer (2019). Horizontal equity effects in energy regulation. *Journal of the Association of Environmental and Resource Economists 6*(S1), S209–S237.

Gugler, K., A. Haxhimusa, and M. Liebensteiner (2021). Effectiveness of climate policies: Carbon pricing vs. subsidizing renewables. *Journal of Environmental Economics and Management 106*, 102405.

Jacobs, B. and F. van den Ploeg (2019). Redistribution and pollution taxes with non-linear engel curves. *Journal of Environmental Economics and Management 95*, 198–226.

Jordahl, H. and L. Micheletto (2005). Optimal utilitarian taxation and horizontal equity. *Journal of Public Economic Theory 7*(4), 681–708.

Kahneman, D. and A. Tversky (1979). Prospect theory: An analysis of decision under risk. *Econometrica 47*(2), 249–266.

Kaplow, L. (1989). Horizontal Equity: Measures in Search of a Principle. *National Tax Journal 42*(2), 139–154.

Kaplow, L. (1992). A note on horizontal equity. *Florida Tax Review 1*, 191.

Kaplow, L. (2008). Optimal policy with heterogeneous preferences. *The B.E. Journal of Economic Analysis & Policy 8*(1), 1539.

Kaplow, L. (2010). Concavity of utility, concavity of welfare, and redistribution of income. *International Tax and Public Finance 17*(1), 25–42.

Klenert, D., G. Schwerhoff, O. Edenhofer, and L. Mattauch (2018). Environmental taxation, inequality and engel's law: The double dividend of redistribution. *Environmental and Resource Economics 71*(3), 605–624.

Levi, S. (2021). Why hate carbon taxes? machine learning evidence on the roles of personal responsibility, trust, revenue recycling, and other factors across 23 european countries. *Energy Research Social Science 73*, 101883.

Mankiw, N. G., M. Weinzierl, and D. Yagan (2009). Optimal taxation in theory and practice. *Journal of Economic Perspectives 23*(4), 147–174.

Musgrave, R. A. (1990). Horizontal equity, once more. *National Tax Journal 43*(2), 113–122.

Nordhaus, W. (2019, June). Climate change: The ultimate challenge for economics. *American Economic Review 109*(6), 1991–2014.

Ohlendorf, N., M. Jakob, J. C. Minx, C. Schröder, and J. C. Steckel (2021). Distributional impacts of carbon pricing: A meta-analysis. *Environmental and Resource Economics 78*(1), 1–42.

Parry, I. (2015). Carbon Tax Burdens on Low-Income Households: A Reason for Delaying Climate Policy? CESifo Working Paper Series 5482, CESifo.

Pigou, A. (1920). *The Economics of Welfare*. London: Macmillan and Co.

Pizer, W. A. and S. Sexton (2019). The distributional impacts of energy taxes. *Review of Environmental Economics and Policy 13*(1), 104–123.

Poterba, J. M. (1991). Is the gasoline tax regressive? *Tax Policy and the Economy 5*, 145–164.

Ramsey, F. P. (1928). A mathematical theory of saving. *The Economic Journal 38*(152), pp. 543–559.

Rausch, S., G. E. Metcalf, and J. M. Reilly (2011). Distributional impacts of carbon pricing: A general equilibrium approach with micro-data for households. *Energy Economics 33*, S20–S33.

Sandmo, A. (1975). Optimal taxation in the presence of externalities. *The Swedish Journal of Economics 77*(1), 86.

Shammin, M. R. and C. W. Bullard (2009). Impact of cap-and-trade policies for reducing greenhouse gas emissions on U.S. households. *Ecological Economics 68*(8-9), 2432–2438.

Slesnick, D. T. (1989). The measurement of horizontal inequality. *The Review of Economics and Statistics 71*(3), 481.

Stiglitz, J. E. (1982). Utilitarianism and horizontal equity. *Journal of Public Economics 18*(1), 1–33.

The Economist (2021). Fighting climate change - the world urgently needs to expand its use of carbon prices. https://www.economist.com/briefing/2020/05/23/the-world-urgently-needs-to-expand-its-use-of-carbon-prices.

Wang, Q., K. Hubacek, K. Feng, Y.-M. Wei, and Q.-M. Liang (2016). Distributional effects of carbon taxation. *Applied Energy 184*, 1123–1131.

Appendix A Proofs for Section 3

A.1 Proof of Proposition 2

Proof. If the government sets $t_E = \frac{\mu}{\gamma}$, then by using (4) equation (11) can be shown to be identical to equation (8). By using (5), equation (11) can be shown to be identical to equation (7).

With $t_E = \frac{\mu}{\gamma}$, Roy's identity $\frac{\partial v^j}{\partial q} = -\lambda^j \widetilde{E}^j$, equation (5) and using (7), equation (10) can be shown to hold.

Using (7) and (8), we can derive two different expressions for W_u^j. Eliminating the latter yields (13). □

A.2 Proof of Corollary 2

Proof. The conditions for the first best to hold are (7) and (8). The latter can be shown to hold by reformulating the government's FOCs (16) and (17). The former, however, is violated as soon as $\Phi_j \neq 0$. To see this, recall (5), which can be plugged into (8) to yield

$$W_u^j u_E^j \alpha_j = \gamma p + \gamma t_E^j$$

If $\Phi_j = 0$, then $t_E^j = \frac{\mu}{\gamma}$ and (7) holds. Otherwise, this condition for the first best allocation does not hold. □

Appendix B Efficiency enhancing investments by households

In the long run, households can make investments in efficiency-enhancing capital, i. e. $x^j \in \mathbb{R}$.

B.0.1 Social planner economy

Analogously to the short-run described above, the social planner now chooses an allocation of numeraire consumption, energy services consumption and investments in energy efficiency to maximize welfare. The Lagrangian, hence, is

$$L^{SP} = W(u^1, ..., u^n) + \mu(\bar{E} - \sum_j \widetilde{E}^j) + \gamma(\sum_j y^j - c^j - p\widetilde{E}^j - x^j), \tag{29}$$

Proposition 1. *In the long-run, the social optimum in our model can be achieved under the condition that*

$$0 = W_u^j u_E^j \alpha_j - \mu - \gamma p \quad \forall j \tag{30}$$

$$0 = W_u^j u_c^j - \gamma \quad \forall j \tag{31}$$

$$0 = W_u^j u_E^j \widetilde{E}^j f'(x_0^j + x^j) - \gamma \quad \forall j \tag{32}$$

B.0.2 Decentralized market economy

Household's optimization then yields the Lagrangian

$$L^H = u(c^j, E^j) + \lambda^j(y - c^j - \frac{q^j E^j}{\alpha^j} - (1 - s^j)x^j), \tag{33}$$

We assume that the government implements a combination of the following household-specific or uniform instruments: energy taxes t_E^j, transfers R^j and subsidies s^j on efficiency-enhancing investments. Household's optimization then yields FOCs

$$u_c^j = \lambda^j \tag{34}$$

$$u_E^j = \frac{\lambda^j q^j}{f(x_0^j + x^j)} \tag{35}$$

$$u_E^j \widetilde{E}^j f' = \lambda^j (1 - s^j) \tag{36}$$

Uniform tax, individual transfers, no subsidy. The government maximizes social welfare $W(u^1, ..., u^n)$, subject to the environmental target $\bar{E} = \sum_j \widetilde{E}^j$ and its budget constraint $\sum_j (t_E \widetilde{E}^j - R^j) = 0$. The government's Lagrangian is now

$$L^G = W(v^1, ..., v^n) + \mu(\bar{E} - \sum_j \widetilde{E}^j) + \gamma \sum_j \left(t_E \widetilde{E}^j - R^j \right) \tag{37}$$

and the FOCs

$$L^G_{R^j} = W_{v^j} v^j_b - \mu \frac{\partial \widetilde{E}^j}{\partial b^j} - \gamma + \gamma t_E \frac{\partial \widetilde{E}^j}{\partial b^j} = 0 \quad \forall j \tag{38}$$

$$L^G_{t_E} = \sum_j W_{v^j} v^j_q - \mu \sum_j \frac{\partial \widetilde{E}^j}{\partial q} + \gamma \sum_j \widetilde{E}^j + \gamma t_E \sum_j \frac{\partial \widetilde{E}^j}{\partial q} = 0 \tag{39}$$

Proposition 2. *In the long-run, the government can achieve the 1st-best optimum by using individual transfers and a uniform tax on energy use. The latter is determined by*

$$t^*_E = \frac{\mu}{\gamma} \tag{40}$$

The optimal transfers are determined indirectly by

$$\frac{\mu}{\gamma} + p = \alpha_j \frac{{u_E}^*}{{u_c}^*} = \alpha_j MRS^* \ \forall j \tag{41}$$

and the governments budget

$$t^*_E \bar{E} = \sum_j {R^j}^*. \tag{42}$$

Proof. The proof is analogous to Proposition 2 □

Uniform tax, uniform transfers, uniform subsidy. The government's Lagrangian is now

$$L^G = W(v^1, ..., v^n) + \mu(\bar{E} - \sum_j \widetilde{E}^j) + \gamma \sum_j \left(t_E \widetilde{E}^j - R - s x^j \right) \tag{43}$$

and the FOCs

$$L_R^G = \sum_j W_{v^j} v_b^j - \mu \sum_j \frac{\partial \widetilde{E}^j}{\partial b^j} - n\gamma + \gamma t_E \sum_j \frac{\partial \widetilde{E}^j}{\partial b^j} - \gamma s \sum_j \frac{\partial x^j}{\partial b^j} = 0 \tag{44}$$

$$L_{t_E}^G = \sum_j W_{v^j} v_q^j - \mu \sum_j \frac{\partial \widetilde{E}^j}{\partial q} + \gamma \sum_j \widetilde{E}^j + \gamma t_E \sum_j \frac{\partial \widetilde{E}^j}{\partial q} - \gamma s \sum_j \frac{\partial x^j}{\partial q} = 0 \tag{45}$$

$$L_s^G = \sum_j W_{v^j} v_s^j - \mu \sum_j \frac{\partial \widetilde{E}^j}{\partial s} + \gamma t_E \sum_j \frac{\partial \widetilde{E}^j}{\partial s} - \gamma \sum_j x^j - \gamma s \sum_j \frac{\partial x^j}{\partial s} = 0 \tag{46}$$

Proposition 3. *With linear tax and transfer, additionally using a subsidy on x^j can be welfare enhancing.*

Proof. The FOCs can be reformulated

$$t_E = \frac{\mu}{\gamma} - \frac{\sum_j W_v^j \frac{\lambda^j}{\gamma}}{\sum_j \widetilde{E}_b^j} + \frac{n}{\sum_j \widetilde{E}_b^j} + s \frac{\sum_j x_b^j}{\sum_j \widetilde{E}_b^j}$$

$$t_E = \frac{\mu}{\gamma} + \frac{\sum_j W_v^j \frac{\lambda^j}{\gamma} \widetilde{E}^j}{\sum_j \widetilde{E}_q^j} - \frac{\sum_j \widetilde{E}^j}{\sum_j \widetilde{E}_q^j} + s \frac{\sum_j x_q^j}{\sum_j \widetilde{E}_q^j}$$

$$t_E = \frac{\mu}{\gamma} - \frac{\sum_j W_v^j \frac{\lambda^j}{\gamma} x^j}{\sum_j \widetilde{E}_s^j} + \frac{\sum_j x^j}{\sum_j \widetilde{E}_s^j} + s \frac{\sum_j x_s^j}{\sum_j \widetilde{E}_s^j}$$

Eliminating t_E by equalizing the first two expressions yields

$$s = \frac{\sum_j \widetilde{E}_q^j \left(\sum_j W_v^j \frac{\lambda^j}{\gamma} - n \right) - \sum_j \widetilde{E}_b^j \left(\sum_j W_v^j \frac{\lambda^j}{\gamma} \widetilde{E}^j + \sum_j \widetilde{E}^j \right)}{\sum_j x^j \sum_j \widetilde{E}_q^j - \sum_j x_q^j \sum_j \widetilde{E}_b^j}$$

In general, the expression for s is non-zero.

□