Optimal redistribution or predistribution? Minimum wages vs income taxes when workers differ in both hourly wages and working hours

Aart Gerritsen*

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The minimum wage has gained in popularity among policy makers and economists alike, partly as a result of new empirical evidence on its mild employment effects. Nevertheless, most theoretical studies have a hard time finding a useful role for the minimum wage alongside the income tax. All of these studies assume a one-to-one correspondence between wages and income. I reconsider the case for the minimum wage when people differ in both wages and preferences for work, such that a given level of income may correspond to different wage rates. This renders the minimum wage unambiguously desirable in a discrete-type labor market à la Stiglitz (1982). But desirability of the minimum wage is ambiguous and subject to a policy trade-off in a continuous-type labor market à la Mirrlees (1971). Compared to the minimum wage, income taxes are less effective in affecting the wage distribution but more effective in redistributing income. The desirability of the minimum wage depends on this trade-off between the "predistributive" benefits of the minimum wage and the "redistributive" benefits of the income tax.

JEL: H21; J38
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1 Introduction

The minimum wage is a politically popular policy instrument meant to improve living standards of the working poor. Virtually all rich countries have a legal minimum wage and political debates over raising the minimum wage are everywhere a recurring phenomenon.[1] Empirical studies on the impact of the minimum wage have greatly contributed to the popularity of the minimum wage among economists, politicians, and the public at large.

*Erasmus University Rotterdam, PO Box 1738, 3000 DR Rotterdam, The Netherlands, agerritsen@ese.eur.nl. Internet: https://sites.google.com/site/aartgerritsen/.

[1] Countries without a legal minimum wage typically have union-negotiated wage floors on sectoral levels. Examples of recently enacted minimum wage policies are the binding minimum wage in Germany, the living wage in the United Kingdom, and minimum-wage increases on state, county, and even city levels in the United States.

In particular, a growing abundance of empirical studies finds that the minimum wage only has modest adverse effects on employment. This abundance stands in stark contrast to the scarcity of theoretical justifications for the minimum wage. Indeed, regardless of the magnitude of the employment response, most theoretical studies have a hard time finding a useful role for the minimum wage if the government can also use income taxes for redistribution.

Existing theoretical studies always assume a one-to-one mapping between wages and income. This implies that low-wage workers could be equivalently targeted by both a minimum wage or a nonlinear income tax. It is perhaps no wonder then that theoretical studies typically fail to see the merits of a minimum wage over taxation. In reality, there is no one-to-one mapping between wages and income because workers differ widely in both wages and working hours. Workers with low earnings may be full-time workers on a minimum wage or part-time workers on a higher wage. This generates a potential justification for the minimum wage that is not captured by previous studies: a government may be better able to target its redistribution towards low-wage workers with the minimum wage than with a nonlinear income tax. In this paper, I determine under what conditions this is indeed the case. That is, I establish conditions – expressed in sufficient statistics – under which a minimum wage is desirable when people differ in both wages and working hours and government cares about raising the utility of low-wage workers.

The first part of the paper considers a highly stylized competitive labor market with a discrete number of worker types. This is meant to illustrate the basic mechanism that makes a minimum wage potentially better targeted than the income tax. I show that a minimum wage is always desirable if rationing is efficient and the lowest income type consists of both minimum-wage and higher-wage workers. This is almost trivially true if there are only two types of workers that differ in wages but earn the same level of income. In that case, an income tax could hardly redistribute from high-wage workers to low-wage workers – while a minimum wage could. Efficient rationing ensures that utility losses from any demand responses to the minimum wage are of second order. More surprisingly, this unambiguous result in favor of the minimum wage survives when considering a discrete number of additional worker types with higher levels of income à la Stiglitz (1982).

This illustration is useful in illuminating the potential role for a minimum wage. However, the assumptions under which a discrete number of worker types pool at the same income level are rather stark. The second part of the paper therefore considers a more realistic case. I assume that individuals differ along a continuum of both preference types and wage rates, yielding a continuous income distribution à la Mirrlees (1971). In this case, the desirability of a minimum wage is *a priori* ambiguous. Intuitively, a minimum wage distorts employment among the lowest-wage workers by reducing demand. The same distortion could be achieved by raising marginal tax rates among the poor. On the one hand, while such an increase in marginal tax rates would raise low wages through general equilibrium effects, it would be less effective in raising low wages than a minimum wage because the tax rate is imperfectly targeted at the lowest wages. On the other hand, unlike the minimum wage, the increase in marginal tax rates raises revenue from higher-income workers, which could be redistributed to the poor. The desirability of the minimum wage hinges on these two opposing welfare effects. Is a minimum wage desirable because it raises low wages more than a tax reform that leads to comparable distortions? Or is a minimum wage undesirable because, unlike a tax reform that leads to comparable distortions, it does not raise revenue from higher-income workers?

Desirability of the minimum wage is unambiguous only in unrealistic special cases when low wages are either uncorrelated or perfectly correlated with income. A minimum wage is unambiguously desirable if wages and income are uncorrelated so that the share of minimum-wage workers at a given income level is the same for all income levels. In that case, income is uninformative about wages and redistribution from high- to low-income workers does not help in raising the utility of the lowest-wage workers. A minimum wage is unambiguously undesirable if wages and income are perfectly correlated so that low levels of income exclusively consist of minimum-wage workers while higher levels of income exclusively consist of higher-wage workers. In that case marginal taxes are equally effective at raising low wages (through general-equilibrium effects) as the minimum wage.

Desirability of the minimum wage is *a priori* ambiguous only if low wages are imperfectly correlated to income. I derive a condition under which a binding minimum wage is desirable and write this condition in terms of a small number of sufficient statistics.

Related literature; Allen (1987); Guesnerie and Roberts (1987); Marceau and Boadway (1994); Boadway and Cuff (2001); Lee and Saez (2012); Blumkin and Danziger (2018); Gerritsen and Jacobs (2020); Hungerbühler and Lehmann (2009); Cahuc and Laroque (2013); Lavecchia (2020); Luttmer (2007); Gerritsen (2018); Hummel and Jacobs (2018), etc.

Roadmap.

2 A model with a discrete number of income levels

2.1 The model

I first consider a highly stylized economy in which a minimum wage is unambiguously optimal. This helps in developing the intuition behind later results. The economy has two types of individuals, denoted by $i = \{A, B\}$. There are n^i individuals of each type and individuals supply h^i hours of work – yielding aggregate labor supply of type i equal to $L^i \equiv n^i h^i$. The two types are imperfect substitutes in production. In particular, I assume that the output of a representative firm is given by the following Cobb-Douglas production function:

$$Y = F(L^A, L^B) = (L^A)^\alpha (L^B)^{1-\alpha}, \tag{1}$$

with $\alpha \in (0,1)$ the share parameter. The Cobb-Douglas form is chosen because of its attractive feature of fixed income shares α and $1-\alpha$ for labor types A and B. Wages for both types are given by w^i and the price of output is normalized to 1. Profit maximization ensures that marginal productivity and wage rates are equated for both types, $F_{L^i}(L^A, L^B) = w^i$. I calibrate the share parameter α to ensure that workers from both types earn the same level of income. Thus, I set $\alpha = n^A/(n^A + n^B)$, such that a share $n^A/(n^A + n^B)$ of income is paid to type-A workers. As a result, equilibrium income is the same for both types of workers and equal to $z^i \equiv w^i h^i = Y/(n^A + n^B)$ for both i.

An individual's tax burden is given by a potentially nonlinear function of income $T(z^i)$. Labor is the only source of income and all income is spent on consumption c^i. The budget constraint of worker i is thus given by $c^i = w^i h^i - T(w^i h^i)$. Individuals enjoy consumption but dislike labor supply. This is reflected by their utility function $U^i = u^i(c^i, h^i)$, which is increasing in c^i, decreasing in h^i, concave in both arguments, homogeneous within types, and heterogeneous across types. Equilibrium labor supply is implied by equating the marginal rate of substitution of leisure for consumption with the marginal net-of-tax wage rate:

$$MRS^i(c^i, h^i) \equiv -u^i_h(c^i, h^i)/u^i_c(c^i, h^i) = (1 - T'(z^i))w^i. \tag{2}$$

I assume that $MRS^B(c,h) > MRS^A(c,h)$ for any given bundle of consumption and labor supply $\{c, h\}$. This implies that type-B individuals value leisure more than type-A individuals. Because profit maximization implies that both types earn the same income, it must therefore be the case that type-B workers earn higher wage rates while working less hours than type-A workers: $h^A > h^B$ and $w^A < w^B$. Intuitively, because type-B individuals have a relative distaste for work, their labor is scarcer and therefore more productive on the margin.

2.2 The government's objective

I assume that the social planner wants to maximize the utility of type-A workers so that we can write the social welfare function as $\mathcal{W} = n^A U^A$. This could be rationalized in different ways. First, there is a long tradition in political philosophy that espouses redistribution to compensate individuals for differences in their opportunities but not their preferences (Dworkin, 2000; Roemer, 1996; Fleurbaey, 2008). As long as type-A workers face a lower wage rate than type-B workers, their opportunities to advance in life are worse because they need to work more hours to obtain the same level of consumption. At the same time, preference heterogeneity should in itself not affect the redistributive preferences of the government. Thus, even though both types of workers earn the same income, workers of type A deserve compensation because their low wage rates keep them from earning a higher income, whereas workers of type B do not deserve compensation because they are materially held back only because of their strong preference for leisure. Second, depending on the cardinalization of the two utility functions, one could argue that type-A workers are worst off because they work more hours than type-B workers even though they enjoy the same amount of consumption. Thus, from a more traditionally welfarist point of view, a Rawlsian (maximin) government would also be intent on maximizing the utility of type-A workers.[2]

The government's budget constraint stipulates that tax revenue must be sufficient to cover some exogenously given revenue requirement R, such that $(n^A + n^B)T(z^A) = R$, where I used the fact that $z^B = z^A$. Denoting the Lagrange multiplier of the budget constraint by λ, the government's objective function is given by the following Lagrangian:

$$\mathcal{L} = n^A u^A(w^A h^A - T(w^A h^A), h^A) + \lambda((n^A + n^B)T(z^A) - R), \tag{3}$$

where I substituted the budget constraint into the utility function. The Lagrangian allows me to decompose the marginal social-welfare effects of any policy reform into utility effects and budgetary effects. The first term in eq. (3) shows that a policy yields utility gains to the extent that it leads to higher type-A wages w^A. On the margin, policy-induced changes in working hours do not directly affect utility as long as workers are free to choose their labor hours so that they are on the margin indifferent between earning more or less. The second term in eq. (3) shows that a policy yields budgetary gains to the extent that it expands the tax bases z^i (provided that marginal taxes are positive, $T'(z^i) > 0$; if $T'(z^i) < 0$, a policy yields budgetary gains if it *reduces* tax bases z^i).

2.3 The desirability of a binding minimum wage

In a competitive labor market, a binding minimum wage causes labor rationing by reducing labor demand to a level below nominal labor supply. I assume that workers are rationed on the intensive margin so that minimum-wage workers are forced to work less hours than they desire. In the absence of rationing, utility maximization implies that workers are on the margin indifferent between working more or less hours. As a result, rationing is efficient in the sense that marginal utility losses associated with rationing are of second order when evaluated at an allocation without rationing. The assumption of efficient rationing is typical in theoretical work on minimum wages but not necessarily

[2]While I assume for simplicity that the government *only* cares about workers of type A, the results below only require that the government attaches a larger marginal social welfare weight to workers of type A than to workers of type B.

realistic. It is therefore important to keep in mind that the utility gains of the minimum wage in this model are likely to be an overestimation.[3]

To determine the desirability of a binding minimum wage, I evaluate the social-welfare effects of a small increase in the low wage rate w^A at an allocation without a binding minimum wage. These welfare effects can be decomposed in utility effects and budgetary effects. I then compare this reform to a tax reform that yields identical budgetary effects. If the increase in the minimum wage yields a larger utility gain than the tax reform, then the government can achieve a social-welfare improvement by implementing a binding minimum wage while offsetting any budgetary effects by an appropriate adjustment in taxes. Moreover, if this holds at the allocation in which taxes are set optimally, then a binding minimum wage must be part of the overall policy optimum. Pursuing this line of analysis in the Appendix yields the following Proposition.

Proposition 1 *In an economy with two types of workers who earn the same income but differ in wages, with efficient rationing, and with a government that values redistribution towards low-wage workers, a binding minimum wage is unambiguously part of the policy optimum.*

Proof. See the Appendix. ■

A marginally binding minimum wage raises the wage rate of type-A workers. As a result, type-A labor demand declines, reducing type-A working hours. This triggers general equilibrium effects as the decline in type-A employment reduces marginal productivity and therefore wages of type-B workers.[4] As long as consumption and leisure are normal goods, the decline in type-B wages reduces labor earnings z^B. And since equilibrium income is the same for both types, the reform also reduces type-A labor earnings z^A. Summing up, a marginal increase in a binding minimum wage yields utility gains through its increase in w^A and budgetary losses through its negative effect on tax bases $z^A = z^B$.[5]

The same reduction in tax bases could be achieved by raising the marginal tax rate $T'(z^i)$ – inducing compensated reductions in labor supply for both types. However, the type-A utility gains associated with such an increase in marginal taxes are always smaller than the utility gains of the increase in the minimum wage. To see this, first consider the case in which compensated labor-supply elasticities are the same for both types of workers. In that case, an increase in the marginal tax rate reduces labor supply of both types in the same proportion, leaving marginal productivity and thus wage rates of both

[3]Empirical evidence on the efficiency of rationing is scarce but Luttmer (2007) finds mixed results. Gerritsen (2018) and Gerritsen and Jacobs (2020) discuss policy implications of inefficient rationing.

[4]This in turn affects type-B labor supply, triggering further general equilibrium effects on marginal productivity and therefore labor demand for type-A workers, etcetera until a new equilibrium is reached.

[5]Assuming that $T'(z^i) > 0$; note that if $T'(z^i) \leq 0$, then a marginal increase in the minimum wage yields utility gains without budgetary losses and is therefore unambiguously desirable.

types unaltered. Thus, with identical elasticities, an increase in marginal taxes yields budgetary losses without any offsetting type-A utility gains. The only case in which the increase in the marginal tax rate could raise type-A wages and utility is if compensated elasticities are larger for type-A workers than for type-B workers. In that case, there is a relatively strong reduction in type-A labor supply, yielding an increase in type-A productivity and wages. Nevertheless, because the increase in the marginal tax rate also discourages labor supply of type-B workers – yielding a countervailing effect on type-A wages – the marginal tax rate can never be as effective as the minimum wage in raising type-A utility. Hence, for the same budgetary effect, a minimum wage always yields greater type-A utility gains than the marginal tax rate. Thus, a binding minimum wage must be part of the policy optimum.

2.4 Three types

It seems almost trivial that a binding minimum wage is a more desirable instrument for redistribution than a marginal tax rate on income when individuals differ in their wage rates but have the same level of income. After all, the *prima facie* case for an income tax is not particularly strong when there is no income inequality. It is therefore useful to show that the results from Proposition 1 are robust to adding another worker type with a higher level of income. In particular, assume that there are also n^C workers of type C. Type-C workers enter the production function linearly, earn a higher wage rate than the other two types ($w^C > w^B > w^A$), and have the same utility function as type-A workers ($u^C(\cdot) = u^A(\cdot)$, i.e., type C also has a relatively weak preference for leisure). As a result, type-C income is higher than that of the other two types ($z^C > z^B = z^A$).

The government still wants to maximize type-A utility subject to a budget constraint that now also includes revenue from type-C workers. In addition to the budget constraint, the government faces an incentive constraint that requires workers of type C to prefer "their" equilibrium level of income over that of types A and B:

$$u^C(z^C - T(z^C), z^C/w^C) \geq u^C(z^A - T(z^A), z^A/w^C). \quad (4)$$

This yields an economy that is comparable to the two-type economy of Stiglitz (1982), except that the low level of income is earned by both high- and low-wage workers. Denoting the Lagrange multiplier of the incentive constraint by γ, the government's objective function is given by the following Lagrangian:

$$\tilde{\mathcal{L}} = \mathcal{L} + \lambda n^C T(z^C) + \gamma(u^C(z^A - T(z^A), z^A/w^C) - u^C(z^C - T(z^C), z^C/w^C)), \quad (5)$$

where $\tilde{\mathcal{L}}$ refers to the new Lagrangian and $\mathcal{L}$ to the original Lagrangian in eq. (3). The following Proposition establishes the robustness of Proposition 1.

Proposition 2 *In an economy with three types of workers where two types earn the same income but differ in wages and a third type earns more, with efficient rationing, and with a government that values redistribution towards low-wage workers, a binding minimum wage is unambiguously part of the policy optimum.*

Proof. See the Appendix. ■

The intuition behind Proposition 2 is straightforward. A binding minimum wage and an increase in the marginal tax rate at $z^A = z^B$ both yield a reduction in taxable income z^A. The incentive constraint in eq. (4) is relaxed by a reduction in taxable income z^A, regardless of whether this reduction took place because of an increase in the minimum wage or an increase in the marginal tax rate. Thus, the minimum wage and marginal taxes are both equally effective in relaxing the incentive constraint. However, as was shown in the proof of Proposition 1, for a given reduction in taxable income z^A, a minimum wage raises $\mathcal{L}$ more than an increase in the marginal tax rate $T'(z^A)$. Thus, even with income inequality, an increase in the minimum wage – evaluated in the absence of a binding minimum wage and at any given tax schedule – is still unambiguously more desirable than an increase in marginal taxes. This implies that a binding minimum wage must be part of the policy optimum.

Proposition 2 stands in stark contrast to the results in Allen (1987) and Guesnerie and Roberts (1987), who conclude that a binding minimum wage is undesirable in the discrete-type optimal tax framework of Stiglitz (1982). The only material difference between their model and mine is that I allow for wage heterogeneity among the low-income type workers. This small alteration of the model yields diametrically opposite results. Instead of being unambiguously undesirable, the minimum wage turns out to be unambiguously desirable.

Nevertheless, the stylized nature of the model – discrete rather than continuous types, and the knife-edge calibration of the production function – means that Propositions 1 and 2 should be interpreted with caution. In particular, the welfare calculus of marginal taxes is quite different in a setting with a continuous rather than a discrete income distribution. With a discrete number of income levels, the marginal tax rate at one income level does not immediately affect the tax burden at the higher income level. This is because any such effect could always be undone by an adjustments of the marginal taxes in between the two levels of income. In contrast, if the income distribution is continuous, an increase in the marginal tax rate *does* mechanically lead to an increase in tax burdens for workers with a higher level of income. The next Section shows that, in that case, desirability of a binding minimum wage is no longer unambiguous.

3 A model with a continuous distribution of income

3.1 The model

I now consider a model with a continuum of income levels as in Mirrlees (1971). I extend the standard Mirrleesian model in two directions. First of all, the standard model features a linear production technology. A minimum wage would in that case simply destroy all jobs for workers whose marginal productivity falls short of the minimum wage (Boadway and Cuff, 2001). This is empirically implausible (e.g., Cengiz et al., 2019). Instead, I assume that production is strictly concave in the supply of a mass of the least productive workers. I do retain the assumption that production is linear in the supply of more productive workers. Second, I allow for heterogeneity in both earning capacity and preferences for work as in Jacquet and Lehmann (2021a). This generates heterogeneity in income even conditional on hourly wage rates.

It is again useful to think of two types of workers $i = \{A, B\}$. Type-A workers are indexed by $a \in \mathcal{A}$, with $\mathcal{A}$ a continuum of mass n^A. Type-B workers are indexed by $b \in \mathcal{B}$, with $\mathcal{B}$ a continuum of mass n^B. I normalize $n^A + n^B = 1$ so that n^i can be thought of as the population share of type-i workers. Type-A individuals supply h^a working hours; type-B individuals supply h^b working hours. Type-A workers are homogeneous in their hourly productivity, so that aggregate effective type-A labor supply is given by $L^A = \int_{\mathcal{A}} h^a \mathrm{d}a$. Type-$B$ workers differ in their hourly productivity θ^b, so that aggregate type-B labor supply is given by $L^B = \int_{\mathcal{B}} \theta^b h^b \mathrm{d}b$. Total production is given by:

$$Y = F(L^A) + L^B, \tag{6}$$

with $F'(\cdot) > 0$ and $F''(\cdot) < 0$. Thus, production is strictly concave in type-A labor supply and linear in type-B labor supply. Profit maximization implies that the equilibrium hourly wage rate for type-A workers is given by $w^a = w^A = F'(L^A)$ for all a. The equilibrium hourly wage rate for type-B workers is given by $w^b = \theta^b$. I only consider equilibriums in which $F'(L^A) < \theta^b$ for all b, so that type-A workers earn the lowest wage rate. Thus, a minimum wage will only be binding for type-A workers. The production function features decreasing returns to scale, which implies equilibrium profits. For simplicity, I assume that these profits are fully taxed away.

Utility of individual $j = \{a, b\}$ is given by $U^j = u^j(c^j, h^j)$, with $c^j = w^j h^j - T(w^j h^j)$ consumption and $T(w^j h^j)$ a nonlinear income tax. For simplicity, I abstract from income effects on labor supply by assuming that $u^j_{cc} = 0$. Workers of type A earn the same wage rate w^A but differ in preferences and thus in hours worked. Workers of type B differ in both wage rates and working preferences. Hence, in equilibrium, there is income inequality among both types of workers. Importantly, despite the multidimensional heterogeneity, I assume that marginal changes in the tax system only cause marginal changes in economic behavior. As a result, given the absence of income effects, a marginal increase of the marginal tax rate around some income level z^* only causes marginal labor supply responses of individuals with income around z^*.[6]

The cumulative distribution function of the resulting income distribution is denoted by $G(z)$ with probability density function $g(z) \equiv G'(z)$. The income-contingent share of type-A earners is denoted by $\sigma(z)$. Thus, a share $\sigma(z)$ of people with income level z earn the minimum wage. I define the highest level of type-A income as $\bar{z} \equiv \max\{z^a\}$.

3.2 The government's objective

As before, the government maximizes type-A utility so that social welfare is given by $\mathcal{W} = \int_{\mathcal{A}} u^a(c^a, h^a)\mathrm{d}a$. Since $u^a_{cc} = 0$, the government does not care about redistribution within the group of type-A workers. This can again be rationalized by theories of justice that emphasize compensation for differences in opportunities but not preferences. Taking into account the budget constraint, the government objective can be written as the following Lagrangian:

$$\text{(7)} \qquad \mathcal{L} = \int_{\mathcal{A}} u^a(c^a, h^a)\mathrm{d}a + \lambda\left(\int_0^\infty T(z)\mathrm{d}G(z) + F(L^A) - w^A L^A - R\right)$$

The first term is aggregate type-A utility, λ denotes the Lagrange multiplier of the budget constraint, and the budget constraint within brackets consists of revenue from the income tax, revenue from the profit tax, and exogenous expenditures R.

3.2.1 Minimum-wage desirability with partially nonlinear taxation

I first consider the desirability of the minimum wage under a set of simplifying assumptions. In particular, I assume that the tax schedule is linear over the range $[0, \bar{z}]$ that $\sigma(z) = \sigma$ for $z \in [0, \bar{z}]$, and that the compensated net wage elasticity of labor supply is identical across households. Denote the compensated net wage elasticity of labor supply by ε^S, the wage elasticity of type-A labor demand by ε^D, and the average income of workers with an income below $\bar{z}$ by z^m. In the Appendix, I prove the following Proposition.

Proposition 3 *In an economy with a continuous income distribution, with efficient rationing, where the highest minimum-wage income is $\bar{z}$, where the share of minimum wage workers at each income level below $\bar{z}$ is constant and given by σ, where the tax schedule is restricted to be linear below $\bar{z}$, and with a government that values redistribution towards low-wage workers, an increase in the minimum wage is desirable if the following condition*

[6] Formally ruling out discrete jumps in labor supply requires additional structure on individual preferences. Here I simply assume that discrete jumps are ruled out without further exploring the necessary structural assumption that would guarantee the absence of such behavioral responses. Also see Jacquet and Lehmann (2021a,b).

holds:

$$\left(\frac{1-\sigma G(\bar{z})}{1-G(\bar{z})}\right)\left(\frac{1-\sigma}{\sigma}\right) > \left(\frac{\bar{z}-z^m}{z^m}\right)\frac{\varepsilon^D}{\varepsilon^S}. \tag{8}$$

Proof. See the Appendix. ■

Two highly unrealistic special cases are helpful in explaining the intuition behind Proposition 3. First consider the case in which minimum wage workers represent a given share of the income distribution at each level of income. In that case $G(\bar{z}) = 1$ and the minimum wage is unambiguously desirable. Intuitively, a minimum wage is better at predistribution while the income tax is better at redistribution of income. But when income is uninformative of whether someone earns the minimum wage, the redistribution obtained by income taxation is useless. As a result, the minimum wage is unambiguously desirable. Second, consider the case in which the income of each minimum-wage worker is lower than that of anyone else. In that case, $\sigma = 1$ and the minimum wage is unambiguously undesirable. In that case, the income tax can achieve the same amount of predistribution as the minimum wage for the same distortions. But on top of that, the income tax also yields redistributional benefits. Thus, the minimum wage is undesirable.

More generally, desirability of the minimum wage:

- increases in $G(\bar{z})$: larger $G(\bar{z})$ means that income is less correlated with wages so that income taxes are worse targeted
- decreases in σ: larger σ means income is more correlated with wages so that income taxes are better targeted
- increases in ε^S: larger supply distortions associated with the income tax
- decreases in ε^D: larger demand distortions associated with the minimum wage
- increases in z^m: income taxes tax more away from minimum wage earners so that income taxes are worse targeted

3.2.2 Minimum-wage desirability with fully nonlinear taxation

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4 Conclusion

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A Proofs

A.1 Proof of Proposition 1

I prove Proposition 1 in four steps. First, I present the key equations of the model and their total derivatives. Second, I use these total derivatives to show that an increase in the minimum wage and an increase in the marginal tax rate both lead to a reduction in individual labor earnings. Third, I take the total derivative of the government's goal function to show that, in a clearing labor market, a given reduction in individual labor earnings yield greater social-welfare gains if achieved through a higher minimum wage rather than a higher marginal tax rate. This implies that the implementation of a binding minimum wage is part of a social-welfare enhancing policy reform for any given tax schedule – including the optimum tax schedule. As a result, a binding minimum wage must be part of the policy optimum.

A.1.1 Comparative statics of key model equations

Write the tax function as $T(w^i h^i) = \tau w^i h^i + \mathcal{T}$, with τ the marginal tax rate and $\mathcal{T}$ a lump-sum component of the tax schedule. This parameterization of the tax schedule is without loss of generalization because there is only one income level. As a result, the relevant features of the tax schedule can be summarized by just two parameters: the marginal tax rate τ, and the intercept of the tax schedule $\mathcal{T}$. The key equations of the model can then be summarized as follows.

$$F(n^A h^A, n^B h^B) = w^A n^A h^A + w^B n^B h^B, \tag{9}$$

$$w^A h^A = w^B h^B, \tag{10}$$

$$MRS^i(c^i, h^i) \equiv \frac{-u_h^i((1-\tau)w^i h^i - \mathcal{T}, h^i)}{u_c^i((1-\tau)w^i h^i - \mathcal{T}, h^i)} = (1-\tau)w^i, \quad i = \{A, B\}. \tag{11}$$

The first equation implies that firms run zero profits, which follows from the competitive nature of the labor market and first-degree homogeneity of the production function (constant returns to scale). The second equation implies that labor income of both types are equal, which follows from the Cobb-Douglas nature of production and the particular calibration of the income shares α. The third equation is the first-order condition for labor supply, in which I have substituted the budget constraint. The latter equation allows me to write equilibrium supply of working hours as $h^i = h^i(w^i, \tau, \mathcal{T})$.

The total derivative of the first equation yields:

$$\frac{\mathrm{d}w^B}{w^B} = -\frac{n^A}{n^B}\frac{\mathrm{d}w^A}{w^A}, \tag{12}$$

where I substituted for the firm's first-order conditions ($F_{L^i} = w^i$) and eq. (10). The percentage change in type-B wages is negatively proportional to the percentage change in type-A wages. Intuitively, any increase in type-A wages are paid for by reductions in type-B wages and vice versa.

The total derivative of the second equation yields:

$$\left(1+\frac{n^A}{n^B}\right)\frac{\mathrm{d}w^A}{w^A}=\frac{\mathrm{d}h^B}{h^B}-\frac{\mathrm{d}h^A}{h^A}, \tag{13}$$

where I substituted for the comparative statics in eq. (12). This equation has two different interpretations. In the case of clearing labor markets, it shows how type-A wages (LHS) adjust to accomodate relative changes in both types' labor supply (RHS). In the case of a binding minimum wage, it shows how type-A labor demand adjusts to accomodate relative changes in the minimum wage and type-B labor supply.

The total derivative of the third equation yields:

$$\frac{\mathrm{d}h^i}{h^i}=-e^i_c\frac{\mathrm{d}\tau}{1-\tau}+e^i_u\frac{\mathrm{d}w^i}{w^i}+\eta^i\left(\mathrm{d}\mathcal{T}+w^ih^i\mathrm{d}\tau\right), \tag{14}$$

with e^i_c and e^i_u the compensated and uncompensated wage elasticities of labor supply, and η^i the income semi-elasticity of labor supply. The (semi-)elasticities are defined as:

$$e^i_c\equiv\left(\frac{\partial h^i(w^i,\tau,\mathcal{T})}{\partial w^i}+(1-\tau)h^i\frac{\partial h^i(w^i,\tau,\mathcal{T})}{\partial\mathcal{T}}\right)\frac{w^i}{h^i} = \left((1-\tau)w^ih^i\frac{MRS^i_c}{MRS^i}+\frac{MRS^i_hh^i}{MRS^i}\right)^{-1}>0, \tag{15}$$

$$\eta^i\equiv\frac{\partial h^i(w^i,\tau,\mathcal{T})}{\partial\mathcal{T}}\frac{1}{h^i}=\frac{MRS^i_c}{MRS^i}e^i_c\geq 0. \tag{16}$$

$$e^i_u\equiv\frac{\partial h^i(w^i,\tau,T)}{\partial w^i}\frac{w^i}{h^i}=e^i_c-(1-\tau)w^ih^i\eta^i. \tag{17}$$

A.1.2 Comparative statics of individual labor earnings

Recall that individual labor earnings are defined as $z^i\equiv w^ih^i=z$ for both i. Taking the total derivative and substituting for eqs. (12) and (14) for $i=B$ yields:

$$\frac{\mathrm{d}z}{z}=\frac{\mathrm{d}h^B}{h^B}+\frac{\mathrm{d}w^B}{w^B}=-\left(1+e^B_u\right)\frac{n^A}{n^B}\frac{\mathrm{d}w^A}{w^A}-e^B_c\frac{\mathrm{d}\tau}{1-\tau}+\eta^B\left(\mathrm{d}\mathcal{T}+z\mathrm{d}\tau\right) \tag{18}$$

When considering an increase in the minimum wage, all right-hand side differentials can be considered exogenous. If leisure and consumption are normal goods, the uncompensated wage elasticities of labor supply must exceed minus one, $e^B_u>-1$. The above equation therefore immediately proves that an increase in the minimum wage $\mathrm{d}w^A>0$ causes a reduction in individual labor earnings $\mathrm{d}z<0$.

When considering a change in taxes, the wage differential $\mathrm{d}w^A$ is endogenous to the tax change. Further substituting for eqs. (13) and (14) yields:

$$(19)\quad \frac{\mathrm{d}z}{z} = -\left(\frac{1+e_u^A+\left(1+e_u^B\right)\frac{n^A}{n^B}\frac{e_c^A}{e_c^B}}{1+e_u^A+\left(1+e_u^B\right)\frac{n^A}{n^B}}\right)e_c^B\frac{\mathrm{d}\tau}{1-\tau} + \left(\frac{1+e_u^A+\left(1+e_u^B\right)\frac{n^A}{n^B}\frac{\eta^A}{\eta^B}}{1+e_u^A+\left(1+e_u^B\right)\frac{n^A}{n^B}}\right)\eta^B\left(\mathrm{d}\mathcal{T}+w^Ah^A\mathrm{d}\tau\right).$$

This proves that any *compensated* increase in the marginal tax rate, such that $\mathrm{d}\tau > 0$ and $\mathrm{d}\mathcal{T} + w^Ah^A\mathrm{d}\tau = 0$, causes a reduction in individual labor earnings $\mathrm{d}z < 0$. Thus both minimum wages and marginal taxes can be used to reduce taxable income. The welfare-relevant question is: which instrument yields a larger welfare gain for a given reduction in income?

A.1.3 Marginal social-welfare effects of a policy reform

Recall that the government objective is given by the following Lagrangian:

$$(20)\qquad \mathcal{L} = n^Au^A((1-\tau)w^Ah^A-\mathcal{T},h^A)+\lambda((n^A+n^B)(\tau z+\mathcal{T})-R).$$

The total derivative of the Lagrangian yields:

$$(21)\quad \mathrm{d}\mathcal{L} = (1-\tau)zn^Au_c^A\left(\frac{\mathrm{d}z}{z}-\frac{\mathrm{d}h^A}{h^A}\right)+\tau z(n^A+n^B)\lambda\frac{\mathrm{d}z}{z} + \left((n^A+n^B)\lambda-n^Au_c^A\right)(z\mathrm{d}\tau+\mathrm{d}\mathcal{T}),$$

where I used the envelope theorem for the derivative of type-A utility and substituted for $\mathrm{d}w^A/w^A = \mathrm{d}z/z - \mathrm{d}h^A/h^A$. There are three terms in this derivative. The first term indicates that an increase in type-A wages raises type-A utility and thus social welfare. The second term indicates that an increase in taxable income yields budgetary gains if the marginal tax rate is positive. And the third term indicates that an increase in individual tax burdens yields both budgetary gains and utility losses.

Further substitute for $\mathrm{d}h^A/h^A$ by using eqs. (12), (13), and (14) for $i = B$ yields:

$$(22)\quad \mathrm{d}\mathcal{L} = \frac{(1-\tau)zn^Bu_c^A}{1+e_u^B}\left(\frac{-\mathrm{d}z}{z}-e_c^B\frac{\mathrm{d}\tau}{1-\tau}\right)-\tau z(n^A+n^B)\lambda\frac{-\mathrm{d}z}{z} + \left((n^A+n^B)\lambda-n^Au_c^A+\frac{(1-\tau)zn^Bu_c^A}{1+e_u^B}\eta^B\right)(z\mathrm{d}\tau+\mathrm{d}\mathcal{T}).$$

Now consider two different reforms. One reform raises the minimum wage ($\mathrm{d}w^A > 0$) and keeps taxes fixed. The other reform raises the marginal tax rate and keeps the tax burden fixed ($\mathrm{d}\tau > 0$ and $\mathrm{d}\mathcal{T} = -z\mathrm{d}\tau$). Substituting this into the derivative of the Lagrangian yields the following two equations for the two different reforms:

$$\frac{1}{-\mathrm{d}z/\mathrm{d}w^A}\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}w^A} = \frac{(1-\tau)n^B u_c^A}{1+e_u^B} - \tau(n^A+n^B)\lambda, \tag{23}$$

$$\frac{1}{-(\mathrm{d}z/\mathrm{d}\tau - z\mathrm{d}z/\mathrm{d}\mathcal{T})}\left(\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\tau} - z\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\mathcal{T}}\right) = \frac{(1-\tau)n^B u_c^A}{1+e_u^B} - \tau(n^A+n^B)\lambda - \frac{zn^B u_c^A e_c^B}{1+e_u^B}\frac{1}{-(\mathrm{d}z/\mathrm{d}\tau - z\mathrm{d}z/\mathrm{d}\mathcal{T})}. \tag{24}$$

The first equation gives us the social-welfare effects of raising the minimum wage such that taxable income declines by one unit (hence the division by $-\mathrm{d}z/\mathrm{d}w^A > 0$ on the left-hand side). The second equation gives us the social welfare effects of raising the marginal tax rate such that taxable income declines by the same one unit (hence the division by $-(\mathrm{d}z/\mathrm{d}\tau - z\mathrm{d}z/\mathrm{d}\mathcal{T}) > 0$ on the left-hand side).

The first equation tells us that the minimum wage raises type-A utility (first term) at the cost of a decline in the tax base and thus tax revenue (second term). The first two terms in the second equation are identical. They tell us that an increase in the marginal tax rate raises type-A wages and utility by reducing type-A labor supply (first term) and reduces the tax base and thus tax revenue (second term). However, there is a strictly negative third term. This term represents the fact that a higher tax rate also reduces type-B labor supply, which has the opposite effect on type-A wages and thus *reduces* type-A utility.

Taken together, the two equations prove Proposition 1. They show us that an increase in the minimum wage and an increase in marginal taxes can yield identical reductions in taxable income. But the minimum wage is more effective in raising type-A wages and thus in raising type-A utility. The reason is that the marginal tax rate not only discourages type-A labor supply (raising type-A wages) but also type-B labor supply (reducing type-A wages). Finally, notice that the second equation must equal zero in the tax optimum. As a result, in the tax optimum we have:

$$\frac{1}{-\mathrm{d}z/\mathrm{d}w^A}\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}w^A} = \frac{zn^B u_c^A e_c^B}{1+e_u^B}\frac{1}{-(\mathrm{d}z/\mathrm{d}\tau - z\mathrm{d}z/\mathrm{d}\mathcal{T})} > 0. \tag{25}$$

Hence, a binding minimum wage can improve upon the tax optimum and is therefore necessarily part of the overall policy optimum.

A.2 Proof of Proposition 2

The tax schedule can now be parameterized such that $T(z^i) = \tau z^i + \mathcal{T}$ for $i = \{A, B\}$ and $T(z^C) = \tau^C z^C + \mathcal{T}^C$, where h^C solves for the first-order condition of type-C labor supply:

$$MRS^C(c^C, h^C) \equiv \frac{-u_h^C((1-\tau^C)z^C - \mathcal{T}^C, z^C/w^C)}{u_c^C((1-\tau^C)z^C - \mathcal{T}^C, z^C/w^C)} = (1-\tau^C)w^C. \tag{26}$$

Again, this parameterization is without loss of generality because, with two levels of income, only four parameters are needed to fully describe the relevant features of the tax schedule: two marginal tax rates and two "virtual" intercepts of the tax schedule.

Recall that the government objective is given by the following Lagrangian:

$$\begin{aligned}\tilde{\mathcal{L}} = \mathcal{L} + \lambda n^C(\tau^C z^C + \mathcal{T}^C) \\ +\gamma\left(u^C\left((1-\tau)z - \mathcal{T}, z/w^C\right) - u^C\left((1-\tau^C)z^C - \mathcal{T}^C, z^C/w^C\right)\right).\end{aligned} \tag{27}$$

The total derivative for given levels of τ^C and $\mathcal{T}^C$ is given by:

$$\mathrm{d}\tilde{\mathcal{L}} = \mathrm{d}\mathcal{L} + \gamma u_c^C\left((1-\tau)z - \mathcal{T}, z/w^C\right)\left((1-\tau)z\frac{\mathrm{d}z}{z} - (z\mathrm{d}\tau + \mathrm{d}\mathcal{T})\right) \tag{28}$$

Consider again an increase in the minimum wage ($\mathrm{d}w^A > 0$) versus a compensated increase in the marginal tax rate at the lower level of income ($\mathrm{d}\tau > 0$ and $\mathrm{d}\mathcal{T} = -z\mathrm{d}\tau$). Substitute this into the derivative of the Lagrangian and rearrange to get:

$$\frac{1}{-\mathrm{d}z/\mathrm{d}w^A}\frac{\mathrm{d}\tilde{\mathcal{L}}}{\mathrm{d}w^A} = \frac{1}{-\mathrm{d}z/\mathrm{d}w^A}\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}w^A} - (1-\tau)\gamma u_c^C\left((1-\tau)z - \mathcal{T}, z/w^C\right) \tag{29}$$

$$\begin{aligned}\frac{1}{-(\mathrm{d}z/\mathrm{d}\tau - z\mathrm{d}z/\mathrm{d}\mathcal{T})}\left(\frac{\mathrm{d}\tilde{\mathcal{L}}}{\mathrm{d}\tau} - z\frac{\mathrm{d}\tilde{\mathcal{L}}}{\mathrm{d}\mathcal{T}}\right) = \frac{1}{-(\mathrm{d}z/\mathrm{d}\tau - z\mathrm{d}z/\mathrm{d}\mathcal{T})}\left(\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\tau} - z\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\mathcal{T}}\right) \\ -(1-\tau)\gamma u_c^C\left((1-\tau)z - \mathcal{T}, z/w^C\right).\end{aligned} \tag{30}$$

From the Proof of Proposition 1, we know that

$$\frac{1}{-\mathrm{d}z/\mathrm{d}w^A}\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}w^A} > \frac{1}{-(\mathrm{d}z/\mathrm{d}\tau - z\mathrm{d}z/\mathrm{d}\mathcal{T})}\left(\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\tau} - z\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\mathcal{T}}\right), \tag{31}$$

such that

$$\frac{1}{-\mathrm{d}z/\mathrm{d}w^A}\frac{\mathrm{d}\tilde{\mathcal{L}}}{\mathrm{d}w^A} > \frac{1}{-(\mathrm{d}z/\mathrm{d}\tau - z\mathrm{d}z/\mathrm{d}\mathcal{T})}\left(\frac{\mathrm{d}\tilde{\mathcal{L}}}{\mathrm{d}\tau} - z\frac{\mathrm{d}\tilde{\mathcal{L}}}{\mathrm{d}\mathcal{T}}\right). \tag{32}$$

The left-hand side of the inequality gives the marginal effect on the Lagrangian of a unit reduction in taxable income caused by an increase in the minimum wage. The right-hand side gives the marginal effect on the Lagrangian of a unit reduction in taxable income caused by an increase in marginal taxes. The inequality implies that the increase in the minimum wage is more desirable than the increase in the tax rate. Hence, by the same reasoning as in Proposition 1, a minimum wage must be part of the policy optimum. This proves Proposition 2.

A.3 Proof of Proposition 3

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