Tax Policies Design in a Hierarchical Two-Side Model with Occupational Decision *

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Abstract

This study incorporates the occupational decision, i.e. dependent and self-employed, into the hierarchical model of Sanchez and Sobel (1993) to investigate distortions in tax policies design. More specifically, this study mainly aimed to explore the audit function, a linear marginal tax rate and the IRS's optimal budget in an economy where evasion is only possible in the self-employment sector. The optimal audit shows that audits are efficient below a cut-off level, and above this level, audits are equal to zero. This result is held under two extensions: either the audit cost is monotonically non-decreasing in the self-employed wage or the fine rate rises in self-employed wage but it is bounded from above. The marginal tax rate is smaller than one, indicating that not considering occupational decisions produces an upward bias on taxes. The optimal IRS's budget does not allow auditing the entire self-employed sector but it is larger than the result from a cost-benefit analysis. Finally, differential taxation is optimal if the marginal tax rate in the self-employed sector is higher than the dependent sector. This result produces that the distortions in the optimal allocation of agents increase in comparison with an environment with only one marginal tax rate.

Keywords: Tax Evasion, Hierarchical Models, Tax Policies Design, Audits, Differential Taxation, Occupational Decision.
JEL Classification: H26, H21, H83.

1 Introduction

Increased tax revenue has continuously been an important issue for governments because of the increasing social expenditures and paying debts or saving for future spending. One option to raise tax collection is avoiding tax evasion since it increases revenue through effectiveness instead of changing the tax policy, which bureaucratically takes more time. For example, after the economic crisis of 2008, developed countries have focused on the fight against tax evasion and in understanding its mechanisms aiming to increase tax revenues (Slemrod, 2019). The necessity to

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increase tax revenue may enlarge after a social or political crisis, even if countries have considerable savings. Therefore, enhancing the effectiveness of enforcement policies appears to be an excellent option to expand expenditure and maintain the institutional background at the same time.

Despite the importance of the increase –or at least it does not decrease– in tax collection, less attention has been paid to distortions made by those policies in more complex and realistic environments. As a matter of fact, some policies in the spirit of increasing tax revenue and improving income inequality end up in distortions over the occupational decision. For instance, in an environment with two possible occupations, a progressive tax schedule may induce an increase in evasion for risk-averse agents (Watson, 1985), or having high taxes changes resources to industries where evasion exists (Kesselman, 1989). Those facts come from the incentives to move into an occupation where evasion is more accessible than others and reflects the analysis's complexity.

The occupational decision appears as a critical issue in distortions made by tax policies. Empirically speaking, some studies reinforce this fact. Firstly, Kleven et al. (2011) and Slemrod (2007) evidence that evasion in the third-party income reports is low, and this problem arises mainly in the income self-report occupations. Additionally, Bárány (2019) shows that high tax rates in the dependent employment sector produce high rates of self-employment. From a theoretical point of view, Kuchumova (2017) demonstrates that the tax collection agency should audit occupations with the least level of information income reports, as this is a profession or field with a self-declaration of their incomes. Bearing in mind these results, taking into account occupational decisions into tax policy models appears to be reasonable.

The current study incorporates the occupational decision into a hierarchical model formerly developed by Sanchez and Sobel (1993) to investigate possible distortions in the tax policy design. As reported by Melumad and Mookherjee (1989), hierarchical models are useful to make normative analysis. Therefore, since a hierarchical model is used throughout this research, this paper focuses on the normative implications of incorporating occupational decisions into a tax policy model. Thus, the main interest is to inquire about the distortion produced by income tax evasion and occupational choice in the tax policy design. Simultaneously, it intends to investigate further into other relevant issues, namely the forces behind the optimal policies given tax evasion and occupational choice, the possible distortion in the optimal allocation of agents driven by tax evasion or the institutional background, and what the mechanisms behind the possible distortions are.

This study draws upon the classic literature on evasion, beginning with the contribution of Allingham and Sandmo (1972) and Yitzhaki (1974). The literature is faced with the question on the relation between optimal taxation, optimal audit scheme and occupational choice before this work. In this line, the seminal paper is Sandmo (1981), which focuses on the impact of tax evasion in the labor market in an economy with two fixed sectors. Pestieau and Possen (1991) concentrate on the differences in risk-aversion among taxpayers who choose between a riskless work and a risky entrepreneurial activity in which evasion is possible. These authors find that the tax rate decreases with audit costs and increases with the concern for equality, and the audit scheme takes only extreme values. Similarly, Boadway et al. (1991) investigate the consequences of occupational choice in a linear income tax, where the occupational choice is between entrepreneurship and wage-earning. They claim that optimal tax rates depend on three effects: efficiency, equity and insurance and heterogeneity in attitude toward risk. Furthermore, Parker (1999) empirically studies linear taxation in the presence of occupational choice; however, he added the possibility of unemployment. He calibrates the model using UK's economy, finding that the optimal tax solution implies high marginal rates and redistribution, and the self-employed sector faces high marginal tax rates but fewer effective tax rates than third-party employees do.

Although some studies incorporate occupational decisions there has not been any studies that focus on the implications of not considering them. Ergo, this paper's main contribution is to contrast the effect of not considering occupational decision into a model of optimal policy design. What is more, this study seeks to shed some light into the consequences for the tax policy design taking into account, at the same time, occupational decision and income tax evasion. The interaction between occupational decision and income tax evasion allows a better understanding of the possible distortions behind the non-optimal policies. To explain further, tax evasion may come from the permission of institutional background instead of purely willingness from agents (Kleven et al., 2011). This fact could be partially explained by not considering both issues tax evasion and occupational choice simultaneously in the tax policy design.

The hierarchical model developed by Sanchez and Sobel (1993) is used as a benchmark. They make a hierarchical tax policy model where the tax collection agency (henceforth the IRS) maximizes tax revenue and interacts with a government in an economy with only one sector and a continuum of risk-neutral agents. These authors' paper shows that the audit scheme takes only extreme values. The IRS audits with the intensity to deter the evasion below some threshold, and above this threshold, audit equals zero. In addition, they show that the budget is insufficient to audit all taxpayers. This fact is given by the agents tax burden's effect, as is well explained in Slemrod and Yitzhaki (1987). Finally, they find that the optimal marginal rate is equal to one, and all the redistribution is through the public goods provision and a subsidy equal for all agents.

Throughout the current study, a three-stage game is built in which the government, the IRS and a continuum of risk-neutral agents interact. This is, each agent chooses between two possible occupations, being an employee or working as a self-employed. Additionally, each agent has one productivity in each occupation, which comes from independent distribution functions. In the dependent sector, the agents cannot collude with their employer to declare less income, avoid taxes and share the gains. On the other hand, in the self-employed sector the agents self-report their income, being able to misreport income and evade taxes. In the first stage, the government maximizes a social welfare function and commits to a linear marginal tax rate, a level of public goods, and the IRS budget. In the second stage, the IRS determines an audit function that seeks to maximize the expected collection subject to its budget constraint. Finally, in the third stage, the risk-neutral agents maximize their utility choosing an occupation. In the case of deciding to work as a self-employed, it also chooses its income declaration. The agents make both decisions observing their productivity (which is private information), the audit function, the marginal tax rate, and the public goods provision.

Since productivity is private information, the revelation principle is used to solve the IRS's problem. Therefore, this study focuses on finding a direct incentive-compatible mechanism composed of a direct audit function and effective taxes, which are the tax level that incentivizes agents to reveal their real income. The audit function takes a cut-off form, as in Sanchez and Sobel (1993). However, in this case, the threshold level relies on the dependent productivity distribution as well. This solution suggests that, if the IRS has an insufficient budget to audit the entire self-employed sector, the self-employed sector face an efficient audit level until the threshold wage, and after this amount, they do not face an audit. Thus, this result produces that effective tax is the same for all agents who do not face an audit. This fact generates a distortion in the occupational decision because agents with self-employed incomes higher than the threshold wage have the incentive to work as self-employed when they are more productive in the dependent sector. Finally, it is demonstrated that this kind of result holds for two extensions: a) if the audit cost can depend on the self-employed wage, it must be monotonically non-decreasing on self-employed's income and b) if the fine rate is increasing in the self-employed wage, it must be bounded from above.

The optimal public good provision is equal as in the first-best solution as agents are risk-neutral. As is well explained in Boadway and Keen (1993), if agents have linear utility for consumption (or are risk-neutral on it), any change through tax evasion, and the consequent increase in the after-tax income, does not produce an impact on the optimal public good provision. These results come from the non-alteration of the marginal rate of substitution between the public good and private consumption. However, for the current study, this result reinforces the idea that tax evasion produces a distortion in the distribution due to the fact that some agents, those who face an audit and those who are dependent workers, increase the burden to finance public good in comparison to a situation without tax evasion.

The optimal marginal tax rate is smaller than one and can be characterized by two effects: welfare and revenue. This result suggests two critical highlights. First, if the government does not consider the occupational decision, taxes have an upward bias. This conclusion comes from the comparison with Sanchez and Sobel (1993), who obtain a marginal tax rate equal to one. Second, the optimal tax must consider, at the same time, the impact on tax revenue and social welfare. It is also demonstrated during this study that differential taxation, i.e., one marginal tax rate in each occupation, is possible but it implies a bigger marginal tax rate in the self-employed sector. This result produces an even more significant distortion in the allocation of agents than the scenario with only one marginal tax rate.

The optimal IRS's budget is insufficient to audit the entire self-employed sector and three effects can characterize this: behavioral, mechanical, and welfare. Even though the budget cannot allow audit all self-employed workers, this level is higher than the result from a cost-benefit analysis. This fact comes from the necessity to take into account the agents tax burden (Slemrod and Yitzhaki, 1987). Finally, this result shows that, in determining the size of the tax collection agency, the government must consider the effect of welfare and revenue (the mechanical effect) and the implication on the occupational decision made by agents (the behavioral effect).

The rest of the current paper is organized as follows. Section 2 presents the model, the requirement for equilibrium, and provides the first-best solution and the decentralization of it. Section 3 solves the IRS's problem, ensuring a second-stage equilibrium, and providing the characterization of the optimal audit policy. Section 4 solves the government's problem, obtaining the optimal tax schedule, the IRS budget, and the optimal provision of public goods. Section 5 briefly discusses some issues and provides a comparative static over the tax rate and the budget for the IRS. Finally, section 6 provides the conclusions of this study.

2 Model, Benchmark and Equilibrium Concepts

2.1 Model

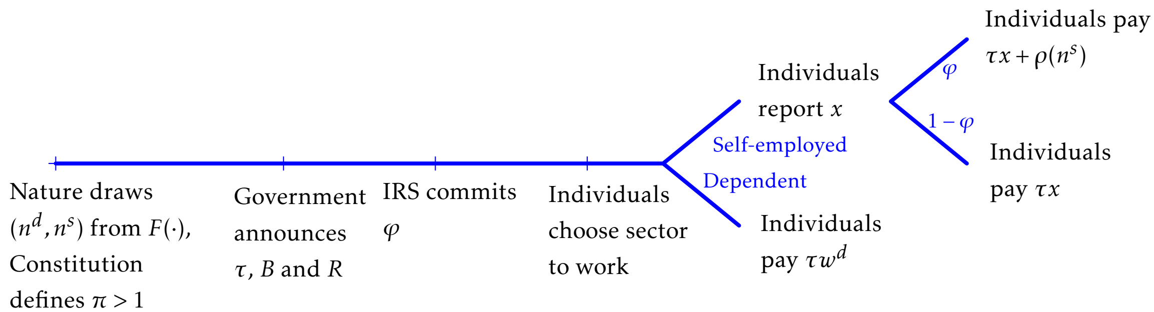
The economy has three agents: the government, the IRS, and a continuum of risk-neutral agents of mass one, which interact in a hierarchical tax administration model. Also, the economy has two sectors, dependent and self-employed, indexed by $i = \{d, s\}$. Since the IRS perfectly knows in what sector each agent works, it is impossible to evade taxes in the dependent sector. However, in the self-employed sector, tax evasion is possible, and the income verification is costly. Let us assume a homogeneous consumption good, which production is linear and uses only labor as an input. For simplicity, the price of this good is normalized to one. Those facts produce that each sector's wage rate is equal to one, and the effective wage is the same as the agent's productivity. In addition, it is implicitly assumed that the government has no resources (the technology, knowledge, expertise among other) to perform the audit process efficiently. What has been previously mentioned produces the necessity of the existence of a specialized agency to look after the tax compliance. Yet, as it is well explained in Melumad et al. (1995), it is possible to attain the same second-best performance level as one without an IRS in this setting.

One interesting way to analyze income tax evasion in an occupational choice model is through differential taxation. Gomes et al. (2017) find the optimal differential taxation into an occupational decision model quite similar to the model developed in this paper. Given that, a natural question is about the possibility of analyzing distortions in tax policy directly through differential taxation. However, Appendix A faces this question and demonstrates that it is not trivial to follow this way.

The timing of the game is the following: in the first stage, the government chooses a linear marginal tax rate, τ, a budget for the IRS, B, and a level of public goods provision, R. The government cannot impose a marginal tax higher than one.[1] This assumption is not restrictive; afterwards, it will be shown that this assumption holds at the optimum. In the second stage, the IRS selects an audit function, φ. Finally, the agents decide the sector in which to work endogenously, after seeing the tax policy decisions chosen by the government and the IRS. If it is decided to work as self-employed, an agent also chooses the income report to the IRS. A graphic exposition of the above explanation is illustrated in Figure 1.

[1] Mirrlees (1971) obtains this fact as a result of an optimal non-linear tax schedule.

Figure 1: Timing of the Game



2.1.1 Agents

Let us assume a continuum of risk-neutral agents of mass one. Nature randomly chooses a pair of productivities $n = (n^d, n^s)$, each term represents the agent's productivity in the dependent and self-employed sector respectively. Let us assume that each productivity is drawn independently from a distribution function. Formally, n^i comes from a cumulative distribution function F^i, twice continuously differentiable, with support $[\underline{n}^i, \overline{n}^i]$ for $i = \{d, s\}$. This function is common knowledge in the model.

The agents decide their occupation considering the government's tax schedule and the audit function committed by the IRS. When it comes to choosing an occupation, the agents compare their utility in each sector and choose which one gives them the highest utility. In each sector, the agents offer an inelastic labor supply equal to one.[2] The agents' utility function is separable between consumption, C, and public goods, R. Since agents are risk-neutral, their utility can be written as a quasi-linear function in consumption. In particular, let ϕ be the benefit for a certain level of public goods, which are increasing and concave (i.e., $\phi' > 0$ and $\phi'' < 0$). The utility for each agent, independently of its occupation, is

$$u(C, R) = C + \phi(R)$$

In the dependent sector, firms declare to the IRS the agent's income.[3] This fact produces that consumption in the dependent sector is equal to the after-tax income, i.e., $C(w^d) = w^d - \tau w^d$. Consequently, the utility in the dependent sector is

$$U^d(w^d) = w^d(1-\tau) + \phi(R)$$

In the self-employed sector, an agent declares to the IRS an income equals to x. Formally $x : [\underline{n}^s, \overline{n}^s] \to \mathbb{R}$. The IRS does not reward over-reporting, implying that any declaration above real productivity is a dominated strategy. Hence, the income declaration will not exceed the real income, $x \leq w^s$. Agents will be audit with probability φ. If the IRS audits a taxpayer, it will be

[2] Implicitly, it has been taken into consideration the fact that effective wage is equal to productivity and write w or n indifferently.

[3] Collusion between firms and workers to underreport wages and divide the evaded amount is impossible. This fact implies that firms always declare each agent's real productivity and tax evasion does not exist.

immediately discovered whether they evaded taxes and the agent must pay the penalty ρ, which is a fine rate $\pi > 1$ over the evaded taxes. The penalty takes the following form

$$\rho(w^s) = \begin{cases} \pi(\tau w^s - \tau x) & \text{if } x < w^s \\ 0 & \text{if } x \geq w^s \end{cases}$$

It can be defined $U^s(w^s)$ as the utility with the optimal income declaration. Formally, this utility level is given by the following

$$U^s(w^s) = \max_{x \leq w^s} w^s - \tau x - \varphi(x) \max\{\pi(\tau w^s - \tau x), 0\} + \phi(R)$$

Let us denote by $x(w^s)$ the solution to this problem. In the above equation, the expected consumption is equal to the expected after-tax income ($C = w^s - \tau x - \varphi(x)\max\{\pi(\tau w^s - \tau x), 0\}$), the same procedure is undertaken in the rest of the article. Anticipating the optimal declaration in the self-employed sector, each agent chooses the sector in which it works. Now, $\mathcal{N}^i$ can be defined as the set of agents that decide to work in the sector i. Formally $\mathcal{N}^i(w^i) = \{(w^d, w^s) \mid U^i(w^i) \geq U^j(w^j)\}$ with $j \neq i$, and $i, j = d, s$.

2.1.2 IRS

The IRS chooses the audit function to maximize the expected tax collection, taking as given both the marginal tax rate and its budget, which are determined by the government, and the fine rate. Formally, define the audit function as $\varphi : [\underline{w}^s, \overline{w}^s] \to [0,1]$. The cost of auditing an agent, c, is linear and constant. The IRS has a budget B to finance the cost of its enforcement policy.[4] If the IRS does not use its entire budget, the excess must be returned to the government as a transfer along with the tax collection. The problem of the IRS is as follows

$$\max_{\varphi:[\underline{w}^s, \overline{w}^s] \to [0,1]} \int_{\mathcal{N}^d(w^d)} \tau w^d dF^d(w^d) dF^s(w^s) + \int_{\mathcal{N}^s(w^s)} [\tau x(w^s) + \varphi(x(w^s))\rho(w^s)] dF^d(w^d) dF^s(w^s)$$

s.t.

$$B \geq \int_{\mathcal{N}^s(w^s)} c \cdot \varphi(x(w^s)) dF^d(w^d) dF^s(w^s)$$

$$x(w^s) \in arg \max_{\tilde{x}(w^s)} \{U^s(w^s)\}$$

2.1.3 Government

The government chooses the marginal tax rate, the provision of public goods, and the IRS's budget to maximize a social welfare function (SWF). The government anticipates the optimal audit scheme obtained by the IRS, the decision made by agents, and uses the known information, i.e., the distribution of productivities and the form of utility in each sector. Let us define G as the social welfare function, and assume that $G' > 0$ and $G'' \leq 0$.

[4]It is assumed that the IRS bears no other costs, like the operation cost.

The government has a budget constraint that states that the IRS's expected tax collection and transfers must go to finance a provision of public goods and the IRS budget. The cost of one unit of a public good is one. Formally, the government chooses a linear marginal tax rate, $\tau \in [0,1]$, a public goods provision, R, and the IRS budget, B. The problem of the government is as follows

$$\max_{\tau,R,B} \int_{\mathcal{N}^d(w^d)} G\left(U^d(w^d)\right) dF^d(w^d) dF^s(w^s) + \int_{\mathcal{N}^s(w^s)} G\left(U^s(w^s)\right) dF^d(w^d) dF^s(w^s)$$

$s.t.$

$$\int_{\mathcal{N}^d(w^d)} \tau w^d dF^d(w^d) dF^s(w^s) + \int_{\mathcal{N}^s(w^s)} [\tau x(w^s) + \varphi(x(w^s))\rho(w^s)] dF^d(w^d) dF^s(w^s)$$

$$+ \left(B - \int_{\mathcal{N}^s(w^s)} c \cdot \varphi(x(w^s)) dF^d(w^d) dF^s(w^s) \right) \geq B + R$$

$$x(w^s) \in arg \max_{\tilde{x}(w^s)} \{U^s(w^s)\}$$

φ solves the problem of the IRS

2.2 Third Stage Equilibrium

Before presenting the first best in this environment, it is necessary to develop the requirements for equilibrium in this model. Since the model has three stages, following backward induction, it is crucial to describe the equilibrium in the third stage. In this sense, each equilibrium characterization requires that each agent not have incentives to change its behavior in equilibrium.

Any equilibrium in the third stage requires a definition of the optimal income declaration and a characterization over the optimal occupational choice. Using the definition of $x(w^s)$ as the optimal income declaration implies that, at the equilibrium, any agent in the self-employed sector declares $x(w^s)$ and has a utility equals to $U^s(w^s)$. Hereafter, wages instead of productivity to clarify the notation and the results are used.

Definition 1 (Occupational Choice Rule) *Define $\mathcal{W}^i(w^i)$ as the occupation set, which is the result of each occupation decision made by each agent, in the following form*

$$\mathcal{W}^s(w^s) = \left\{(w^s, w^d) \in [\underline{w}^s, \overline{w}^s] \times [\underline{w}^d, \overline{w}^d] \mid U^s(w^s) \geq U^d(w^d)\right\}$$
$$\mathcal{W}^d(w^d) = \left\{(w^s, w^d) \in [\underline{w}^s, \overline{w}^s] \times [\underline{w}^d, \overline{w}^d] \mid U^s(w^s) < U^d(w^d)\right\}$$

This rule establishes, implicitly, a threshold function that defines a critical productivity/wage limit in the self-employed sector. This is up to what wage level some agents prefer to work as a self-employed.

Lemma 1 (Threshold Function) *There exists a threshold function, $\kappa : [\underline{w}^s, \overline{w}^s] \to \mathbb{R}_+$, such that*

$$(w^s, w^d) \in \mathcal{W}^s(w^s) \Leftrightarrow w^d \leq \kappa(w^s)$$

Proof.

See Appendix D ■

Considering the above analysis, a requirement for a third-stage equilibrium must contain two elements: 1) an optimal agents' income declaration, and 2) its occupational decision should reflect the sector in which they obtain the largest utility from. These concepts are defined as follows.

Definition 2 *A third-stage equilibrium is given by:*

1. *Each agent in the self-employed sector chooses* $x(w^s) \in arg\max U^s(w^s)$.
2. *The occupational decision made for the agents is optimal and satisfy conditions in Definition 1.*
3. *The threshold function exists and holds with Lemma 1.*

2.3 Second Stage Equilibrium

In the second stage, the IRS maximizes the expected tax collection choosing an audit strategy anticipating the optimal decision made by the agents. Agents in the self-employed sector can hide information from the IRS and optimally choosing a smaller declaration than their real income ($x(w^s) \leq w^s$). Therefore, it is necessary to use a method that makes agents declare their real wage; this method is a mechanism design approach. Considering the revelation principle (Myerson, 1979, 1981), finding a direct incentive-compatible (IC) mechanism $\mathcal{M} : \{\alpha, \mathcal{T}\}$ is without loss of generality. Being α the probability to audit an agent and $\mathcal{T}$ the effective tax paid by an agent, namely $\mathcal{T}(w^s) = \tau x(w^s)$.[5] Formally, the direct IC mechanism is as follows

$$\mathcal{M} = \begin{cases} \mathcal{T}: & [\underline{w}^s, \overline{w}^s] \rightarrow [\underline{w}^s, \overline{w}^s] \\ \alpha: & [\underline{w}^s, \overline{w}^s] \rightarrow [0,1] \end{cases}$$

A direct IC mechanism in this setting produces that the optimal income declaration of every self-employed agent is equal to the real income earned ($x(w^s) = w^s$). Thus, in equilibrium, evasion does not exist.

The mechanism need to be composed of an audit function and a level of taxes, the effective taxes, which together produce that agents reveal their real income. Unless the mechanism follows the implementable requirements (declared below) some self-employed workers may have the incentive to cheat the IRS and hide part of their, or their whole, income. This approximation produces a simplification of the problem and allowing seeing the tax policy implication to incorporate the occupational choice.

Let us define $V^s(w^s, \mathcal{M})$ as the indirect utility in the self-employed sector given a mechanism $\mathcal{M}$. Any implementable IC mechanism has the following characteristics.

Lemma 2 *The IC mechanism* $\mathcal{M}$ *is implementable if and only if*

1. $\alpha(w^s)$ *is non-increasing in* w^s

[5] In the dependent sector, agents cannot misreport their income; implying that it is not necessary to define an IC mechanism there.

2. $\mathcal{T}(w^s) = \frac{w^s(1-\tau\alpha(w^s)\pi)}{1-\pi\alpha(w^s)} - \frac{\int\limits_{j=\underline{w}^s}^{w^s}(1-\alpha(j)\pi\tau)\,dj}{1-\pi\alpha(w^s)} - \frac{V^s(\underline{w}^s)}{1-\pi\alpha(w^s)}$

Proof.

See Appendix E ■

In the case that the audit function takes the value of $1/\pi$, the agent always reveals its real productivity.[6] This result was first formally established by Scotchmer (1987), and it allows redefining the support for audit function in the mechanism as $\alpha(w^s) \in [0, 1/\pi]$.[7]

Lemma 2 establishes the requirements for any mechanism to be implementable. Hence, it characterizes a direct IC mechanism entirely in this setting. Keeping in mind these requirements, it is possible to redefine the occupational choice rule for the second stage equilibrium and the threshold function that results from this.

Definition 3 (Occupational Choice Rule in an IC Mechanism) *For a direct IC mechanism, it can be defined $\mathcal{W}^i(\mathcal{M})$ as the occupational set which is the result of each agent's occupational decision by*

$$\mathcal{W}^s(\mathcal{M}) = \left\{(w^s, w^d) \in [\underline{w}^s, \overline{w}^s] \times [\underline{w}^d, \overline{w}^d] \mid V^s(w^s, \mathcal{M}) \geq U^d(w^d)\right\}$$
$$\mathcal{W}^d(\mathcal{M}) = \left\{(w^s, w^d) \in [\underline{w}^s, \overline{w}^s] \times [\underline{w}^d, \overline{w}^d] \mid V^s(w^s, \mathcal{M}) < U^d(w^d)\right\}$$

Lemma 3 (IC Threshold Function) *For any direct IC mechanism $\mathcal{M}$, there exists a threshold function $\kappa : [\underline{w}^s, \overline{w}^s] \to \mathbb{R}_+$, that establishes the wage limit to work as a self-employed*

$$w^d \leq \kappa(w^s)$$

Moreover, in equilibrium the threshold function is characterized by

$$\kappa'(w^s) = \frac{V^s_{w^s}(w^s, \mathcal{M})}{U^d_{\kappa(w^s)}(\kappa(w^s))}$$

where X_i denotes to the derivative of variable X with respect to i.

Proof.

See Appendix F ■

[6]To see this, note that when $\varphi(w^s) = 1/\pi$ the utility in the self-employed sector is $U^s(w^s) = w^s - \tau w^s$, implying that $x(w^s) = w^s$.

[7]Additionally, condition 1 in Lemma 2 gives a restriction on marginal taxes which is the same as the assumption made early:

$$\begin{aligned} 1 - \alpha(w^s)\pi\tau &\geq 0 \\ \frac{1}{\alpha(w^s)\pi} &\geq \tau \end{aligned}$$

The upper limit of the audit function is $1/\pi$ and the marginal tax is limited from above with a decreasing function in the audit. This fact implies that the most restrictive limit is when the audits take it upper limit, in this case it is $1 \geq \tau$. The preceding implies that the marginal tax never exceeds one.

The second stage equilibrium establishes the conditions over the IRS's choice and the allocation of agents in equilibrium. Furthermore, it is of great importance to highlight that, by definition, any equilibrium that fulfills the following conditions also holds with Definition 2 and, consequently, is a third stage equilibrium.

Definition 4 *The second-stage equilibrium is given by:*

1. *The IRS maximizes the expected tax collection in the set of direct IC mechanisms, which follow the conditions in Lemma 2.*
2. *The occupational decision made by each agent is optimal and satisfies the conditions in Definition 3.*
3. *The threshold function exists, and holds with Lemma 3.*

2.4 Full-Information Solution

A central element to compare distortions made by evasion and the occupational choice is obtain the fist best solution. This solution is the result of a centralized problem where the government has complete information about agents and the economy. This solution gives the benchmark in this work.

The government decides the sector in which each agent will work as well as their consumption level. In this setting, the productivity of each agent is used instead of their wages. Recall that, the utility of each agent, independently of the sector in which they work, is $u(C_i(n^i), R) = C_i(n^i) + \phi(R)$. Therefore, the government must choose a consumption function, $C : [\underline{n}^s, \overline{n}^s] \times [\underline{n}^d, \overline{n}^d] \to \mathbb{R}_+$, a public good provision R, and an occupational choice function, $Z : [\underline{n}^s, \overline{n}^s] \times [\underline{n}^d, \overline{n}^d] \to \{d, s\}$. The occupational choice function determines the sector where each agent will optimally work. The problem that the government solves is as follows

$$\max_{\mathcal{Z}_s, \mathcal{Z}_d, C_d(n^s,n^d), C_s(n^s,n^d), R} \int_{\mathcal{Z}_d} \left[G\left(u(C_d(n^s, n^d), R)\right)\right] dF^d(n^d) dF^s(n^s) + \int_{\mathcal{Z}_s} \left[G\left(u(C_s(n^s, n^d), R)\right)\right] dF^d(n^d) dF^s(n^s)$$

$s.t.$

$$\int_{\mathcal{Z}_d} n^d dF^d(n^d) dF^s(n^s) + \int_{\mathcal{Z}_s} n^s dF^d(n^d) dF^s(n^s) \geq \int_{\mathcal{Z}_d} C_d(n^s, n^d) dF^d(n^d) dF^s(n^s) + \int_{\mathcal{Z}_s} C_s(n^s, n^d) dF^d(n^d) dF^s(n^s) + R$$

where Z_i is the set of agents on the work side i and C_i is the consumption function for agents in the sector i, with $i = \{d, s\}$. The government has a resource constraint, which states that production in both sectors must be equal to the consumption and spending on public goods. The following proposition formalizes the government's solution to the centralized problem.

Proposition 1 *The solution to this problem is* $\{\mathcal{Z}_s^*, \mathcal{Z}_d^*, R^*, C_i^*(n^i, n^j)\}$, *with* $i \neq j$ *and* $i, j = \{d, s\}$, *and takes the following form*

$$
\begin{aligned}
\mathcal{Z}_s^* &= \{s \mid \forall (n^s, n^d) \in \mathcal{Z}_s^* \Rightarrow n^s \geq n^d\} \\
\mathcal{Z}_d^* &= \{d \mid \forall (n^s, n^d) \in \mathcal{Z}_d^* \Rightarrow n^s < n^d\} \\
1 &= \int_{\mathcal{Z}_d^*} MRS(R^*, C_d^*) dF^d(n^d) dF^s(n^s) + \int_{\mathcal{Z}_s^*} MRS(R^*, C_s^*) dF^d(n^d) dF^s(n^s) \\
R^* &= \int_{\mathcal{Z}_d^*} \left(n^d - C_d^*(n^d, n^s)\right) dF^d(n^d) dF^s(n^s) + \int_{\mathcal{Z}_s^*} \left(n^s - C_s^*(n^d, n^s)\right) dF^d(n^d) dF^s(n^s) \\
\frac{dG\left(u(C_d(n^d, n^s), R)\right)}{du(C_d(n^d, n^s), R)} &= \frac{dG\left(u(C_s(n^d, n^s), R)\right)}{du(C_s(n^d, n^s), R)}
\end{aligned}
$$

where MRS is the marginal rate of substitution and

$$MRS(R^* C_i^*) = \left. \frac{du/dR}{du/dC_i} \right|_{R^*, C_i^*}$$

Proof.

See Appendix B ■

The result in Proposition 1 shows the first best solution for this setting. The first and second equation reflect the occupational choice rule and show that each agent should work in their most productive occupation. After that, equation three reflects the optimal provision of public goods, which tells us that the marginal cost of providing the public good is equal to the sum among agents of the marginal rate of substitution between the public goods and the consumption good. This result is known as the Bowen-Lindahl-Samuelson (henceforth BLS) rule (Samuelson, 1954, 1955). The last two equations show the characterization for the optimal consumption bundle.

2.5 Decentralized Solution

The first best can be decentralized showing that the central issue in this context is evading taxes. One can assume that the government knows each agent's real income in each sector and, therefore, evasion is impossible. In this framework, the government maximizes the social welfare function subject to a resource constraint, but now it considers wages instead of productivities. The government does not choose a consumption function, it chooses a lump-sum taxes that induces the agent to decide the sector to work in and a level of consumption. Formally, the government solves the following problem

$$\max_{T_s(w^s),T_d(w^d),R} \int_{\mathcal{N}_d(w^d)} G\left(U^d(w^d,R)\right)dF^d(w^d)dF^s(w^s) + \int_{\mathcal{N}_s(w^s)} G(U^s(w^s,R))\,dF^d(w^d)dF^s(w^s)$$

$s.t$

$$\int_{\mathcal{N}_d(w^d)} T_d(w^d)dF^d(w^d)dF^s(w^s) + \int_{\mathcal{N}_s(w^s)} T_s(w^s)dF^d(w^d)dF^s(w^s) \geq R$$

$$U^i(w^i,R) = w^i - T_i(w^i) + \phi(R), \text{ for } i = d,s$$

where $\mathcal{N}_i$ is the set of taxpayers who choose work in sector $i = \{d,s\}$. The first constraint is the resource constraint that shows that the tax revenue must be at least the same as the expenditure in public goods provision. The second constraint is the definition of utility in each sector.

Proposition 2 *The solution to this problem is $\{T_i^*(w^i), \mathcal{N}_i^*, R^*\}$, with $i = \{d,s\}$, and takes the following form*

$$\begin{aligned}
\mathcal{N}_s^* &= \{w^s \mid \forall (w^s,w^d) \in \mathcal{N}_s^* \Rightarrow w^s \geq w^d\} \\
\mathcal{N}_d^* &= \{w^s \mid \forall (w^s,w^d) \in \mathcal{N}_d^* \Rightarrow w^s < w^d\} \\
1 &= \int_{\mathcal{N}_d^*} MRS(R^*,C_d^*)dF^d(w^d)dF^s(w^s) + \int_{\mathcal{N}_s^*} MRS(R^*,C_s^*)dF^d(w^d)dF^s(w^s) \\
R^* &= \int_{\mathcal{N}_d^*} T_d^*(w^d)dF^d(w^d)dF^s(w^s) + \int_{\mathcal{N}_s^*} T_s^*(w^s)dF^d(w^d)dF^s(w^s) \\
C_i^* &= w^i - T_i^*(w^i), \textit{ for } i = s,d \\
&\frac{dG\left(U^d(C_d(w^d),R)\right)}{dU^d(C_d(w^d),R)} = \frac{dG(U^s(C_s(w^s),R))}{dU^s(C_s(w^s),R)}
\end{aligned}$$

where MRS is the marginal rate of substitution between the public good and the private consumption, and

$$MRS(R^*,C_i^*) = \left.\frac{du/dR}{du/dC_i}\right|_{R^*,C_i^*}$$

Proof.
See Appendix C ■

Proposition 2 shows that it is possible to attains the first best in an environment with lump-sum taxes. This result comes from the second welfare theorem and identifies that the crucial issue in this framework is tax evasion in the presence of occupational choice. Therefore, not only can evasion distort taxes but also the optimal and efficient allocation of workers.

3 IRS Problem

The IRS's problem involves finding a direct IC mechanism to incentive agents in the self-employed sector to reveal their real productivity. Thus, the focus is on finding and characterize a second stage equilibrium. The IRS finds an audit schedule that maximizes the expected collection and meets conditions in Lemma 2. For this reason, those conditions are in the IRS problem. Formally, the problem of the IRS is as follows

$$\max_{\alpha(w^s)} \int\limits_{\mathcal{W}^d(\mathcal{M})} \tau w^d dF^d(w^d)dF^s(w^s) + \int\limits_{\mathcal{W}^s(\mathcal{M})} [\tau x(w^s) + \alpha(w^s)\pi(\tau w^s - \tau x(w^s))]\, dF^d(w^d)dF^s(w^s)$$

$s.t.$

$$B \geq \int\limits_{\mathcal{W}^s(\mathcal{M})} c \cdot \alpha(w^s)dF^d(w^d)dF^s(w^s)$$

$$\alpha(w^s) \text{ non-increasing in } w^s$$

$$\mathcal{T}(w^s) = \frac{w^s(1-\tau\alpha(w^s)\pi)}{1-\pi\alpha(w^s)} - \frac{\int_{j=\underline{w}^s}^{w^s} (1-\alpha(j)\pi\tau)\,dj}{1-\pi\alpha(w^s)} - \frac{V^s(\underline{w}^s)}{1-\pi\alpha(w^s)} = \tau x(w^s)$$

$$U^d(\kappa(w^s)) = V^s(w^s,\mathcal{M})$$

The first restriction is the budget constraints, which asserts that the IRS audits until equalizing the expected audit cost with its budget. The second and third conditions come from the requirement for a direct IC mechanism. Finally, the fourth condition is the threshold function's definition, which defines the occupational choice rule.

Some procedures to simplify the problem are made before solving it. Let us assume that the threshold function is weakly-increasing in the income of the self-employed. In addition, since at the equilibrium by the direct IC mechanism agents do not misreport their wages, the expected penalty is zero. The objective function and the budget constrain are rewritten as

$$\max_{\alpha(w^s)} \int_{\underline{w}^s}^{\overline{w}^s}\int_{\kappa(w^s)}^{\overline{w}^d} \tau w^d dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} \mathcal{T}(w^s)dF^d(w^d)dF^s(w^s)$$

$$B \geq \int_{\underline{w}^s}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} c\cdot\alpha(w^s)dF^d(w^d)dF^s(w^s)$$

The second term in the objective function does not rely on productivity in the dependent sector, and the same occurs in the budget constraint. This evidence implies that it is possible to simplify both expressions taking into consideration only the mass of agents with productivity until $\kappa(w^s)$.

To solve this problem, firstly one has to replace the audit function using the IC condition, $dV^s(w^s,\mathcal{M})/dw^s = V^s_{w^s}(w^s,\mathcal{M}) = 1 - \pi\alpha(w^s)\tau$. Secondly, it can be used the fact that $\mathcal{T}(w^s) = w^s - V^s(w^s,\mathcal{M})$. With those changes it is possible to formulate the problem of the IRS using the indirect utility in the self-employed sector, $V^s(w^s, \mathcal{M})$, as the state variable, and its derivation with regard to productivity, $V^s_{w^s}(w^s, \mathcal{M})$, as the control variable. This specification suggests that the condition over the audit scheme is replaced for a condition over the marginal indirect utility. More specifically, the marginal indirect utility must be non-decreasing and its support is $[1-\tau, 1]$. The Lagrangian of this problem is

$$\begin{aligned}\mathcal{L}(V^s(w^s,\mathcal{M}); V^s_{w^s}(w^s,\mathcal{M})) = & \int_{\underline{w}^s}^{\overline{w}^s} \int_{\kappa(w^s)}^{\overline{w}^d} \tau w^d dF^d(w^d) dF^s(w^s) \\ & + \int_{\underline{w}^s}^{\overline{w}^s} [w^s - V^s(w^s,\mathcal{M})] F^d(\kappa(w^s)) f^s(w^s) dw^s + p(w^s) V^s_{w^s}(w^s,\mathcal{M}) \\ & - \mu \left(\int_{\underline{w}^s}^{\overline{w}^s} c \cdot \left[\frac{1 - V^s_{w^s}(w^s,\mathcal{M})}{\pi\tau} \right] F^d(\kappa(w^s)) f^s(w^s) dw^s - B \right)\end{aligned}$$

where $p(w^s)$ is the adjoint function associated with the state variable, and μ the Lagrange multiplier associated with the budget constraint. The necessary conditions to solve this problem are the following:

1. $\dfrac{d\mathcal{L}^*(V^s(w^s,\mathcal{M}); V^s_{w^s}(w^s,\mathcal{M}))}{dV^s_{w^s}(w^s,\mathcal{M})} = p(w^s) + \mu \left(\int_{\underline{w}^s}^{\overline{w}^s} \left[\dfrac{c}{\pi\tau} \right] F^d(\kappa(w^s)) f^s(w^s) dw^s \right)$

2. $\dfrac{dp(w^s)}{dw^s} = -\dfrac{d\mathcal{L}^*(V^s(w^s,\mathcal{M}); V^s_{w^s}(w^s,\mathcal{M}))}{dV^s(w^s,\mathcal{M})} = \int_{\underline{w}^s}^{\overline{w}^s} F^d(\kappa(w^s)) f^s(w^s) dw^s$

3. $V^s(\overline{w}^s)$ is free, implying that $p(\overline{w}^s) = 0$.

4. $\mu \geq 0$, $\mu \left(\int_{\underline{w}^s}^{\overline{w}^s} c \cdot \left[\dfrac{1 - V^s_{w^s}(w^s,\mathcal{M})}{\pi\tau} \right] F^d(\kappa(w^s)) f^s(w^s) dw^s - B \right) = 0$

The first necessary condition shows that the solution is linear in the control variable, implying that the control variable takes only its support's extreme values. The second condition is positive, which indicates that the adjoint function is increasing, and the third condition suggests that it comes from the negatives to zero. Those three conditions reflect that the budget constraint is crucial for the identification of the solution. As a matter of fact, in the case that $\mu = 0$, or the budget is enough to audit the entire self-employed sector, the solution should be auditing all agents in the self-employed sector. Finally, the solution shows that the IRS always uses its entire budget.

When the solution does not imply full auditing, the candidate for a solution is the wage that equalizes the budget constraint. The solution to this problem is defined as w^*; this level separates the audit function between its extreme values. Below the wage w^*, the IRS audits efficiently, or $\alpha(w^s) = 1/\pi$. This level is efficient in the sense that deters evasion for every indirect mechanism, i.e., every mechanism that does not produce that the income declaration set is equal as income set.

The following equation characterizes the wage level that solves this problem.

$$\int_{\underline{w}^s}^{w^*} F^d(\kappa(w^s)) f^s(w^s) dw^s = \frac{B\pi}{c} \tag{1}$$

The first necessary condition guarantees the existence of a solution since the adjoint function exists, and the second term is always non-negative. Also, since the budget constraint is strictly increasing, the solution must be unique.

The audit scheme indicates that, below w^*, the IRS audits efficiently and, after the threshold wage, audits are equal to zero. The audit schedule takes the same form as in Sanchez and Sobel (1993) and fulfills the condition over the audit scheme in Lemma 2. Furthermore, for agents with productivity below the cut-off wage, the effective tax is equal to the taxes defined by the government. In this sense, for those agents the effective tax takes the form as a poll tax. For the rest of taxpayers the effective tax is different. Agents with productivity bigger than w^* pay taxes equal as the agent with the cut-off wage because they do not face an audit. The following proposition formalizes these results as the direct IC mechanism for this problem.

Proposition 3 *The direct IC mechanism $\mathcal{M}$ which solves this problem is*

$$\alpha(w^s) = \begin{cases} 1/\pi & \text{if} \quad \underline{w}^s \leq w^s < w^* \\ 0 & \text{if} \quad w^* \leq w^s \leq \overline{w}^s \end{cases} \tag{2}$$

$$\mathcal{T}(w^s) = \begin{cases} \tau w^s & \text{if} \quad \underline{w}^s \leq w^s < w^* \\ \tau w^* & \text{if} \quad w^* \leq w^s \leq \overline{w}^s \end{cases} \tag{3}$$

where the threshold w^* is defined by

$$\int_{\underline{w}^s}^{w^*} F^d(\kappa(w^s)) f^s(w^s) dw^s = \frac{B\pi}{c}$$

Corollary 1 *If the IRS has not audit cost (i.e., $c = 0$) the direct IC mechanism $\mathcal{M}$ is*

$$\alpha(w^s) = 1/\pi$$
$$\mathcal{T}(w^s) = \tau w^s$$

Proof.
See Appendix G ■

Figure 2: IRS Problem's Results

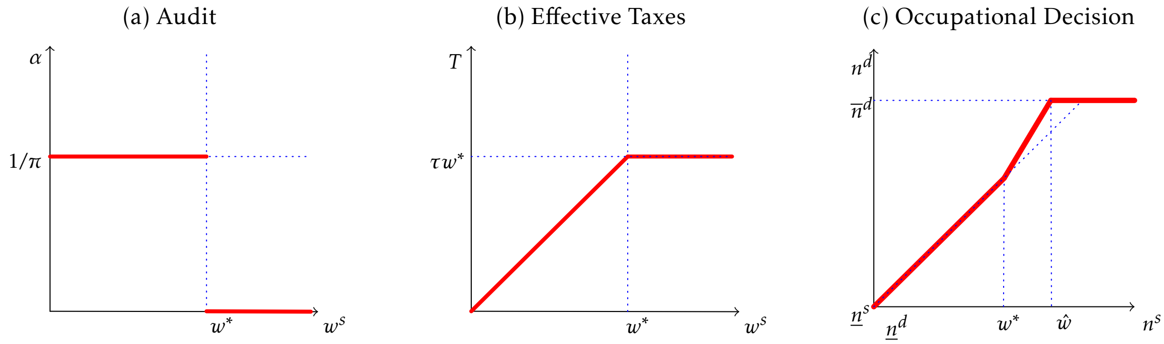


Figure 2 shows the results from Proposition 3. Panel (a) shows that all agents with an income higher than the threshold level face no audit. This fact produces that, to incentive/encourage truth-telling, those agents face a zero effective marginal tax rate, or pay the same taxes, as seen in panel (b). Finally, the results on audit and the effective taxes produce a distortion in the occupational choice for self-employed workers with wage higher than the cut-off. Distortions increase the incentive for being self-employed, this arise mainly from the audit schedule. Therefore, if the solution implies audit the entire self-employed sector, the distortion will vanish.

The audit function means that the IRS use audit for twofold: to examine tax compliance and as a threat to avoid income misreport. In a normative sense, and in line with Kuchumova (2017), an audit should be stronger in those income levels where it is likely to find evaders. As it is shown in Almunia and Lopez-Rodriguez (2018) for firms, agents strategically declare less income (revenue) to avoid more forceful audits and take advantage of this. Hence, not only audits must be used as a tool for tax compliance but also as a threat to strategic behavior, which results in tax evasion. In this case, the agent's incentive is to under-report income to pay fewer taxes. For this reason, the audit must be higher at the low and middle-wage levels if the IRS cannot audit the entire self-employed sector.[8]

A partial characterization of the threshold function is necessary to give a partial result of the second stage equilibrium, and to understand the distortion shown in Figure 2 panel (c) better. The following equation partially characterizes the threshold function, but for a complete result, it is necessary to solve the government problem.

$$\kappa'(w^s) = \begin{cases} 1 & \text{if } \underline{w}^s \leq w^s < w^* \\ \\ \dfrac{1}{1-\tau} & \text{if } w^* \leq w^s \leq \overline{w}^s \end{cases}$$

The possibility of evading taxes makes that the best course of action for the IRS is to audit efficiently until some level, defined as w^*. This level depends on three exogenous variables for the

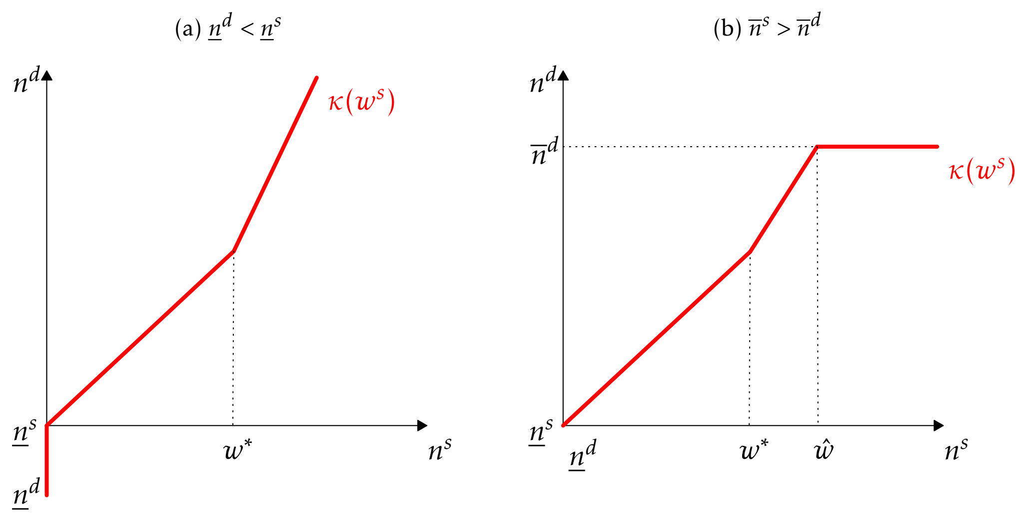
[8]This result can change with an upward incentive scheme, in other words, if the agent has the incentive to declare larger productivity. If low-productivity agents have the motivation to report larger productivity, or income, Bigio and Zilberman (2011) and Zilberman (2016) show that the audit schedule is increasing in the agent's productivity, producing the opposite result to the one shown in the equation 2.

IRS: the audit cost, the fine rate, and its budget. If those variables' conjugation implies that it is not optimal to audit all self-employed sector, distortions in the occupational decision appear and play only for agents with wages higher than the cut-off solution. These distortions produce that more agents decide to become self-employed as larger is the marginal tax rate. The explanation for this fact is simple: since the IRS must incentive taxpayers to tell their real wages, the effective tax for agents who do not face audits is a poll tax, or must be equal. This fact produces an increase in the utility for work as a self-employed in this productivity zone, provoking distortions to appear.

4 Government Problem

Before dealing with the government's problem, some relevant aspects of the productivities distribution in this step are clarified. Bearing in mind the assumption of independence in the distribution function, there exist four feasible scenarios related to the possible non-common support in the productivity distribution. The cases are: either one of the lower bound is smaller than the other ($\underline{n}^d < \underline{n}^s$ or vice versa) or one upper bound is larger than the other ($\overline{n}^s > \overline{n}^d$ or vice versa). This paper focuses on the latter case, as is shown in Figure 3 panel (b). The first case, shown in Figure 3 panel (a), only produces that the threshold function takes the value of the lower bound self-employed productivity, which does not depend on taxes or the budget for the IRS. Whereas, the second case produce the existence of a limit level since the threshold function derivative is zero, and potentially, impact on the optimal results.

Figure 3: Productivity Distribution Cases



Let us define the threshold which solve the IRS's problem as w^* and the point where the threshold function takes the upper bound of the dependent distribution as $\hat{w}$.[9] Given this, the threshold

[9]In order to clarify the another case, Appendix K shows the solution for $\overline{n}^d > \overline{n}^s$.

function takes the following form

$$\kappa(w^s) = \begin{cases} w^s & \text{if } w^s < w^* \\ \dfrac{w^s - \tau w^*}{1-\tau} & \text{if } w^* \leq w^s < \hat{w} \\ \overline{w}^d & \text{if } \hat{w} \leq w^s \end{cases} \tag{4}$$

This function only depends on taxes in the zone where the IRS does not audit. Thus, any change in the IRS's budget affects the threshold function through the cut-off wage w^* in this zone. Both instruments, taxes and IRS's budget, modify not only the shape of the threshold function but also the level where it takes the maximum (i.e., $\hat{w}$). Those facts produce two effects: 1) distortions in the incentives to be in some occupations, and 2) changes in the mass of agents that, regardless of their dependent productivity, always prefer being self-employed. More specifically, Figure 3 panel (b) shows that all agents with productivity larger than $\hat{w}$ always prefer being self-employed, even if they have the most significant productivity in the dependent sector.

With those specifications and incorporating the results of the IRS's problem to ensure a direct IC mechanism, the government's problem is as follows

$$\max_{\tau,R,B} \int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\overline{w}^d} G\big(U^d(w^d)\big) dF^d(w^d) dF^s(w^s) + \int_{w^*}^{\hat{w}} \int_{\kappa(w^s)}^{\overline{w}^d} G\big(U^d(w^d)\big) dF^d(w^d) dF^s(w^s) + \int_{\underline{w}^s}^{w^*} \int_{\underline{w}^d}^{\kappa(w^s)} G(V^s(w^s,\mathcal{M})) dF^d(w^d) dF^s(w^s)$$

$$\int_{w^*}^{\hat{w}} \int_{\underline{w}^d}^{\kappa(w^s)} G(V^s(w^s,\mathcal{M})) dF^d(w^d) dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} G(V^s(w^s,\mathcal{M})) dF^d(w^d) dF^s(w^s)$$

$s.t$

$$\tau \int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) + \tau \int_{w^*}^{\hat{w}} \int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) + \tau \int_{\underline{w}^s}^{w^*} w^s F^d(\kappa(w^s)) dF^s(w^s) + \tau w^* \int_{w^*}^{\hat{w}} F^d(\kappa(w^s)) dF^s(w^s)$$

$$+ \tau w^* \int_{\hat{w}}^{\overline{w}^s} F^d(\kappa(w^s)) dF^s(w^s) \geq B + R$$

$$w^* \text{ solves } \int_{\underline{w}^s}^{w^*} F^d(\kappa(w^s)) f^s(w^s) dw^s = \frac{B\pi}{c}$$

$$\alpha(w^s) = \begin{cases} 1/\pi & \text{if } \underline{w}^s \leq w^s < w^* \\ 0 & \text{if } w^* \leq w^s \leq \overline{w}^s \end{cases}$$

$$\mathcal{T}(w^s) = \begin{cases} \tau w^s & \text{if } \underline{w}^s \leq w^s < w^* \\ \tau w^* & \text{if } w^* \leq w^s \leq \overline{w}^s \end{cases}$$

$$U^d(w^s) \geq 0, \; V^s(w^s,\mathcal{M}) \geq 0$$

The government faces five restrictions. First, the budget constraint consists of a balanced budget rule, which means that the expected tax collection must be at least equal to the expenses in the public goods provision and the IRS's budget. In the government budget constraint, the first two terms reflect the dependent sector, the third term represents the self-employed, who are audited efficiently, and the other terms reflect those agents who do not face audits. There exist three restrictions that come from the IRS's problem: 1) the definition of the productivity threshold w^*, 2) the definition of audit schedule and 3) effective taxes. Finally, the fifth restriction is the limited liability, which states that any agent has a non-negative utility.

The government's problem is to choose the marginal tax rate, the IRS's budget and the provision of public goods, to maximize the social welfare function subject to its budget constraint. The procedure is to use the budget constraint and incorporate the rest of the restriction into the problem. Let δ be the Lagrange multiplier for the government budget constraint, the Lagrangian for this problem is

$$\begin{aligned}\mathcal{L} = &\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} G(V^s(w^s,\mathcal{M}))dF^d(w^d)dF^s(w^s)\\ &+\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)} G(V^s(w^s,\mathcal{M}))dF^d(w^d)dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} G(V^s(w^s,\mathcal{M}))dF^d(w^d)dF^s(w^s) - \delta\left\{B + R - \tau\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s)\right.\\ &\left.-\tau\int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) - \tau\int_{\underline{w}^s}^{w^*} w^s F^d(\kappa(w^s))dF^s(w^s) - \tau w^*\int_{w^*}^{\hat{w}} F^d(\kappa(w^s))dF^s(w^s) - \tau w^*\int_{\hat{w}}^{\overline{w}^s} F^d(\kappa(w^s))dF^s(w^s)\right\}\end{aligned}$$

4.1 Optimal Public Goods Provision

To describe the optimal BLS rule is followed Section 2.4, using the notation $MRS(RC)$ for the marginal rate of substitution between the public good and the private consumption good. The following proposition formalizes the optimal BLS rule in this model.

Proposition 4 *The Bowen-Lindahl-Samuelson rule is*

$$\begin{aligned}\phi'(R) = &\int_{\underline{w}^s}^{\overline{w}^s}\int_{\kappa(w^s)}^{\overline{w}^d} MRS(RC)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} MRS(RC)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)} MRS(RC)dF^d(w^d)dF^s(w^s)\\ &+\int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} MRS(RC)dF^d(w^d)dF^s(w^s) = 1 \qquad (5)\end{aligned}$$

Proof.
See Appendix H.1 ■

It is important to highlight that the BLS rule is the same as the first best, and this result comes from the agent's utility. In particular, as is well established in Boadway and Keen (1993), since consumption is linear in the agent's utility, evasion does not distort the optimal provision of public goods. This is due to the fact that all agents have the same MRS, implying that variations in consumption due to evasion does not generate distortion on it.

This result implies two conclusions to be taken into account. Firstly, the government only has two instruments to improve social welfare: taxes and the IRS budget. Secondly, giving the audit solution, some agents pay fewer taxes than in the case without evasion but have the same amount of public good. This difference produces a worsened and more unequal distribution, because some taxpayers raise their utilities, and others bear, percentage-wise, more burden to finance the public goods.

4.2 Optimal Linear Tax Rate

The optimal tax rate partially characterizes the threshold function and shows the implications in occupational choice. When it comes to the threshold function, the effect of taxes is twofold. First, it produces a change in the threshold function's slope, modifying the incentive to being in one occupation. Second, taxes change the level where the threshold function takes its upper level, affecting the mass of agents who always prefer being in one occupation.

In order to give a better interpretation of the marginal tax rate, $g(w)$ can be defined as the social value of consumption for an agent with productivity w expressed in terms of the public funds (Saez, 2001; Saez and Stantcheva, 2016). Formally, $g(w) = {}^{(G' \times U'_C)}/_{\delta}$. By definition, the sum of g among agents is equal to one. This expression indicates the importance of an increase in agent's consumption concerning public funds.

Proposition 5 *The optimal marginal tax rate is characterized by*

$$\begin{aligned}
\frac{\tau}{1-\tau}\int_{w^*}^{\hat{w}}(w^s-w^*)\kappa_\tau(w^s)f^d(\kappa(w^s))dF^s(w^s) &= \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d}(1-g(w^d))w^d dF^d(w^d)dF^s(w^s)+\int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d}(1-g(w^d))w^d dF^d(w^d)dF^s(w^s)\\
&+\int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)}(1-g(w^s))w^s dF^d(w^d)dF^s(w^s)+\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}(1-g(w^s))w^* dF^d(w^d)dF^s(w^s)\\
&+\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}(1-g(w^s))w^* dF^d(w^d)dF^s(w^s) \qquad (6)
\end{aligned}$$

Proof.
See Appendix H.2. ■

The marginal tax formula can be decomposed into two effects: welfare and revenue. The welfare effect shows the impact of changes in the agent's utility through the effect of taxes on consumption. When taxes rise, all agents face more significant taxes than before, therefore, consume less. However, the reduction in their consumption depends on the wage that each consumer had. This effect must be measured in public funds terms; hence, the term $g(w)w$ captures it. The revenue effect has two components. First, when taxes rise, all agents pay more taxes, which is equal to each taxpayer's wage. Second, an increase in taxes produces that some agents change their occupation and pay a different tax level. When taxes rise, a taxpayer with a former wage $\kappa(w^s)$ bigger than w^*, moves from the dependent to the self-employed sector and pays τw^*. The term which captures this effect is $\tau(\kappa(w^s) - w^*)\kappa_\tau(w^s)f^d(\kappa(w^s))$. In equilibrium, the sum of those effects must be zero. Figure 4 shows the forces behind the marginal tax formula.

Figure 4: Effect of Raising Tax Rate

(a) Agents Distribution (b) Taxes Paid

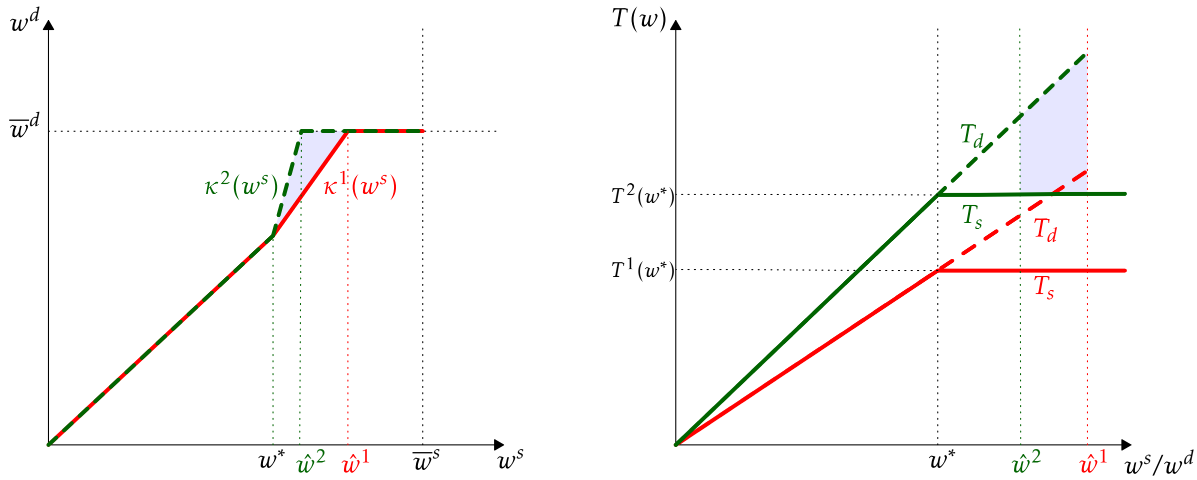


Figure 4 panel (a) shows the consequences of an increase in the marginal tax in the threshold function and occupation distribution. The red line indicates the initial situation, whereas the dashed green line illustrates a raise in the marginal tax. All agents with a productivity tuple situated behind the red (green) line (dashed line) are self-employed. An increase in taxes produces that agents with productivity between w^* and $\hat{w}^1$ change their occupational decision; this is depicted as the blue zone. However, the threshold wage does not change because the IRS's budget does not change. [10] Those facts provide an increase in the mass of self-employed agents. Additionally, an increase in the marginal tax rate produces that the productivity level $\hat{w}$ falls, from $\hat{w}^1$ to $\hat{w}^2$. This change produces the mass of agents that always prefer to be self-employed increase. This effect captures two pieces of evidence. Firstly, a collective of agents changes their occupation, changing the proportion of agents in each occupation. Secondly, those agents who change their occupation face new taxes, producing an impact on tax revenue and an impact on the agents' utility.

Figure 4 panel (b) shows the consequences of an increase in marginal tax in the taxes paid by

[10] To portray this fact much more efficiently, let us differentiate the definition of the threshold level with respect to τ and w^* and equalize to zero:

$$\int_{\underline{w}^s}^{w^*} \left(\frac{\partial \kappa}{\partial w^*} dw^* + \frac{\partial \kappa}{\partial \tau} d\tau \right) f^d(\kappa(w^s)) dF^s(w^s) + F^d(w^*) f^s(w^*) dw^* dw^s = 0$$

By dividing both side by $d\tau$ obtaining:

$$\int_{\underline{w}^s}^{w^*} \left(\frac{\partial \kappa}{\partial w^*} \frac{dw^*}{d\tau} + \frac{\partial \kappa}{\partial \tau} \right) f^d(\kappa(w^s)) dF^s(w^s) + F^d(w^*) f^s(w^*) \frac{dw^*}{d\tau} dw^s = 0$$

The only way to obtain $\frac{dw^*}{d\tau} = 0$ is if $\frac{d\kappa}{d\tau} = 0$. This hold from the IRS's solution, which says that $\kappa(w^s) = w^s$ for $w^s < w^*$ and $\kappa(w^*) = w^*$.

agents and in tax revenue. Let us keep in mind that the red line represents the initial situation, and the green line refers to the tax rise. Moreover, it is also shown those taxes paid by the two occupations. The solid line is for the taxes paid in the self-employed sector and the dashed line in the dependent one. When taxes rise, all agents pay higher taxes than before; this is represented by the difference between the red and green lines, increasing tax revenue. However, some agents change their occupation and face different taxes. The agents who change their occupation are those who do not face audits, hence, bear taxes equal to τw^* instead of τw^d. This change produces a revenue loss for the government, depicted by the blue zone. Moreover, the more significant the increase in taxes, the bigger the revenue loss is. Those forces, the increase in revenue vs. welfare and revenue losses, produce the equilibrium in the tax rate.

The inclusion of occupational choice produces taxes smaller than one due to possible losses from changes in occupation. This finding evidences the forces behind the Laffer curve. If total revenue gains are more significant than welfare and occupational revenue losses, the marginal tax will increase. In the opposite case, the marginal tax will decrease. Furthermore, this result is opposed to Sanchez and Sobel (1993), who obtain a tax rate that is equal to one, showing that not considering occupational decision induces an upward bias in the tax rate, and induces agents to work as self-employed. For this reason, it is relevant to consider different occupations, and also different possibilities to evade.

The relationship between the marginal tax rate and the threshold function may explain the empirical relation between tax rates and income tax evasion. If the marginal tax falls, the threshold function will take smaller values, producing a reduction in the incentive to evade taxes. Empirically, Bárány (2019), and Berger et al. (2016) show a positive relationship between high tax rates and high evasion rates. This relationship indicates that to deter evasion, the government must reduce incentives to evade taxes through falling taxes. This mechanism is similar to previous findings in the general equilibrium literature (Kesselman, 1989; Watson, 1985). All this evidence reinforces the importance of taxes as a tool in discouraging tax evasion.

4.3 Optimal IRS Budget

Changes in the IRS's budget produce changes in the agency's ability to audit more (less) agents, changing the threshold wage and the threshold function's upper level. Moreover, those changes produce a modification in the self-employed tax paid. For this reason, a modification in the amount of the IRS's budget induce changes in occupation decisions, but also in tax revenue. Those facts are into the forces behind the optimal budget for the IRS.

Proposition 6 *The optimal budget for the IRS is characterized by*

$$\tau\int_{w^*}^{\hat{w}}(w^*-\kappa(w^s))\frac{dw^*}{dB}\kappa'_B f^d(\kappa(w^s))dF^s(w^s)+\tau\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s)+\tau\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s)$$

$$=1+\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}g(w^s)\tau\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s)+\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}g(w^s)\tau\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s) \quad (7)$$

Proof.

See Appendix H.3. ■

It is possible to decompose this equation into three effects: behavioral, mechanical and welfare. To start with, the behavioral effect relates to the capacity to audit more (less) taxpayers, producing that a mass of agents changes their occupational decision. As for the mechanical effect, this one reflects the impact on agents' taxes and the cost of changing the budget, which is equal to one. Since the threshold level changes, the taxpayer who does not face audit pays more taxes than before, and this effect is purely mechanical. This change produces that their consumption falls in the amount that budget affects taxes, producing a welfare loss, which must be measure in public fund terms. In equilibrium, the sum of the three effects must be zero, and this provides the characterization in Proposition 6.

This equation shows three relevant results. First, similar to Sanchez and Sobel (1993), the IRS always has a budget smaller than that which allows auditing the entire self-employed sector, because the marginal cost of increasing the budget is always one and the marginal gain is zero when the IRS audit the entire self-employed sector. Second, in determining the optimal budget, it is necessary to consider the effect on welfare. This result produces a distance from the cost-benefit analysis. Finally, as established in Slemrod and Yitzhaki (1987), the government must consider the tax burden when deciding the IRS's budget. The increase in the budget attempts to decrease distortions in the tax burden.

5 Discussion

Although the equilibrium in this model is already shown, some questions are still open. Those questions are related to differential taxation, different assumptions over elements in the IRS's problem, and comparative statics. Firstly, this section looks into the optimality to impose two different linear tax rates, one for each occupation. This extension allows contemplating the possibility of recovering efficiency in the allocation of workers. Regarding the audit result, the problem of achieving the same result pattern with non-linear audit costs and imposing an increasing fine rate is resolved. Finally, the effect on changing the audit costs on the tax rate and the IRS's budget is shown.

5.1 Differential Taxation

Differential taxation allows to seek efficiency in the allocation of agents, even if the audit schedule produces incentives to be self-employed. However, because of the linear marginal tax rate, it is difficult to recover the first best as every action taken to get rid of the incentive in the high-income agents through tax rate ends up affecting the rest of them. Also, it is critical for the design of both marginal tax rates that each of one of them must meet with the IC requirement, in specific both marginal tax rates cannot be higher than one. By using differential taxation, the threshold function takes the following form

$$\kappa(w^s) = \begin{cases} \dfrac{w^s(1-t_s)}{1-t_d} & \text{if } w^s < w^* \\ \dfrac{w^s - t_s w^*}{1-t_d} & \text{if } w^* \leq w^s < \hat{w} \\ \overline{w}^d & \text{if } \hat{w} \leq w^s \end{cases}$$

In the above formulation, t_s and t_d are the tax rate in the self-employed and the dependent sector, respectively. In addition, the budget constraint change, now takes the following form

$$t_d \int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) + t_d \int_{w^*}^{\hat{w}} \int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) + t_s \int_{\underline{w}^s}^{w^*} w^s F^d(\kappa(w^s)) dF^s(w^s) + t_s w^* \int_{w^*}^{\hat{w}} F^d(\kappa(w^s)) dF^s(w^s)$$
$$+ t_s w^* \int_{\hat{w}}^{\overline{w}^s} F^d(\kappa(w^s)) dF^s(w^s) \geq B + R$$

Although this specification changes the threshold function, it does not alter the form of the optimal audit. This result comes from the necessary conditions for a solution to the IRS's problem. Since the strategy to solve the IRS's problem, differential taxation only changes the applicable tax rate for the determination of the threshold. This is to say, either audit the entire self-employed sector or until the level determined by the budget.[11] For this reason, the same optimal audit pattern is valid for this specification.

The government solves a Lagrangian similarly as in Section 4, but, in this case, it chooses both tax rates and the budget for the IRS. As it has been previously explained, the public goods provision is the same as in the first best. This result is maintained in this framework because choosing two different tax rates does not alter this result. Also, since the focus is only in differential taxation, the optimal budget for the IRS is not obtained.

Proposition 7 *In a hierarchical model with occupational choice where the IRS maximizes expected collection and the government maximizes the social welfare, differential taxation implies a higher marginal tax rate in the sector where evasion is possible.*

[11] To see this, note that in the Lagrangian of the IRS's problem the marginal indirect utility is used as a control variable, thus conditions 2 and 3 do not change. Only condition 1 changes, but this change is used t_s instead of τ and they do not change the form of the solution.

Proof.
See Appendix I. ■

This result comes from the possibility to distort sufficiently enough the occupational decision for agents who face zero audits to compensate for the losses in tax collection in the zone where audits are efficient. That is to say, the increase in tax revenue from agents who decide work as a dependent worker in the location where audits are zero is more significant than the losses from agents who made the same change in the zone where audits are positive. This effect happens at the margins; hence, the government compares only those agents who change its occupation due to tax changes.

Figure 5: Differential Taxation

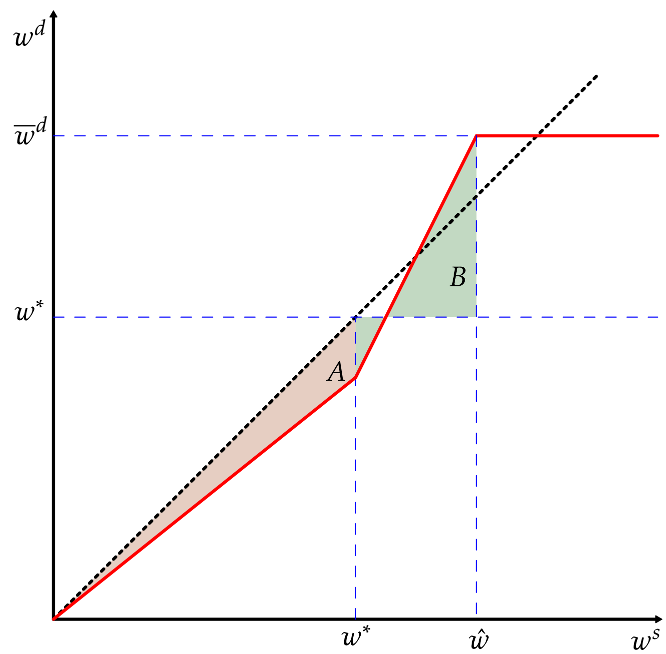


Figure 5 shows the basic idea behind Proposition 7. The red line represents the threshold function, and the black dotted line represents the same wage in both sectors. Differential taxation is possible only if zone A is smaller than zone B. Both zones represent the difference in taxable income for agents who change its occupation due to changes in the marginal tax rate. Zone A represent the losses from agents who decide to be a dependent worker, but paid higher taxes and declared higher income as a self-employed worker. In contrast, zone B shows the increase in tax collection from agents who paid higher taxes in the dependent sector because they paid a constant tax in the self-employed sector. Thus, differential taxation happens if the increase in tax collection from agents above w^* is bigger than the losses from agents below it. When the difference in the marginal taxes rise, the threshold function fall, increasing A and decreasing B; thus, the government do not have the incentive to separate both marginal taxes a lot. Consequently, differential taxation aims to solve tax collection losses due to zero audits rather than improve the allocation of agents, ending up distorting the entire workers' occupational decision.

The higher tax for a self-employed worker in an occupational decision model goes in the opposite direction as the traditional literature in optimal taxation, which recommends smaller taxes for self-employed workers (Mirrlees, 1971, 1976). Although the differences in policy recommendations, the meaning of both arguments is the same: using tax incentives to vanish the motivation to evade. When an occupational decision is incorporated into the optimal taxation model, a rise in self-employment tax rate differential produces, simultaneously, vanishing the incentive to evade and increase welfare due to the differential increase in the dependent sector consumption. For this reason, both arguments go in the same way but with different recommendations.

The possibility of differential taxation produces a distortion in the allocation of agents, resulting in distance from the first best. Ergo, this solution produces a larger distortion in the workers' allocation, as one with only one marginal tax rate equal for both occupations. For this reason, in this model, differential taxation is useful for increasing revenue rather than increasing the efficiency in allocating workers. Moreover, if the government decides to impose differential taxation, distortion will be higher than if it has only one marginal tax rate.

Corollary 2 *It is not possible in a hierarchical model with occupational choice and linear tax schemes to recover the efficiency of workers' allocation.*

This result is essential for policymakers to design tax and audit policies. The possible solution for the audit scheme's inefficiencies through differential taxation allows the government to improve economic efficiency and raise revenue. However, Corollary 2 reflects that it is impossible to reconcile incentives for taxpayers to tell the truth with the incentive to choose their occupations efficiently. Hence, the government needs to find other policies to improve efficiency and raise revenue at the same time.

5.2 Audit Results

5.2.1 Audit Costs

As in the model, it is assumed that the audit cost depends on the agent's productivity in the self-employed sector and it is known for the IRS when the agent reveals its type. The audit cost is defined as $c(w^s)$. From Section 3, the condition that determines the form of the solution is the first necessary condition from the Lagrangian of the IRS's problem. In this condition, only the audit cost changes because neither the control variable changes nor does the state variable. The new first necessary condition is

$$\frac{d\mathcal{L}}{dV^s_{w^s}} = p(w^s) + \mu\left(\int_{\underline{w}^s}^{\overline{w}^s}\left[\frac{c(w^s)}{\pi\tau}\right]F^d(\kappa(w^s))f^s(w^s)dw^s\right)$$

A condition similar to a single-crossing is found, in the sense that the above function should crosses one time the zero line. To obtain this condition, let us derive the equation above with regard to productivity in the self-employed sector and use two facts. First, by the linear income tax, the second derivative of tax is equal to zero. Second, the derivative of the adjunct function is positive. The following equation establishes the condition in which the threshold function as a solution to the new problem of the IRS is not obtained.

$$-\frac{c'(w)}{c(w)} > \frac{\pi\tau}{\mu c(w)} + \frac{f^d(k(w))k'(w)}{F^d(k(w))} + \frac{f^{s\prime}(w)}{f^s(w)}$$

where w is the level that equalizes the derivative $d\mathcal{L}/dV^s_{w^s}$ to zero. The threshold condition is fulfilled with a positive marginal cost, or a strictly increasing cost because the right side is positive by definition (let us assume distribution functions with a positive second derivative). Hence, if the audit cost is monotone non-decreasing in the self-employed agent's income, the threshold solution is maintained.

An exciting example is if the IRS has countervailing incentives as ones developed in Lewis and Sappington (1989). One case is assuming that the IRS has fixed costs that decrease with an increasing marginal cost. This form is the same as in Lewis-Sappington's model. In this case, the IRS audits only agents in the middle of the productivity distribution because at the extreme levels the cost is higher than in the middle. Zilberman (2016) reaches a similar conclusion, but he uses a participation constraint that depends on the productivity of agents, simiar as Jullien (2000). This fact produces the countervailing incentives for the agent's declaration, and the IRS must choose a non-increasing audit scheme. Both examples indicate the possibility of obtaining a different solution than the traditional threshold one.

5.2.2 Fine Rate

Another interesting result is obtained when the fine rate is not linear. This issue regards the intensity of the penalty or the intention to deter high tax evasion levels. Two options are analyzed: 1) a fine rate increasing in the evaded amount and 2) a fine rate rising in the reported income. These specifications may allow the IRS to audit more taxpayers at less cost. This effect comes from the efficient audit form because a higher fine rate replaces the audit's deter spirit, allowing fewer levels and, consequently, fewer costs. For this reason, it is reasonable to take this kind of fines into consideration.

Given the direct mechanism, evasion does not exist in equilibrium, and any instrument that depends on an evaded level vanishes. Thus, a fine rate that depends on the amount of evaded taxes collapses to a linear fine rate in equilibrium. This situation is the same as the one proposed in the model and resulting in an equal optimal audit as one presented earlier.

As for a fine rate that depends on the reported wage, this is different. In this case, a similar analysis that one realized with audit cost is necessary. Now, the fine rate depends on the self-employed wage, and the first necessary condition became in

$$\frac{d\mathcal{L}}{dV^s_{w^s}} = p(w^s) + \mu\left(\int_{\underline{w}^s}^{\overline{w}^s}\left[\frac{c}{\pi(w^s)\tau}\right]F^d(\kappa(w^s))f^s(w^s)dw^s\right)$$

Deriving this condition with respect to the self-employed wage allows us to obtain the condition to maintain the single-crossing.

$$\frac{\pi'(w)}{\pi(w)} > \frac{\pi(w^s)\tau}{\mu c} + \frac{f^d(k(w))k'(w)}{F^d(k(w))} + \frac{f^{s\prime}(w)}{f^s(w)}$$

As it is stated before, w is the wage level that equalizes the $^{d\mathcal{L}}/_{dV^s_{w^s}}$ to zero. This expression shows that the growth in the fine rate must not be higher than the right side. In other words, to obtain the threshold form in the audit function, the marginal fine rate must be bounded from above. This result allows the imposition of an increasing fine rate but restricts the form that it raises. Therefore, it is possible to impose an increasing fine rate to put more penalties in higher incomes; and consequently, audit more agents and improve the IRS's effectiveness.

5.3 Comparative Statics

The effects are decomposed into direct and indirect effects. The former is the effect on taxes and the latter is the effect on the threshold level. It is possible to separate the effect on the impact on the tax collection and the impact on social welfare in each effect. The derivation of the effect on taxes is in Appendix J.1 and the effect in the budget is in Appendix J.2.

The overall effect of changes in audit cost on the tax rate depends on the size and impact on agents who face no audits. When the audit costs decrease, the threshold level raises, producing that all agents who do not face an audit pay higher taxes than the others. However, if marginal tax raises, the group of agents with productivity below $\hat{w}$ will have the incentive to work as self-employed, pay fewer taxes than the dependent workers, and increase their welfare. Both effects make the overall impact depend crucially on what happens with those agents. Furthermore, the result relies on which force is more prominent. If the increase in revenue is more significant than the effect on welfare, the overall impact of audit cost on taxes is positive. This result is very intuitive, this is to say, if the government faces an increase in audit cost, taxes only increase when the revenue increases more significantly than the welfare loss.

When it comes to the budget for the IRS, the situation is similar. When the budget rises, it is feasible to audit more agents, increasing the threshold level. As the mass of agents who face zero audit decreases, it is possible to impose higher tax rates because fewer agents have the incentive to change their occupation. Nevertheless, this impacts on welfare through a smaller after-tax income and a decrease in its consumption. The welfare for some taxpayers in the self-employed sector decreases, and as taxes rise, the entire welfare falls. Those effects produce that the entire effect in the budget is not apparent and depends on which force is more prominent.

6 Conclusion

By and large, this paper investigates the distortions produced by the introduction of occupational choice in a hierarchical model similar to Sanchez and Sobel (1993). In this sense, these authors' model is extended, incorporating the possibility for agents to choose between two occupations; dependent and self-employed. Income tax evasion is possible only in the self-employed sector because agents self-report their income. The focus is on distortion in the audit function, decided by the IRS, and in a linear marginal tax rate, the budget for the IRS, and public good provision, decided by the government.

The IRS maximizes expected tax collection subject to a budget constraint, with a constant audit cost, and considers that it must find a direct incentive-compatible mechanism. Such mechanism is composed of a direct audit and an effective tax, which is the tax paid by a self-employed agent who optimally declares its real income. It is of great importance to recall that the direct incentive-compatible mechanism has the intention to encourage the self-employed sector to reveal its real income. The optimal audits follow a threshold form. Before a cut-off wage, the audit level is efficient, due to the fact that it deters evasion in every indirect mechanism, and above this level, audits are equal to zero. This form states that the IRS must keep all efforts, using the audit to eliminate the incentive to under-declare income, which is the agent's incentive modeled in this paper. This audit pattern provokes that effective tax is equals for agents with a wage higher than the cut-off wage level. Afterward, it is shown that this result holds in two realistic extensions. First, if the audit cost depends on the self-employed reported income, the critical condition must be monotonously non-decreasing in the self-employed reported income. Second, the fine rate could be increased in the self-employed reported income, but its growth must be bounded from above.

The government maximizes social welfare subject to a resource constraint anticipating the incentive-compatible requirements and the IRS's solution. Because of risk-neutral agents, the optimal public good provision is the same as in the first best, producing that the government only has two instruments to attain its objectives. The marginal tax rate is smaller than one and is composed of two effects: welfare and revenue. In regard to the optimal IRS's budget, it is lower than the level that allows auditing all self-employed but is higher than the result from a cost-benefit analysis. The optimal IRS's budget can be characterized through three effects: behavioral, mechanical, and welfare. If the audit cost raises, both instruments, i.e. the marginal tax rate and the IRS's budget, will face undefined changes. The crucial fact is if revenue increases (decreases) are bigger/smaller than welfare losses (gains). In this setting, differential taxation is optimal only if the marginal tax in the self-employed sector is higher than in the dependent sector. This result goes in the opposite direction than the traditional optimal taxation literature.

The under-budgeted situation gives the result of the audit. On the one hand, if the IRS has sufficient budget to audit the entire self-employed sector, it will do so. However, on the other hand, audits introduce distortion in the occupational decision; this means that agents do not work in the sector where they are more productive purely because of the institutional policy. Hence, agents work in an occupation that increases its utility because the institutional policy allows so. Moreover, although differential taxation is optimal, this instrument does not solve the distortion in agents' allocation. Differential taxation increases this problem because it makes marginal tax in the self-employed sector be higher than in the dependent sector.

The comparison with the results over the tax rate seen in Sanchez and Sobel (1993) conveys that occupational decision produces a fall in the marginal tax rate, for the forces introduced by occupational decisions. Specifically, the marginal tax rate is smaller than one, which is the result in Sanchez and Sobel (1993). This solution shows that not considering occupational decisions induce an upward bias in taxes. This problem may increase the distortion in workers' allocation from the audit result because they introduce more incentives to work as a self-employed in those agents who face smaller audits.

The two previously mentioned results - this is to say a marginal tax rate smaller than one and the distortion in the allocation of agents - show the relevance of taking into account the occupational decision made by agents in any audit or taxation model. Since the problems rely on the optimal policies, it is necessary to inquire over new instruments, mechanisms and strategies to increase enforcement policies' effectiveness. Even when the government chooses its tax administration policy optimally, this paper demonstrates that distortions exist, and it is necessary to incorporate other course of actions to solve them.

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Appendix

A Occupational Choice Model vs. Evasion Model

The following section demonstrates that although it is possible to nest a model of occupational choice akin to the one developed in Gomes et al. (2017) with a hierarchical model similar to one built by Sanchez and Sobel (1993), the solution diverges in some cases. In order to do this, the same model as in Gomes et al. (2017) is used along with the solution in the audit scheme obtained in Sanchez and Sobel (1993), and finally, it is also shown how the solution diverges.

There exist two sectors in economy, self-employed and dependent. Each agent has an ability tuple (n_d, n_s), which represents the productivity in the dependent and the self-employed sector, respectively. Agents choose the labor supply h_i with $i = \{d, s\}$, implying an effective labor in sector i equals to $h_i n_i$. Income for work in sector i is $y_i = w_i h_i n_i$, which w_i is the wage rate in sector $i = \{d, s\}$. The government can impose two different tax schemes, one for each sector. Let T_i be the income tax schedule in sector i. Agent's utility is

$$u_i(n_i) = w_i h_i n_i - \psi(h_i) - T_i(w_i h_i n_i) \text{ for } i = \{d, s\}$$

where ψ is the convex cost of labor. To nest this frame with a hierarchical model with tax evasion, make the following assumption. In the dependent sector, labor supply is inelastic and equals to one, this is $h_d = \overline{h}_d = 1$. The first-order conditions with respect to the labor supply in the self-employed sector and the derivatives with respect to productivity in each sector respectively are

$$\begin{array}{lrcl} \langle h_s \rangle & w_s n_s \left[1 - T_s'(w_s h_s n_s)\right] & = & \psi'(h_s) \\ \langle n_s \rangle & w_s h_s \left[1 - T_s'(w_s h_s n_s)\right] & = & u_s'(n_s) \\ \langle n_d \rangle & w_d \left[1 - T_d'(w_d n_d)\right] & = & u_d'(n_d) \end{array}$$

The derivatives regarding productivity in each sector are obtained by simplifying the threshold function explained below. The threshold function reflects the ability level until any agent decides to work in the self-employed sector. Let $c : [\underline{n}_s, \overline{n}_s] \to [\underline{n}_d, \overline{n}_d]$ be the threshold function. By definition, this function holds with $u_s(n_s) = u_d(c(n_s))$ implying the following condition

$$c'(n_s) = \frac{u_s'(n_s)}{u_d'(c(n_s))} = \frac{w_s h_s \left[1 - T_s'(w_s h_s n_s)\right]}{w_d \left[1 - T_d'(w_d c(n_s))\right]} \tag{8}$$

This function is the same threshold function obtained in Gomes et al. (2017). Therefore, this as a result of a model with differential taxation and no evasion.

For the model with tax evasion, one can assume that in the self-employed and dependent sectors, the labor supply is inelastic and equal to one. Let x_s be the income report in the self-employed sector. This assumption produces migration to a model with evasion. In this case, the government also decides the audit strategy and imposes only one income tax schedule for all agents, T. Let ρ be the monitory strategy that depends on the declared income in the self-employed sector. If the government discovers an evader agent, they must pay a linear fine on the evaded amount with a penalty rate $\pi > 1$. The agent's utility in each sector is

$$\begin{array}{rcl} u_d(n_d) & = & w_d n_d - T(w_d n_d) - \psi(\overline{h}_d) \\ u_s(n_s) & = & w_s n_s - T(x_s) - \rho(x_s)\pi\left[T(w_s n_s) - T(x_s)\right] - \psi(\overline{h}_s) \end{array}$$

In the case with evasion, the government chooses the audit probability and the tax schedule, and in the model without evasion, it decides two tax schedules, one for each occupation. In each formulation, the government chooses two instruments; hence, in this sense, the problem is similar.

In the model with evasion, only the derivatives with respect to each sector's ability are obtained. These derivatives are as follow

$$\begin{array}{lccc} \langle n_s \rangle & w_s \left[1 - \rho(x_s)\pi T_s'(w_s n_s)\right] & = & u_s'(n_s) \\ \langle n_d \rangle & w_d \left[1 - T_d'(w_d n_d)\right] & = & u_d'(n_d) \end{array}$$

The threshold function in this case is

$$c'(n_s) = \frac{u_s'(n_s)}{u_d'(c(n_s))} = \frac{w_s \left[1 - \rho(x_s)\pi T_s'(w_s n_s)\right]}{w_d \left[1 - T_d'(w_d c(n_s))\right]} \tag{9}$$

For a clear comparison, the same wage rate is assumed in each model. The critical fact is to compare equation 8 with equation 9. The dependent sector has the same condition in each model; however, since self-employed agents face a fine and an audit probability, the self-employed sector's situation is different. Suppose that the taxes in the model without evasion are optimal for a particular distribution of ability. Now, let us proceed to find the conditions under which both threshold functions will be the same.

First, the dependent sector is the same in both models, so one can define the same tax schedule for each one, implying $T = T_b$. Also, it is assumed that c is equal in both models. With those, the numerator in equation 8 and equation 9 must be the same. This equality happens only if the following condition hold.

$$\rho(x_s) = \frac{1 - h_s \left[1 - T_s'(w_s h_s n_s)\right]}{\pi T_b'(w_s n_s)}$$

where h_s is the labor supply in the model without evasion. In the case that the above condition are met, both problems produce the same results and are correctly nested. However, if the audit strategy takes other forms, both solutions are not the same. Suppose the following audit scheme.

$$\rho(x_s) = \begin{cases} 1/\pi & \text{if } n_s < n^* \\ 0 & \text{if } n^* \leq n_s \end{cases}$$

where n^* is the threshold for the audit scheme. This strategy, which is optimal under the hierarchical model by Sanchez and Sobel (1993), shows that not always the same solution between a model with an occupational decision and one with tax evasion is obtained. Hence, it is not trivial to assume it is possible to nest both models and achieve similar results. In addition, the entire problem is different, principally for two facts. First, the government faces a cost for audit, which produces more expenditure and, consequently, a more significant revenue than in the absence of evasion. Second, the existence of evasion evidences how imperative is for the government to consider more incentive-compatible conditions to incentivize agents to declare their real type.

B Proof of Proposition 1

The first two expressions in Proposition 1 are the occupational choice rules and states that each agent works in their more productive sector. To prove them, suppose that a mass of agents greater than zero is assigned to work in their less productive sector. This mass of agents has $w^s > w^d$, but works in the sector d. From the resource constraint, redirecting all these agents to the sector s produces and increases economic resources, which can be assigned to increase the consumption of some agents or increase the provision of public goods, resulting in a social welfare increase. This means that the initial solution cannot be optimal because there exists a reallocation of agents that provide higher social welfare. Therefore, each agent works in the sector with bigger productivity.

The fifth expression comes from the maximization of the government problem with respect to the consumption in each sector. The first-order condition (henceforth FOC) over consumption is

$$\frac{dG(u(C_i(n^i),R))}{du(C_i(n^i),R)} f^d(n^d)f^s(n^s) - \gamma f^d(n^d)f^s(n^s) = 0, \qquad \text{for } i = d,s$$

where γ is the Lagrange multiplier for the resource constraint. Assuming an interior solution and by using γ to equalize both conditions and the fifth expression is obtained.

The third expression comes from the maximization of the social welfare function with respect to public goods. The first-order condition with respect to the public good is

$$\int_{\mathcal{Z}_d^*} \frac{dG}{du} \times \frac{d\phi(R)}{dR} dF^d(n^d)dF^s(n^s) + \int_{\mathcal{Z}_s^*} \frac{dG}{du} \times \frac{d\phi(R)}{dR} dF^d(n^d)dF^s(n^s) - \gamma$$

Both sides can be divided by γ and replaced by the FOC from consumption in each sector.

$$\int_{\mathcal{Z}_d^*} \frac{\frac{dG}{du} \times \frac{d\phi(R)}{dR}}{\frac{dG}{du} \times \frac{du}{dC_d}} dF^d(n^d)dF^s(n^s) + \int_{\mathcal{Z}_s^*} \frac{\frac{dG}{du} \times \frac{d\phi(R)}{dR}}{\frac{dG}{du} \times \frac{du}{dC_s}} dF^d(n^d)dF^s(n^s) = 1$$

It can be defined $MRS(R^*C_i^*)$ with $i = \{d,s\}$, as the marginal rate of substitution between public good and private consumption, evaluated at the optimum. It is critical to note that, the MRS is equal for all agents, because of $du/dC_i = 1$ and $du/dR = \phi'(R)$. This condition gives us the following, which is the third expression.

$$\phi'(R) = \int_{\mathcal{Z}_d^*} MRS(R^*C_d^*)dF^d(n^d)dF^s(n^s) + \int_{\mathcal{Z}_s^*} MRS(R^*C_s^*)dF^d(n^d)dF^s(n^s) = 1$$

Finally, the fourth expression shows the resources needed to finance the optimal provision of public goods R^* and comes from the resource constraint.

C Proof of Proposition 2

Before a more structured proof, it is possible to demonstrate the decentralization of the first-best invoking the second welfare theorem. In this demonstration, it is sufficient to note that the lump-sum taxes are the transfer vector in a quasi-equilibrium with transfers.[12]

The structured proof begins deriving the social welfare function concerning public goods. Let η be the Lagrange multiplier for resource constraint. The first-order condition, assuming an interior solution, is

$$\int_{\mathcal{N}_d} \frac{dG}{dU^i} \times \frac{d\phi(R)}{dR} dF^d(w^d)dF^s(w^s) + \int_{\mathcal{N}_s} \frac{dG}{dU^i} \times \frac{d\phi(R)}{dR} dF^d(w^d)dF^s(w^s) = \eta$$

[12] For a complete explanation of this see Mas-Colell et al. (1995) Chapter 16.

and the first-order condition (FOC) for the consumption is

$$\frac{dG\left(U^i(C_i^*(w^i),R^*)\right)}{dU^i(C_i^*(w^i),R^*)}f^d(w^d)f^s(w^s)-\eta f^d(w^d)f^s(w^s)=0$$

With what it is expressed above, it is possible to obtain the third expression.

$$\int\limits_{\mathcal{N}_d^*} MRS(R^*,C_d^*)dF^d(w^d)dF^s(w^s)+\int\limits_{\mathcal{N}_s^*} MRS(R^*,C_s^*)dF^d(w^d)dF^s(w^s)=1$$

Now, by using the FOC of consumption yields

$$\frac{dG\left(U^d(C_d(w^d),R)\right)}{dU^d(C_d(w^d),R)}=\frac{dG\left(U^s(C_s(w^s),R)\right)}{dU^s(C_s(w^s),R)}$$

The definition of consumption in each sector is $C_i = w^i - T_i^*(w^i)$, and the resource constraint require that

$$R^*=\int\limits_{\mathcal{N}_d^*} T_d^*(w^d)dF^d(w^d)dF^s(w^s)+\int\limits_{\mathcal{N}_s^*} T_s^*(w^s)dF^d(w^d)dF^s(w^s)$$

Hence, the optimal lump-sum transfer must finance the public good and equalize the marginal social welfare in both sectors for all wages. Finally, from the optimal lump-sum transfer, and the fact that $G' \geq 0$ and $G'' < 0$, agents prefer to work in the more productive sector.

D Proof of Lemma 1

Suppose there exist two agents, A and B. Agent A has a bigger productivity in the dependent sector but the same in the self-employed one in comparison with agent B. This is $w_A^d > w_B^d$ and $w_A^s = w_B^s$. This produces that both agents have the same utility in the self-employed sector. However, agent A has a more considerable utility in the dependent sector than agent B. What has been mentioned before comes from the assumption that marginal tax cannot be higher than one, which produces that $dU^d/dw^d = 1-\tau \geq 0$, or the utility in the dependent sector is increasing in the dependent productivity. Let us assume that the difference in agent B's utilities is that such agent decided to work as a dependent worker. Hence, since agent A has more utility in the dependent sector than agent B, agent A also works in the dependent sector. Therefore, every agent with more significant productivity in the dependent sector, and the same productivity in the self-employed sector, than agent B, makes the same occupational decision.

Now, let us assume that there is also an agent C, who is less productive in the dependent sector and has the same productivity in the self-employed sector than agent B. This is $w_B^d > w_C^d$ and $w_B^s = w_C^s$. Agent C has productivities such that the utility in both sectors is equal, i.e., $U^d(w_C^d) = U^s(w_C^s)$. By Definition 1, agent C works as a self-employed; therefore, every agent with less productivity in the dependent sector and the same in the self-employed one than agent C, makes the same occupational decision than such agent. This is, as all those agents have a strictly bigger utility in the self-employed sector than in the dependent one. Because utility in the dependent sector is increasing in the dependent productivity, and agent C has equal utility in both sectors, every agent with less productivity in the dependent sector has a strictly bigger productivity in the self-employed sector.

The last two arguments show that for the same productivity in the self-employed sector there is a level of productivity in the dependent sector, which equalizes utility in both sectors and determines a switch in the occupational decision. Formally, this switching level can be named as κ, or in the example $\kappa(w^s) = w^d_C$. Hence, every agent with $w^d \leq \kappa(w^s)$, works in the self-employed sector. This result is also accurate for every possible self-employed productivity. Therefore, the threshold function exists and is valid for every potential productivity in the self-employed sector.

E Proof of Lemma 2

To obtain an incentive-compatible mechanism is necessary to analyze two restrictions on a self-employed agent, participation and incentive compatibility. In this case, the participation constraint is the limited liability condition, which refers to the non-negative utility in each sector.

$$\begin{aligned} U^s(w^s) &= w^s - \mathcal{T}(w^s) - \alpha(w^s)\pi\left[\tau w^s - \mathcal{T}(w^s)\right] \geq 0 \\ U^d(w^d) &= w^d - \tau w^d \geq 0 \end{aligned}$$

The incentive-compatible restriction can be replaced by the following

$$V(w^s, \mathcal{M}) = \max_x \{w^s - \mathcal{T}(x) - \alpha(x)\pi(\tau w^s - \mathcal{T}(x))\}$$

By using the envelop theorem, one can derive with respect to w^s the above expression.

$$\frac{\partial V(w^s, \mathcal{M})}{\partial w^s} = 1 - \alpha(x)\pi\tau - \frac{dx}{dw^s}\left[\mathcal{T}'(x) + \alpha'(x)\pi(\tau w^s - \mathcal{T}(x)) - \alpha(x)\pi\mathcal{T}'(x)\right]$$

The term in brackets is zero from optimal income declaration.

$$\frac{\partial V(w^s, \mathcal{M})}{\partial x} = \mathcal{T}'(x) + \alpha'(x)\pi(\tau w^s - \mathcal{T}(x)) - \alpha(x)\pi\mathcal{T}'(x) = 0$$

As truth-telling is seeking, let us impose this condition, i.e., $x = w^s$, in the derivative and obtain

$$\frac{\partial V(w^s, \mathcal{M})}{\partial w^s} = 1 - \alpha(w^s)\pi\tau$$

Now, by using the fundamental theorem of calculus, rewrite the indirect utility for self-employed agents and use their definition to obtain the following equality.

$$\int_{j=\underline{w}^s}^{w^s} (1 - \alpha(j)\pi\tau) + V(\underline{w}^s) = V^s(w^s) = w^s - \mathcal{T}(w^s) - \alpha(w^s)\pi(\tau w^s - \mathcal{T}(w^s))$$

By doing algebra the effective tax formula is obtained

$$\mathcal{T}(w^s) = \frac{w^s(1 - \tau\alpha(w^s)\pi)}{1 - \pi\alpha(w^s)} - \frac{\int_{j=\underline{w}^s}^{w^s} (1 - \alpha(j)\pi\tau)\, dj}{1 - \pi\alpha(w^s)} - \frac{V^s(\underline{w}^s)}{1 - \pi\alpha(w^s)}$$

Now, in order to demonstrate the first statement, Guesnerie and Laffont (1984) are followed. First, obtain the first-order condition for the optimal income declaration from the utility in the self-employed agent.

$$\frac{\partial V(w^s,\mathcal{M})}{\partial x} = -\mathcal{T}'(x) - \alpha'(x)\pi(\tau w^s - \mathcal{T}(x)) + \alpha(x)\pi\mathcal{T}'(x) = 0$$

Second, it can be obtained the second-order condition (SOC), which must be zero or negative from the quasi-concavity of the utility.

$$\frac{\partial^s V(w^s,\mathcal{M})}{\partial x^2} = -\mathcal{T}''(x) - \alpha''(x)\pi(\tau w^s - \mathcal{T}(x)) + 2\alpha'(x)\pi\mathcal{T}'(x) + \alpha(x)\pi\mathcal{T}''(x) \leq 0$$

Third, the FOC can be derived with respect to w^s and assumed truth-telling.

$$\frac{\partial^s V(w^s,\mathcal{M})}{\partial x \partial w^s} = -\alpha'(w^s)\pi\tau + [-\mathcal{T}''(w^s) - \alpha''(w^s)\pi(\tau w^s - \mathcal{T}(w^s)) + 2\alpha'(w^s)\pi\mathcal{T}'(w^s) + \alpha(w^s)\pi\mathcal{T}''(w^s)] = 0$$

The term in brackets is negative or zero from the SOC. This fact implies that the first term must be positive or zero. The marginal tax is positive by assumption; hence, marginal audit must be negative or zero. Therefore, the audit must be non-increasing in the self-employed wage.

F Proof of Lemma 3

The first part of this Lemma is the same as Lemma 1 because the form of the utility and the central assumption does not change. Hence, the existence of the threshold also holds in this case.

The second part comes from deriving the equality $V^s(w^s,\mathcal{M}) = U^d(\kappa(w^s))$ with respect to w^s.

$$\begin{aligned} V^s_{w^s}(w^s,\mathcal{M}) =& U^d_{\kappa(w^s)}(\kappa(w^s))\kappa'(w^s) \\ \kappa'(w^s) =& \frac{V^s_{w^s}(w^s,\mathcal{M})}{U^d_\kappa(\kappa(w^s))} \end{aligned}$$

where the expression X_i refers to the derivative of X with respect to i.

G Proof of Corollary 1

If the audit cost is zero ($c = 0$), the first necessary condition for an optimum depends only on the adjoint function, and it does not rely on the audit cost. From the second necessary condition, the adjoint function is increasing, and from the third necessary condition, it comes from the negative numbers to zero. Those facts imply that the first necessary condition is equal to zero only when the productivity in the self-employed sector takes the upper bound value, i.e., $w^* = \overline{w}^s$. This result shows that the IRS audit all the taxpayers in the self-employed sector, and consequently, all agents in this industry have effective taxes equal to the taxes designed by the government.

H Derivation of the Optimal Government Policies

H.1 Optimal BLS Rule

The optimal BLS rule is obtained from the government's problem deriving the Lagrangian with respect to the public goods. The Lagrange of the government's problem is

$$\mathcal{L} = \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s,\mathcal{M})\right) dF^d(w^d)dF^s(w^s)$$
$$+ \int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s,\mathcal{M})\right) dF^d(w^d)dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s,\mathcal{M})\right) dF^d(w^d)dF^s(w^s) - \delta\Bigg\{B + R - \tau\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s)$$
$$-\tau\int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) - \tau\int_{\underline{w}^s}^{w^*} w^s F^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau w^*\int_{w^*}^{\hat{w}} F^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau w^*\int_{\hat{w}}^{\overline{w}^s} F^d\left(\kappa(w^s)\right) dF^s(w^s)\Bigg\}$$

The first-order condition with respect to R is

$$\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} G'(U^d(w^d))\phi'(R)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} G'\left(U^d(w^d)\right)\phi'(R)dF^d(w^d)dF^s(w^s)$$
$$+\int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} G'\left(V^s(w^s,\mathcal{M})\right)\phi'(R)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)} G'\left(V^s(w^s,\mathcal{M})\right)\phi'(R)dF^d(w^d)dF^s(w^s)$$
$$+\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} G'\left(V^s(w^s,\mathcal{M})\right)\phi'(R)dF^d(w^d)dF^s(w^s) - \delta = 0$$

By doing algebra yields

$$\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} \frac{G'(U^d(w^d))}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} \frac{G'\left(U^d(w^d)\right)}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s)$$
$$+\int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} \frac{G'\left(V^s(w^s,\mathcal{M})\right)}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)} \frac{G'\left(V^s(w^s,\mathcal{M})\right)}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s)$$
$$+\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} \frac{G'\left(V^s(w^s,\mathcal{M})\right)}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s) = 1$$

The derivative of the Lagrangian with respect to each agent's consumption in each sector are

$$G'(U^d(w^d))f^d(w^d)f^s(w^s) - \delta f^d(w^d)f^s(w^s) = 0$$
$$G'\left(V^s(w^s,\mathcal{M})\right)f^d(w^d)f^s(w^s) - \delta f^d(w^d)f^s(w^s) = 0$$

where the definition of consumption, $C(w) = w - \tau w$, is used to replaces taxes for consumption. By using the previous equations to replace the Lagrange multiplier in the FOC of the public good yields

$$\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} \phi'(R)dF^d(w^d)dF^s(w^s)+\int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} \phi'(R)dF^d(w^d)dF^s(w^s)+\int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} \phi'(R)dF^d(w^d)dF^s(w^s)$$
$$+\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)} \phi'(R)dF^d(w^d)dF^s(w^s)+\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} \phi'(R)dF^d(w^d)dF^s(w^s)=1$$

Finally, and in order to obtain the same expression as the first-best, use the fact that the marginal utility of consumption is equal to one. Hence, the marginal rate of substitution is only expressed as $\phi'(R)$. The final expression is

$$\phi'(R)=\int_{\underline{w}^s}^{\overline{w}^s}\int_{\kappa(w^s)}^{\overline{w}^d} MRS(RC)dF^d(w^d)dF^s(w^s)+\int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} MRS(RC)dF^d(w^d)dF^s(w^s)+\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)} MRS(RC)dF^d(w^d)dF^s(w^s)$$
$$+\int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} MRS(RC)dF^d(w^d)dF^s(w^s)=1$$

H.2 Optimal Marginal Tax Rate

The optimal marginal tax rate solves the Lagrangian in the government's problem. The Lagrangian of the government's problem is

$$\mathcal{L}=\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right)dF^d(w^d)dF^s(w^s)+\int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right)dF^d(w^d)dF^s(w^s)+\int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} G(V^s(w^s,\mathcal{M}))dF^d(w^d)dF^s(w^s)$$
$$+\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)} G(V^s(w^s,\mathcal{M}))dF^d(w^d)dF^s(w^s)+\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} G(V^s(w^s,\mathcal{M}))dF^d(w^d)dF^s(w^s)-\delta\left\{B+R-\tau\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s)\right.$$
$$\left.-\tau\int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s)-\tau\int_{\underline{w}^s}^{w^*} w^s F^d\left(\kappa(w^s)\right)dF^s(w^s)-\tau w^*\int_{w^*}^{\hat{w}} F^d\left(\kappa(w^s)\right)dF^s(w^s)-\tau w^*\int_{\hat{w}}^{\overline{w}^s} F^d\left(\kappa(w^s)\right)dF^s(w^s)\right\}$$

The first-order condition with respect to τ is

$$
\begin{aligned}
&\int_{\underline{w}^s}^{w^*} (-\kappa'_\tau) G\left(U^d(\kappa(w^s))\right) f^d(\kappa(w^s)) dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} G'_{U^d} U^d_C \frac{dC_d}{d\tau} dF^d(w^d) dF^s(w^s) + \hat{w}'_\tau \int_{\kappa(\hat{w})}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d) f^s(\hat{w}) \\
&+ \int_{w^*}^{\hat{w}} (-\kappa'_\tau) G\left(U^d(\kappa(w^s))\right) f^d(\kappa(w^s)) dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} G'_{U^d} U^d_C \frac{dC_d}{d\tau} dF^d(w^d) dF^s(w^s) + \int_{\underline{w}^s}^{w^*} \kappa'_\tau G(V^s(w^s,\mathcal{M})) f^d(\kappa(w^s)) dF^s(w^s) \\
&+ \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} G'_{V^s} V^s_C \frac{dC_s}{d\tau} dF^d(w^d) dF^s(w^s) + \hat{w}'_\tau \int_{\underline{w}^d}^{\kappa(\hat{w})} G(V^s(\hat{w},\mathcal{M})) dF^d(w^d) f^s(\hat{w}) + \int_{w^*}^{\hat{w}} \kappa'_\tau G(V^s(w^s,\mathcal{M})) f^d(\kappa(w^s)) dF^s(w^s) \\
&+ \int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)} G'_{V^s} V^s_C \frac{dC_s}{d\tau} dF^d(w^d) dF^s(w^s) + (-\hat{w}'_\tau) \int_{\underline{w}^d}^{\kappa(\hat{w})} G(V^s(\hat{w},\mathcal{M})) dF^d(w^d) f^s(\hat{w}) + \int_{\hat{w}}^{\overline{w}^s} \kappa'_\tau G(V^s(w^s,\mathcal{M})) f^d(\kappa(w^s)) dF^s(w^s) \\
&+ \int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} G'_{V^s} V^s_C \frac{dC_s}{d\tau} dF^d(w^d) dF^s(w^s) - \delta \Bigg\{ -\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) - \tau \int_{\underline{w}^s}^{w^*} (-\kappa'_\tau)\kappa(w^s) f^d(\kappa(w^s)) dF^s(w^s) \\
&- \int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) - \tau \hat{w}'_\tau \int_{\kappa(\hat{w})}^{\overline{w}^d} w^d dF^d(w^d) f^s(\hat{w}) - \tau \int_{w^*}^{\hat{w}} (-\kappa'_\tau)\kappa(w^s) f^d(\kappa(w^s)) dF^s(w^s) - \int_{\underline{w}^s}^{w^*} w^s F^d(\kappa(w^s)) dF^s(w^s) \\
&- \tau \int_{\underline{w}^s}^{w^*} w^s \kappa'_\tau f^d(\kappa(w^s)) dF^s(w^s) - \int_{w^*}^{\hat{w}} w^* F^d(\kappa(w^s)) dF^s(w^s) - \tau \hat{w}'_\tau w^* F^d(\kappa(w^s)) dF^s(w^s) - \tau \int_{w^*}^{\hat{w}} w^* \kappa'_\tau f^d(\kappa(w^s)) dF^s(w^s) \\
&- \int_{\hat{w}}^{\overline{w}^s} w^* F^d(\kappa(w^s)) dF^s(w^s) - \tau(-\hat{w}'_\tau) w^* F^d(\kappa(w^s)) dF^s(w^s) - \tau \int_{\hat{w}}^{\overline{w}^s} w^* \kappa'_\tau f^d(\kappa(w^s)) dF^s(w^s) \Bigg\} = 0
\end{aligned}
$$

To simplify the previous expression, use several results and assumptions. First, $g = (G_{V^i} \times V^i_C)/\delta$ is defined as the social valuation of agent's consumption. Second, it is necessary to recall that $U^d(\kappa(w^s)) = V^s(w^s, \mathcal{M})$, implying that some terms are canceling among them. Third, by assumption $\kappa(\hat{w}) = \overline{w}^d$, hence, some integrals collapse to zero. Fourth, as a result of the linearity in tax and the audit schedule, $\kappa'_\tau = 0$ for $w^s \leq w^*$ and for $w^s \geq \hat{w}$, resulting in some term to be zero. Finally, the derivatives of consumption respect to marginal tax in each sector are.

$$
\frac{dC_d}{d\tau} = -w^d, \qquad \frac{dC_s}{d\tau} = \begin{cases} -w^s & \text{if } w^s < w^* \\ -w^* & \text{if } w^* \leq w^s \end{cases}
$$

By using those facts and doing algebra yields

$$\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} g(w^d)(-w^d)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} g(w^d)(-w^d)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} g(w^s)(-w^s)dF^d(w^d)dF^s(w^s)$$
$$+\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)} g(w^s)(-w^*)dF^d(w^d)dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} g(w^s)(-w^*)dF^d(w^d)dF^s(w^s) = -\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s)$$
$$-\int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) + \tau\int_{w^*}^{\hat{w}} \kappa_\tau' \kappa(w^s) f^d(\kappa(w^s))dF^s(w^s) - \int_{\underline{w}^s}^{w^*} w^s F^d\left(\kappa(w^s)\right) dF^s(w^s)$$
$$-\int_{w^*}^{\hat{w}} w^* F^d\left(\kappa(w^s)\right) dF^s(w^s) - \tau\int_{w^*}^{\hat{w}} w^*\kappa_\tau' f^d\left(\kappa(w^s)\right) dF^s(w^s) - \int_{\hat{w}}^{\overline{w}^s} w^* F^d\left(\kappa(w^s)\right) dF^s(w^s)$$

Let us note that, $\kappa(w^s) - w^* = \dfrac{w^s - w^*}{1-\tau}$. By replacing it in the above equation and doing algebra, the optimal marginal tax formula is

$$\frac{\tau}{1-\tau}\int_{w^*}^{\hat{w}}(w^s - w^*)\kappa_\tau(w^s) f^d(\kappa(w^s))dF^s(w^s) = \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} (1-g(w^d))w^d dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} (1-g(w^d))w^d dF^d(w^d)dF^s(w^s)$$
$$+\int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} (1-g(w^s))w^s dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)} (1-g(w^s))w^* dF^d(w^d)dF^s(w^s)$$
$$+\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} (1-g(w^s))w^* dF^d(w^d)dF^s(w^s)$$

H.3 Optimal Budget for the IRS

The Lagrangian of the government's problem is

$$\mathcal{L} = \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} G(V^s(w^s,\mathcal{M}))dF^d(w^d)dF^s(w^s)$$
$$+\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)} G(V^s(w^s,\mathcal{M}))dF^d(w^d)dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} G(V^s(w^s,\mathcal{M}))dF^d(w^d)dF^s(w^s) - \delta\left\{B + R - \tau\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s)\right.$$
$$\left.-\tau\int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) - \tau\int_{\underline{w}^s}^{w^*} w^s F^d\left(\kappa(w^s)\right)dF^s(w^s) - \tau w^*\int_{w^*}^{\hat{w}} F^d\left(\kappa(w^s)\right)dF^s(w^s) - \tau w^*\int_{\hat{w}}^{\overline{w}^s} F^d\left(\kappa(w^s)\right)dF^s(w^s)\right\}$$

The first-order condition with respect to B is

$$w_B^* \int_{\kappa(w^*)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d) f^s(w^*) + \int_{\underline{w}^s}^{w^*} (\kappa_B') G\left(U^d(\kappa(w^s))\right) f^d(\kappa(w^s)) dF^s(w^s) + (-w_B^*) \int_{\kappa(w^*)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d) f^s(w^*)$$

$$+ \hat{w}_B' \int_{\kappa(w^s)}^{\overline{w}^d} G\left(U^d(w^d)\right) dF^d(w^d) dF^s(w^s) + \int_{w^*}^{\hat{w}} (-\kappa_B') G\left(U^d(\kappa(w^s))\right) f^d(\kappa(w^s)) dF^s(w^s) + w_B^* \int_{\underline{w}^d}^{\kappa(w^*)} G(V^s(w^*, \mathcal{M})) dF^d(w^d) f^s(w^*)$$

$$+ \int_{\underline{w}^s}^{w^*} \kappa_B' G(V^s(w^s, \mathcal{M})) f^d(\kappa(w^s)) dF^s(w^s) + (-w_B^*) \int_{\underline{w}^d}^{\kappa(w^*)} G(V^s(w^*, \mathcal{M})) dF^d(w^d) f^s(w^*) + \hat{w}_B' \int_{\underline{w}^d}^{\kappa(\hat{w})} G(V^s(\hat{w}, \mathcal{M})) dF^d(w^d) f^s(\hat{w})$$

$$+ \int_{w^*}^{\hat{w}} \kappa_B' G(V^s(w^s, \mathcal{M})) f^d(\kappa(w^s)) dF^s(w^s) + \int_{w^*}^{\hat{w}} \int_{\underline{w}^d}^{\kappa(w^s)} G_{V^s}' V_C^s \frac{dC_s}{dB} dF^d(w^d) dF^s(w^s) + (-\hat{w}_B') \int_{\underline{w}^d}^{\kappa(\hat{w})} G(V^s(\hat{w}, \mathcal{M})) dF^d(w^d) f^s(\hat{w})$$

$$+ \int_{\hat{w}}^{\overline{w}^s} \kappa_B' G(V^s(w^s, \mathcal{M})) f^d(\kappa(w^s)) dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} G_{V^s}' V_C^s \frac{dC_s}{dB} dF^d(w^d) dF^s(w^s) - \delta \Bigg\{ 1 - \tau w_B^* \int_{\kappa(w^*)}^{\overline{w}^d} w^d dF^d(w^d) f^s(w^*)$$

$$- \tau \int_{\underline{w}^s}^{w^*} (-\kappa_B') \kappa(w^s) f^d(\kappa(w^s)) dF^s(w^s) - \tau(-w_B^*) \int_{\kappa(w^*)}^{\overline{w}^d} w^d dF^d(w^d) f^s(w^*) - \tau \hat{w}_B' \int_{\kappa(\hat{w})}^{\overline{w}^d} w^d dF^d(w^d) f^s(\hat{w})$$

$$- \tau \int_{w^*}^{\hat{w}} (-\kappa_B') \kappa(w^s) f^d(\kappa(w^s)) dF^s(w^s) - \tau w_B^* w^* F^d(\kappa(w^*)) f^s(w^s) - \tau \int_{\underline{w}^s}^{w^*} w^s \kappa_B' f^d(\kappa(w^s)) dF^s(w^s) - \tau(-w_B^*) w^* F^d(\kappa(w^*)) f^s(w^*)$$

$$- \tau \hat{w}_B' w^* F^d(\kappa(\hat{w})) f^s(\hat{w}) - \tau \kappa_B' w^* f^d(\kappa(w^s)) dF^s(w^s) - \tau \int_{w^*}^{\hat{w}} \frac{dw^*}{db} F^d(\kappa(w^s)) dF^s(w^s) - \tau(-\hat{w}_B') w^* F^d(\kappa(\hat{w})) f^s(\hat{w})$$

$$- \tau \int_{\hat{w}}^{\overline{w}^s} w^* \kappa_B' f^d(\kappa(w^s)) dF^s(w^s) - \tau \int_{\hat{w}}^{\overline{w}^s} \frac{dw^*}{dB} F^d(\kappa(w^s)) dF^s(w^s) \Bigg\}$$

Use some results and assumptions to simplify the above expression. First, the social valuation of the agent's consumption can be defined as $g = (G_V' \times V_C)/\delta$. Second, by definition $U^d(\kappa(w^s)) = V^s(w^s, \mathcal{M})$, makes some terms cancel among them. Third, from the result in the IRS's problem, $\kappa_B' = 0$ where $w^s \leq w^*$ and $w^s > \hat{w}$, result in some terms to be equal to zero. Fourth, by definition $\kappa(\hat{w}) = w^d$ collapsing some terms to zero. Finally, the derivative of consumption in the self-employed sector with respect to the budget is

$$\frac{dC_s}{dB} = \begin{cases} 0 & \text{if } w^s < w^* \\ -\tau \dfrac{dw^*}{dB} & \text{if } w^* \leq w^s \end{cases}$$

Let us recall that, in the dependent sector, an increase in the budget for the IRS does not affect taxes. By using those facts and doing algebra yields

$$\tau\int_{w^*}^{\hat{w}}(w^*-\kappa(w^s))\frac{dw^*}{dB}\kappa'_B f^d(\kappa(w^s))dF^s(w^s)+\tau\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s)+\tau\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s)$$
$$=1+\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}g(w^s)\tau\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s)+\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}g(w^s)\tau\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s)$$

I Differential Taxation

Let us define t_s and t_d as the tax rate in the self-employed and the dependent sector, respectively. As it has been explained before, the solution of the optimal audit maintains because differential taxation does not alter the problem. Now, the Lagrangian of the government's problem is

$$\begin{aligned}\mathcal{L}=&\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d}G\Big(U^d(w^d)\Big)dF^d(w^d)dF^s(w^s)+\int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d}G\Big(U^d(w^d)\Big)dF^d(w^d)dF^s(w^s)+\int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)}G(V^s(w^s,\mathcal{M}))dF^d(w^d)dF^s(w^s)\\&+\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}G(V^s(w^s,\mathcal{M}))dF^d(w^d)dF^s(w^s)+\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}G(V^s(w^s,\mathcal{M}))dF^d(w^d)dF^s(w^s)-\delta\Bigg\{B+R-t_d\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d}w^d dF^d(w^d)dF^s(w^s)\\&-t_d\int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d}w^d dF^d(w^d)dF^s(w^s)-t_s\int_{\underline{w}^s}^{w^*}w^s F^d(\kappa(w^s))dF^s(w^s)-t_s w^*\int_{w^*}^{\hat{w}}F^d(\kappa(w^s))dF^s(w^s)-t_s w^*\int_{\hat{w}}^{\overline{w}^s}F^d(\kappa(w^s))dF^s(w^s)\Bigg\}\end{aligned}$$

For simplicity, the same notation in the Lagrange multiplier for the budget constraint is maintained. At this point, it is of great importance to recall that, the threshold function changes, and takes the following form

$$\kappa(w^s)=\begin{cases}\dfrac{w^s(1-t_s)}{1-t_d} & \text{if } w^s<w^*\\ \dfrac{w^s-t_s w^*}{1-t_d} & \text{if } w^*\le w^s<\hat{w}\\ \overline{w}^d & \text{if } \hat{w}\le w^s\end{cases}$$

As it has been explained before, the public good provision is the same as in the first-best, because the differential taxation does not alter the marginal rate of substitution between the public goods and consumption. For this reason, the government only chooses both tax rates and the budget for the IRS. First, the optimal tax rate in the dependent sector is obtained. The first-order condition regarding t_d is

$$
\begin{aligned}
& w^{*\prime}_{t_d} \int_{\kappa(w^*)}^{\overline{w}^d} G\big(U^d(w^d)\big)\, dF^d(w^d) f^s(w^*) + \int_{\underline{w}^s}^{w^*} (-\kappa'_{t_d}) G\big(U^d(\kappa(w^s))\big) f^d(\kappa(w^s)) dF^s(w^s) + \int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\overline{w}^d} G'_{U^d} U^d_C \frac{dC_d}{dt_d} dF^d(w^d) dF^s(w^s) \\
& + (-w^{*\prime}_{t_d}) \int_{\kappa(w^*)}^{\overline{w}^d} G\big(U^d(w^d)\big) dF^d(w^d) f^s(w^*) + \hat{w}'_{t_d} \int_{\kappa(\hat{w})}^{\overline{w}^d} G\big(U^d(w^d)\big) dF^d(w^d) f^s(\hat{w}) + \int_{w^*}^{\hat{w}} (-\kappa'_{t_d}) G\big(U^d(\kappa(w^s))\big) f^d(\kappa(w^s)) dF^s(w^s) \\
& + \int_{w^*}^{\hat{w}} \int_{\kappa(w^s)}^{\overline{w}^d} G'_{U^d} U^d_C \frac{dC_d}{dt_d} dF^d(w^d) dF^s(w^s) + w^{*\prime}_{t_d} \int_{\underline{w}^d}^{\kappa(w^*)} G(V^s(w^*, \mathcal{M}))\, dF^d(w^d) f^s(w^*) + \int_{\underline{w}^s}^{w^*} \kappa'_{t_d} G(V^s(w^s, \mathcal{M}))\, f^d(\kappa(w^s)) dF^s(w^s) \\
& + \int_{\underline{w}^s}^{w^*} \int_{\underline{w}^d}^{\kappa(w^s)} G'_{V^s} V^s_C \frac{dC_s}{dt_d} dF^d(w^d) dF^s(w^s) + (-w^{*\prime}_{t_d}) \int_{\underline{w}^d}^{\kappa(w^*)} G(V^s(w^*, \mathcal{M}))\, dF^d(w^d) f^s(w^*) + \hat{w}'_{t_d} \int_{\underline{w}^d}^{\kappa(\hat{w})} G(V^s(\hat{w}, \mathcal{M}))\, dF^d(w^d) f^s(\hat{w}) \\
& + \int_{w^*}^{\hat{w}} \kappa'_{t_d} G(V^s(w^s, \mathcal{M}))\, f^d(\kappa(w^s)) dF^s(w^s) + \int_{w^*}^{\hat{w}} \int_{\underline{w}^d}^{\kappa(w^s)} G'_{V^s} V^s_C \frac{dC_s}{dt_d} dF^d(w^d) dF^s(w^s) + (-\hat{w}'_{t_d}) \int_{\underline{w}^d}^{\kappa(\hat{w})} G(V^s(\hat{w}, \mathcal{M}))\, dF^d(w^d) f^s(\hat{w}) \\
& + \int_{\hat{w}}^{\overline{w}^s} \kappa'_{t_d} G(V^s(w^s, \mathcal{M}))\, f^d(\kappa(w^s)) dF^s(w^s) + \int_{w^*}^{\hat{w}} \int_{\underline{w}^d}^{\kappa(w^s)} G'_{V^s} V^s_C \frac{dC_s}{dt_d} dF^d(w^d) dF^s(w^s) - \delta \Bigg\{ - \int_{\hat{w}}^{\overline{w}^s} \int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) \\
& - t_d w^{*\prime}_{t_d} \int_{\kappa(w^*)}^{\overline{w}^d} w^d dF^d(w^d) f^s(w^*) - t_d \int_{\underline{w}^s}^{w^*} (-\kappa'_{t_d}) \kappa(w^s) f^d(\kappa(w^s)) dF^s(w^s) - \int_{w^*}^{\hat{w}} \int_{\kappa(w^s)}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) \\
& - t_d(-w^{*\prime}_{t_d}) \int_{\kappa(w^*)}^{\overline{w}^d} w^d dF^d(w^d) f^s(w^*) - t_d(-\hat{w}'_{t_d}) \int_{\kappa(\hat{w})}^{\overline{w}^d} w^d dF^d(w^d) f^s(\hat{w}) - t_d \int_{w^*}^{\hat{w}} (-\kappa'_{t_d}) \kappa(w^s) f^d(\kappa(w^s)) dF^s(w^s) \\
& - t_s(w^{*\prime}_{t_d}) w^* F^d(\kappa(w^*)) f^s(w^*) - t_s \int_{\underline{w}^s}^{w^*} w^s \kappa'_{t_d} f^d\,(\kappa(w^s))\, dF^s(w^s) - t_s w^{*\prime}_{t_s} \int_{w^*}^{\hat{w}} F^d(\kappa(w^s)) dF^s(w^s) - t_s w^*(-w^{*\prime}_{t_d}) F^d(\kappa(w^*)) f^s(w^*) \\
& - t_s w^* \hat{w}'_{t_d} F^d\,(\kappa(\hat{w}))\, dF^s(w^s) - t_s w^* \int_{w^*}^{\hat{w}} \kappa'_{t_d} f^d\,(\kappa(w^s))\, dF^s(w^s) - t_s(w^{*\prime}_{t_d}) \int_{\hat{w}}^{\overline{w}^s} F^d(\kappa(w^s)) dF^s(w^s) - t_s w^*(-\hat{w}'_{t_d}) F^d\,(\kappa(\hat{w}))\, dF^s(w^s) \\
& - t_s w^* \int_{\hat{w}}^{\overline{w}^s} \kappa'_{t_d} f^d\,(\kappa(w^s))\, dF^s(w^s) \Bigg\} = 0
\end{aligned}
$$

To simplify the above expression, use some assumptions and results. First, the social value of the agent's consumption is defined as $g = (G' \times U^d_C)/\delta$. Second, by definition $U^d(\kappa(w^s)) = V^s(w^s, \mathcal{M})$ makes some terms cancel among them. Third, by definition $\kappa(\hat{w}) = \overline{w}^d$, collapsing some terms to zero. Fourth, from the result in the IRS's problem, $\kappa'_{t_d} = \kappa(w^s)/(1 - t_d)$ for $w^s \leq w^*$, $\kappa'_{t_d} = -\dfrac{t_s w^{*\prime}_{t_d}}{1 - t_d} + \kappa(w^s)/(1 - t_d)$ for $w^* < w^s \leq \hat{w}$, and $\kappa'_{t_d} = 0$ for $w^s > \hat{w}$. Fifth, $dC_d/dt_d = -w^d$. Sixth, $dC_s/dt_d = 0$ for $w^s \leq w^*$, and $dC_s/dt_d = -\dfrac{dw^*}{dt_d} t_s$ for $w^* < w^s \leq \hat{w}$. By using those facts and doing algebra yields

$$t_d = \frac{\overbrace{\left(\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d}(1-g(w^d))w^d dF^d(w^d)dF^s(w^s)+\int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d}(1-g(w^d))w^d dF^d(w^d)dF^s(w^s)\right)}^{A}(1-t_d)}{\underbrace{\int_{\underline{w}^s}^{w^*}\kappa(w^s)^2 f^d(\kappa(w^s))dF^s(w^s)+\int_{w^*}^{\hat{w}}\left(\kappa(w^s)-t_s w^{*\prime}_{t_d}\right)\kappa(w^s)f^d(\kappa(w^s))dF^s(w^s)}_{B}}$$

$$+\frac{\overbrace{w^{*\prime}_{t_d}\left(\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}(1-g(w^s))dF^d(w^d)dF^s(w^s)+\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}(1-g(w^s))dF^d(w^d)dF^s(w^s)\right)}^{D}(1-t_d)}{\underbrace{\int_{\underline{w}^s}^{w^*}\kappa(w^s)^2 f^d(\kappa(w^s))dF^s(w^s)+\int_{w^*}^{\hat{w}}\left(\kappa(w^s)-t_s w^{*\prime}_{t_d}\right)\kappa(w^s)f^d(\kappa(w^s))dF^s(w^s)}_{B}}$$

$$+\frac{\overbrace{t_s\left(\int_{\underline{w}^s}^{w^*}w^s\kappa(w^s)f^d(\kappa(w^s))dF^s(w^s)+\int_{w^*}^{\hat{w}}w^*\left(\kappa(w^s)-t_s w^{*\prime}_{t_d}\right)f^d(\kappa(w^s))dF^s(w^s)\right)}^{C}}{\underbrace{\int_{\underline{w}^s}^{w^*}\kappa(w^s)^2 f^d(\kappa(w^s))dF^s(w^s)+\int_{w^*}^{\hat{w}}\left(\kappa(w^s)-t_s w^{*\prime}_{t_d}\right)\kappa(w^s)f^d(\kappa(w^s))dF^s(w^s)}_{B}}$$

To simplify the exposure of the result, some parts with letters can be denoted. The optimal tax rate is

$$t_d = \frac{A + t_s D}{A + B + t_s D} + t_s \frac{C}{A + B + t_s D}$$

In order to fulfill the limited liability constraint the marginal tax rate in the dependent sector cannot be higher than one, thus, it is necessary that $t_s \leq B/C$. In in other words, to obtain a marginal tax rate in the dependent sector not bigger than one, the marginal tax rate in the self-employed sector cannot be higher than the ratio B/C.

Now, the tax rate in the self-employed sector will be obtained. The first-order condition with respect to

t_s is

$$w^{*\prime}_{t_s}\int_{\kappa(w^*)}^{\overline{w}^d} G\Big(U^d(w^d)\Big)dF^d(w^d)f^s(w^*)+\int_{\underline{w}^s}^{w^*}(-\kappa'_{t_s})G\Big(U^d(\kappa(w^s))\Big)f^d(\kappa(w^s))dF^s(w^s)+(-w^{*\prime}_{t_s})\int_{\kappa(w^*)}^{\overline{w}^d} G\Big(U^d(w^d)\Big)dF^d(w^d)f^s(w^*)$$

$$+\hat{w}'_{t_s}\int_{\kappa(\hat{w})}^{\overline{w}^d} G\Big(U^d(w^d)\Big)dF^d(w^d)f^s(\hat{w})+\int_{w^*}^{\hat{w}}(-\kappa'_{t_s})G\Big(U^d(\kappa(w^s))\Big)f^d(\kappa(w^s))dF^s(w^s)+w^{*\prime}_{t_s}\int_{\underline{w}^d}^{\kappa(w^*)} G(V^s(w^*,\mathcal{M}))dF^d(w^d)f^s(w^s)$$

$$+\int_{\underline{w}^s}^{w^*}\kappa'_{t_s}G(V^s(w^s,\mathcal{M}))f^d(\kappa(w^s))dF^s(w^s)+\int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} G'_{V^s}V^s_C\frac{dC_s}{dt_s}dF^d(w^d)dF^s(w^s)+(-w^{*\prime}_{t_s})\int_{\underline{w}^d}^{\kappa(w^*)} G(V^s(w^*,\mathcal{M}))dF^d(w^d)f^s(w^s)$$

$$+\hat{w}'_{t_s}\int_{\underline{w}^d}^{\kappa(\hat{w})} G(V^s(\hat{w},\mathcal{M}))dF^d(w^d)f^s(\hat{w})+\int_{w^*}^{\hat{w}}\kappa'_{t_s}G(V^s(w^s,\mathcal{M}))f^d(\kappa(w^s))dF^s(w^s)+\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)} G'_{V^s}V^s_C\frac{dC_s}{dt_s}dF^d(w^d)dF^s(w^s)$$

$$+(-\hat{w}'_{t_s})\int_{\underline{w}^d}^{\kappa(\hat{w})} G(V^s(\hat{w},\mathcal{M}))dF^d(w^d)f^s(\hat{w})+\int_{\hat{w}}^{\overline{w}^s}\kappa'_{t_s}G(V^s(w^s,\mathcal{M}))f^d(\kappa(w^s))dF^s(w^s)+\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} G'_{V^s}V^s_C\frac{dC_s}{dt_s}dF^d(w^d)dF^s(w^s)$$

$$-\delta\Bigg\{-t_d w^{*\prime}_{t_s}\int_{\kappa(w^*)}^{\overline{w}^d} w^d dF^d f^s(w^*)-t_d\int_{\underline{w}^s}^{w^*}(-\kappa'_{t_s})\kappa(w^s)f^d(\kappa(w^s))dF^s(w^s)-t_d(-w^{*\prime}_{t_s})\int_{\kappa(w^*)}^{\overline{w}^d} w^d dF^d f^s(w^*)$$

$$-t_d\hat{w}'_{t_s}\int_{\kappa(\hat{w})}^{\overline{w}^d} w^d dF^d(w^d)f^s(\hat{w})-t_d\int_{w^*}^{\hat{w}}\kappa'_{t_s}\kappa(w^s)f^d(\kappa(w^s))dF^s(w^s)-\int_{\underline{w}^s}^{w^*} w^s F^d(\kappa(w^s))dF^s(w^s)-t_s w^{*\prime}_{t_s}w^*F^d(\kappa(w^*))f^s(w^*)$$

$$-t_s\int_{\underline{w}^s}^{w^*} w^s\kappa'_{t_s}f^d(\kappa(w^s))dF^s(w^s)-w^*\int_{w^*}^{\hat{w}} F^d(\kappa(w^s))dF^s(w^s)-t_s w^{*\prime}_{t_s}\int_{w^*}^{\hat{w}} F^d(\kappa(w^s))dF^s(w^s)-t_s w^*(-w^{*\prime}_{t_s})F^d(\kappa(w^*))f^s(w^*)$$

$$-t_s w^*\hat{w}'_{t_s}F^d(\kappa(\hat{w}))dF^s(w^s)-t_s w^*\int_{w^*}^{\hat{w}}\kappa'_{t_s}f^d(\kappa(w^s))dF^s(w^s)-w^*\int_{\hat{w}}^{\overline{w}^s} F^d(\kappa(w^s))dF^s(w^s)-t_s w^{*\prime}_{t_s}\int_{\hat{w}}^{\overline{w}^s} F^d(\kappa(w^s))dF^s(w^s)$$

$$-t_s w^*(-\hat{w}'_{t_s})F^d(\kappa(\hat{w}))dF^s(w^s)-t_s w^*\int_{\hat{w}}^{\overline{w}^s}\kappa'_{t_s}f^d(\kappa(w^s))dF^s(w^s)\Bigg\}=0$$

To simplify the expression, use some assumptions and results. First, one can be defined the social value of the agent's consumption as$g = (G' \times V^s_C)/\delta$. Second, by definition $U^d(\kappa(w^s)) = V^s(w^s,\mathcal{M})$ makes some terms cancel among them. Third, by definition $\kappa(\hat{w}) = \overline{w}^d$, collapsing some terms to zero. Fourth, from the result in the IRS's problem, $\kappa'_{t_s} = (-w^s)/(1-t_d)$ for $w^s \le w^*$, $\kappa'_{t_s} = (-w^* - t_s w^{*\prime}_{t_s})/(1-t_d)$ for $w^s \in (w^*, \hat{w})$, and $\kappa'_{t_s} = 0$ for $w^s > \hat{w}$. Fifth, $dC_s/dt_s = -w^s$ for $w^s < w^*$ and $dC_s/dt_s = -w^* - t_s w^{*\prime}_{t_s}$ for $w^s \ge w^*$. By using those facts and doing algebra yields

$$t_s = \frac{\overbrace{\left[\int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)}(1-g(w^s))w^s dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}(1-g(w^s))(w^* + t_s w^{*\prime}_{t_s})dF^d(w^d)dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}(1-g(w^s))(w^* + t_s w^{*\prime}_{t_s})dF^d(w^d)dF^s(w^s)\right]}^{E}(1-t_d)}{\underbrace{\int_{\underline{w}^s}^{w^*}(w^s)^2 f^d(\kappa(w^s))dF^s(w^s) + \int_{w^*}^{\hat{w}} w^*(w^* + t_s w^{*\prime}_{t_s})f^d(\kappa(w^s))dF^s(w^s)}_{F}}$$

$$+ \frac{t_d \overbrace{\left(\int_{\underline{w}^s}^{w^*}\kappa(w^s)w^s f^d(\kappa(w^s))dF^s(w^s) + \int_{w^*}^{\hat{w}}\kappa(w^s)(w^* + t_s w^{*\prime}_{t_s})f^d(\kappa(w^s))dF^s(w^s)\right)}^{C}}{\underbrace{\int_{\underline{w}^s}^{w^*}(w^s)^2 f^d(\kappa(w^s))dF^s(w^s) + \int_{w^*}^{\hat{w}} w^*(w^* + t_s w^{*\prime}_{t_s})f^d(\kappa(w^s))dF^s(w^s)}_{F}}$$

Again, some parts with letters can be denoted to simplify the exposure of the result. Note that the term C is equal to one in the derivation of t_d. Rearranging terms

$$t_s = \frac{E}{F} - t_d \frac{(E-C)}{F}$$

In this case, $t_s \leq 1$ is needed to fulfill the requirement for a direct incentive-compatible mechanism. Applying this condition to the above equation yields

$$\frac{E-F}{E-C} \leq t_d \leq 1 \Rightarrow F \geq C$$

Let us analyze if each possible combination of the marginal tax rates hold the condition for an optimum marginal tax rate obtained above. First, if $t_s < t_d$, $\kappa(w^s) > w^s$ for $w^s \leq \hat{w}$, hence $C > F$ producing a contradiction with the condition to obtain a solution, therefore, it is not a solution. Second, if $t_s = t_d$, $\kappa(w^s) = w^s$ for $w^s \leq w^*$ and $\kappa(w^s) > w^s$ for $w^* \leq w^s \leq \hat{w}$, resulting in $C > F$. Therefore, $t_s = t_d$ cannot be a solution for differential taxation either. Third, if $t_s > t_d$, $\kappa(w^s) < w^s$ for $w^s \leq w^*$ and $\kappa(w^s) \lesseqgtr w^s$ for $w^* \leq w^s \leq \hat{w}$, producing that for some cases $F \geq C$. Hence, the only possible solution for differential taxation is $t_s > t_d$.

J Comparative Statics

J.1 Taxes

For simplicity, the tax rate formula can be rewritten as $t = {}^{A}\!/\!_{(A+X)}$, where A takes the value of

$$
\begin{aligned}
A = & \int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\overline{w}^d} (1-g(w^d))w^d dF^d(w^d) dF^s(w^s) + \int_{w^*}^{\hat{w}} \int_{\kappa(w^s)}^{\overline{w}^d} (1-g(w^d))w^d dF^d(w^d) dF^s(w^s) \\
& + \int_{\underline{w}^s}^{w^*} \int_{\underline{w}^d}^{\kappa(w^s)} (1-g(w^s))w^s dF^d(w^d) dF^s(w^s) + \int_{w^*}^{\hat{w}} \int_{\underline{w}^d}^{\kappa(w^s)} (1-g(w^s))w^* dF^d(w^d) dF^s(w^s) \\
& + \int_{\hat{w}}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} (1-g(w^s))w^* dF^d(w^d) dF^s(w^s)
\end{aligned}
$$

and X the value of

$$
X = \int_{w^*}^{\hat{w}} (w^s - w^*)\kappa_\tau(w^s) f^d(\kappa(w^s)) dF^s(w^s)
$$

With this notation, the partial derivative of the tax rate with respect to the audit cost, c, which is

$$
\frac{d\tau}{dc} = \frac{\frac{dA}{dc}X - \frac{dX}{dc}A}{(A+X)^2}
$$

This formula simplifies how the overall effect is obtained. First, one can obtain the characterization of the partial derivative of the term A with respect to c

$$
\begin{aligned}
\frac{dA}{dc} = & \frac{dw^*}{dc} \int_{\kappa(w^*)}^{\overline{w}^d} (1-g(w^d))w^d dF^d(w^d) f^s(w^*) + \int_{\underline{w}^s}^{w^*} (-\kappa_c')(1-g(\kappa(w^s)))\kappa(w^s) f^d(\kappa(w^s)) dF^s(w^s) + \int_{\underline{w}^s}^{w^*} \int_{\kappa(w^s)}^{\overline{w}^d} (-g' U_c^d) w^d dF^d(w^d) dF^s(w^s) \\
& + (-\frac{dw^*}{dc}) \int_{\kappa(w^*)}^{\overline{w}^d} (1-g(w^d))w^d dF^d(w^d) f^s(w^*) + \hat{w}_c' \int_{\kappa(\hat{w})}^{\overline{w}^d} (1-g(w^d))w^d dF^d(w^d) f^s(\hat{w}) + \int_{w^*}^{\hat{w}} (-\kappa_c')(1-g(\kappa(w^s)))\kappa(w^s) f^d(\kappa(w^s)) dF^s(w^s) \\
& + \int_{w^*}^{\hat{w}} \int_{\kappa(w^s)}^{\overline{w}^d} (-g_c' U_c^d) w^d dF^d(w^d) dF^s(w^s) + \frac{dw^*}{dc} \int_{\underline{w}^d}^{\kappa(w^*)} (1-g(w^*))w^* dF^d(w^d) f^s(w^*) + \int_{\underline{w}^s}^{w^*} \kappa_c'(1-g(w^s))w^s f^d(\kappa(w^s)) dF^s(w^s) \\
& + \int_{\underline{w}^s}^{w^*} \int_{\underline{w}^d}^{\kappa(w^s)} (-g' V_c^s) w^s dF^d(w^d) dF^s(w^s) + (-\frac{dw^*}{dc}) \int_{\underline{w}^d}^{\kappa(w^*)} (1-g(w^*))w^* dF^d(w^d) f^s(w^*) + \hat{w}_c' \int_{\underline{w}^d}^{\kappa(\hat{w})} (1-g(\hat{w}))w^* dF^d(w^d) f^s(\hat{w}) \\
& + \int_{w^*}^{\hat{w}} \kappa_c'(1-g(w^s))w^* f^d(\kappa(w^s)) dF^s(w^s) + \int_{w^*}^{\hat{w}} \int_{\underline{w}^d}^{\kappa(w^s)} \left[-g' V_c^s w^* + (1-g(w^s)) \frac{dw^*}{dc} \right] dF^d(w^d) dF^s(w^s) + (-\hat{w}_c') \int_{\underline{w}^d}^{\kappa(\hat{w})} (1-g(\hat{w}))w^* dF^d(w^d) f^s(\hat{w}) \\
& + \int_{\hat{w}}^{\overline{w}^s} \kappa_c'(1-g(w^s))w^* f^d(\kappa(w^s)) dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} \left[-g' V_c^s w^* + (1-g(w^s)) \frac{dw^*}{dc} \right] dF^d(w^d) dF^s(w^s)
\end{aligned}
$$

It is possible to simplify the above expression using the following facts. First, by definition $\kappa(\hat{w}) = \overline{w}^d$, this produces that some terms are equal to zero. Second, by definition $\kappa_c' = 0$ for $w^s > \hat{w}$, and for $w^s < w^*$, producing that some terms are equal to zero. Finally, some terms are canceled among them because they are the same but have the opposite sign. The simplified expression is

$$\begin{aligned}\frac{dA}{dc} =& \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\overline{w}^d} (-g'U_c^d) w^d dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\hat{w}}\int_{\kappa(w^s)}^{\overline{w}^d} (-g_c'U_c^d) w^d dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} (-g'V_c^s) w^s dF^d(w^d)dF^s(w^s) \\ &+ \int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)} \left[-g'V_c^s w^* + (1-g(w^s))\frac{dw^*}{dc}\right] dF^d(w^d)dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} \left[-g'V_c^s w^* + (1-g(w^s))\frac{dw^*}{dc}\right] dF^d(w^d)dF^s(w^s) \\ &+ \int_{w^*}^{\hat{w}} (-\kappa_c')(1-g(\kappa(w^s)))\kappa(w^s) f^d(\kappa(w^s))dF^s(w^s) + \int_{w^*}^{\hat{w}} \kappa_c'(1-g(w^s))w^* f^d(\kappa(w^s))dF^s(w^s)\end{aligned}$$

Let us use the following results to simplify the solution. First, from the threshold function

$$\kappa_c = \frac{(\frac{d\tau}{dc}(w^s - w^*) - \frac{dw^*}{dc}(t-t^2))}{(1-t)^s}$$

for $w^s \in (w^*, \hat{w})$. Second, from the agent's utility, $U_c^d = -w^d(d\tau/dc)$, and $V_c^s = -w^s(d\tau/dc)$ for $w^s < w^*$ and $V_c^s = -w^*(d\tau/dc) - \tau(d\tau/dc)$ for $w^* \leq w^s$. By using those facts, grouping terms by $d\tau/dc$ and dw^*/dc, and joining together the integrals with $\hat{w}$, which reflect the same effect. Those fact yields

$$\begin{aligned}\frac{dA}{dc} =& \frac{d\tau}{dc}\left\{\int_{\underline{w}^s}^{\overline{w}^s}\int_{\kappa(w^s)}^{\overline{w}^d} (w^d)^2 g_{U^d}^d dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} (w^s)^2 g_{V^s}^s dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} (w^*)^2 g_{V^s}^s dF^d(w^d)dF^s(w^s)\right. \\ &\left. + \int_{w^*}^{\hat{w}} \frac{(w^s - w^*)}{(1-\tau)^2}(w^* - \kappa(w^s))(1-g^s(w^s))f^d(w^d)dF^s(w^s)\right\} + \frac{dw^*}{dc}\left\{\int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} ((1-g^s(w^s)) + tw^* g_{V^s}^s) dF^d(w^s)dF^s(w^s)\right. \\ &\left. - \int_{w^*}^{\hat{w}} \frac{\tau}{1-\tau}(w^* - \kappa(w^s))(1-g^s(w^s))f^d(w^d)dF^s(w^s)\right\}\end{aligned}$$

For simplicity, the above equation can be denoted as $dA/dc = (d\tau/dc)a_1 + (dw^*/dc)a_2$. Let us recall that, $g_V \leq 0$, hence $a_1 \leq 0$ and $a_2 \lesseqgtr 0$. Now, obtain the derivative of the term X with respect to c

$$\begin{aligned}\frac{dX}{dc} =& (-w^*)(w^* - w^*)\kappa_\tau'(w^*)f^d(\kappa(w^*))f^s(w^*) + \hat{w}_c'(\hat{w} - w^*)\kappa_\tau'(\hat{w})f^d(\kappa(\hat{w}))f^s(\hat{w}) \\ &+ \int_{w^*}^{\hat{w}} \left[-\frac{dw^*}{dc}\kappa_\tau' f^d(\kappa(w^s)) + (w^s - w^*)\kappa_{\tau c}' f^d(\kappa(w^s)) + (w^s - w^*)\kappa_\tau' f^{d\prime}(\kappa(w^s))\kappa_c'\right] dF^s(w^s)\end{aligned}$$

Use the following result to simplify the expression. First

$$\kappa_{\tau c} = -\frac{dw^*}{dc}\frac{1}{(1-\tau)^2} + \frac{d\tau}{dc}\frac{2}{1-\tau}\kappa_t$$

for $w^s \in (w^*, \hat{w})$. Second, $\hat{w} = \overline{w}^d(1-\tau) + \tau w^*$, hence $\hat{w}_c' = -\frac{d\tau}{dc}(\overline{w}^d - w^*) + \frac{dw^*}{dc}\tau$. By using this fact, rearranging and grouping by $d\tau/dc$ and dw^*/dc. Those fact yields

$$\frac{dX}{dc}=\frac{d\tau}{dc}\left\{\int_{w^*}^{\hat{w}}\left((w^s-w^*)\frac{2}{1-\tau}\kappa'_\tau f^d(\kappa(w^s))+(w^s-w^*)(\kappa'_\tau)^2 f^{d\prime}(\kappa(w^s))\right)dF^s(w^s)-(\overline{w}^d-w^*)(\hat{w}-w^*)\kappa'_\tau(\hat{w})f^d(\kappa(\hat{w}))f^s(\hat{w})\right\}$$

$$-\frac{dw^*}{dc}\left\{\int_{w^*}^{\hat{w}}\left(\kappa_\tau f^d(\kappa(w^s))+\frac{(w^s-w^*)}{(1-\tau)^2}f^d(\kappa(w^s))+(w^s-w^*)\kappa_\tau\frac{\tau}{1-\tau}f^{d\prime}(\kappa(w^s))\right)dF^s(w^s)+\tau(\hat{w}-w^*)\kappa'_\tau(\hat{w})f^d(\kappa(\hat{w}))f^s(\hat{w})\right\}$$

For simplicity, the above equation is denoted as $dX/dc=(d\tau/dc)x_1-(dw^*/dc)x_2$, where $x_1\lesseqgtr 0$ and $x_2>0$.

By using the characterization of dA/dc and dX/dc, is possible to obtain the overall effect on the audit cost in the tax rate. Replace the first formula using the simplified version of each characterization and resolve for the effect of the audit cost in the tax rate, obtaining

$$\frac{d\tau}{dc}=\frac{dw^*}{dc}\frac{a_2X+x_2A}{(A+X)^2-(a_1X-x_1A)}$$

Note that, the denominator of the previous equation is positive; thus, the effect on audit cost in tax rate depends on the sign of the numerator, which is the effect for changes in audit cost on revenue and social welfare for productivities $w^s\in(w^*,\hat{w})$.

J.2 Budget for the IRS

Let us derive both sides of equation 7 with regard to the marginal audit cost and simplify it to obtain an expression for this effect. First, the derivative with respect to the audit cost is

$$\frac{d\tau}{dc}\int_{w^*}^{\hat{w}}(w^*-\kappa(w^s))\frac{dw^*}{dB}\kappa'_B f^d(\kappa(w^s))dF^s(w^s)+\tau(-\frac{dw^*}{dc})(w^*-\kappa(w^*))\frac{dw^*}{dB}\kappa'_B f^d(\kappa(w^*))f^s(w^*)+\tau\hat{w}'_c(w^*-\kappa(\hat{w}))\frac{dw^*}{dB}\kappa'_B f^d(\kappa(\hat{w}))f^s(\hat{w})$$

$$\tau\int_{w^*}^{\hat{w}}\Big[(\frac{dw^*}{dc}-\kappa'_c)\frac{dw^*}{db}\kappa'_B f^d(\kappa(w^s))+(w^*-\kappa(w^s))\frac{dw^*}{dB}\frac{dB}{dc}\kappa'_B f^d(\kappa(w^s))+(w^*-\kappa(w^s))\frac{dw^*}{dB}\kappa''_B\frac{dB}{dc}f^d(\kappa(w^s))$$

$$+(w^*-\kappa(w^s))\frac{dw^*}{dB}\kappa'_B\kappa'_c f^d(\kappa(w^s))\Big]dF^s(w^s)+\frac{d\tau}{dc}\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s)+\tau(-\frac{dw^*}{dc})\int_{\underline{w}^d}^{\kappa(w^*)}\frac{dw^*}{dB}dF^d(w^d)f^s(w^*)$$

$$+\tau\hat{w}'_c\int_{\underline{w}^d}^{\kappa(\hat{w})}\frac{dw^*}{dB}dF^d(w^d)f^s(\hat{w})+\tau\int_{w^*}^{\hat{w}}\kappa'_c\frac{dw^*}{dB}f^d(\kappa(w^s))dF^s(w^s)+\frac{d\tau}{dc}\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}\frac{dw^*}{dB}\frac{dB}{dc}dF^d(w^d)dF^s(w^s)$$

$$+\frac{d\tau}{dc}\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s)+\tau(-\hat{w}'_c)\int_{\underline{w}^d}^{\kappa(\hat{w})}\frac{dw^*}{dB}dF^d(w^d)f^s(\hat{w})+\tau\int_{\hat{w}}^{\overline{w}^s}\kappa'_c\frac{dw^*}{dB}f^d(\kappa(w^s))dF^s(w^s)$$

$$+\tau\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}\frac{dw^*}{dB}\frac{dB}{dc}dF^d(w^d)dF^s(w^s)=(-\frac{dw^*}{dc})\int_{\underline{w}^d}^{\kappa(w^*)}g(w^*)\tau\frac{dw^*}{dB}dF^d(w^d)f^s(w^*)+(\hat{w}'_c)\int_{\underline{w}^d}^{\kappa(\hat{w})}g(\hat{w})\tau\frac{dw^*}{dB}dF^d(w^d)f^s(\hat{w})$$

$$+\int_{w^*}^{\hat{w}}\kappa'_c g(w^s)f^d(\kappa(w^s))dF^s(w^s)+\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}\left[g'V^s_c\tau\frac{dw^*}{dB}+g(w^s)\frac{d\tau}{dc}\frac{dw^*}{dB}+g(w^s)\tau\frac{dw^*}{dB}\frac{dB}{dc}\right]dF^s(w^s)+(-\hat{w}'_c)\int_{\underline{w}^d}^{\kappa(\hat{w})}g(\hat{w})\tau\frac{dw^*}{dB}dF^d(w^d)f^s(\hat{w})$$

$$+\int_{\hat{w}}^{\overline{w}^s}\kappa'_c g(w^s)f^d(\kappa(w^s))dF^s(w^s)+\int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}\left[g'V^s_c\tau\frac{dw^*}{dB}+g(w^s)\frac{d\tau}{dc}\frac{dw^*}{dB}+g(w^s)\tau\frac{dw^*}{dB}\frac{dB}{dc}\right]dF^s(w^s)$$

To simplify this expression, use some assumptions and results. First, $\kappa(w^*)=w^*$. Second, $\kappa'_c=0$ for

$w^s > \hat{w}$. Third, $\hat{w}'_c = \frac{dw^*}{dc}\tau + \frac{d\tau}{dc}(\overline{w}^d - w^*)$. Fourth, $V^s_c = -\frac{dw^*}{dc}\tau - \frac{d\tau}{dc}w^*$. Fifth, $\kappa'_c = \frac{d\tau}{dc}\frac{w^s - w^*}{(1-\tau)^2} - \frac{\tau}{1-\tau}\frac{dw^*}{dc}$. Finally, the terms can be grouped by the effect on taxes and threshold level. By using those facts, the total impact on audit cost in the budget for the IRS is

$$\frac{dB}{dc}\left\{\int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}\tau\frac{dw^*}{dB}(1-g(w^s))dF^d(w^d)dF^s(w^s) + \int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}\tau\frac{dw^*}{dB}(1-g(w^s))dF^d(w^d)dF^s(w^s) + \tau\int_{w^*}^{\hat{w}}(w^* - \kappa(w^s))\frac{dw^*}{dB}(\kappa'_B + \kappa''_B)f^d(\kappa(w^s))dF^s(w^s)\right\}$$

$$= \frac{d\tau}{dc}\left\{\tau(\overline{w}^d - w^*)(w^* - \kappa(w^s))\frac{dw^*}{dB}\kappa'_B f^d(\kappa(w^s))dF^s(w^s) + \int_{w^*}^{\hat{w}}\left[\tau\frac{w^s - w^*}{(1-\tau)^2}(1 - w^* + \kappa(w^s))\frac{dw^*}{dB}\kappa'_B f^d(\kappa(w^s)) - (w^* - \kappa(w^s))\frac{dw^*}{dB}\kappa'_B f^d(\kappa(w^s))\right.\right.$$

$$\left.\left.\tau\frac{w^s - w^*}{(1-\tau)^2}\frac{dw^*}{dB}(1 - g(w^s))f^d(\kappa(w^s))\right]dF^s(w^s) - \int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}\frac{dw^*}{dB}(1 - g'w^*\tau + g(w^s))dF^d(w^d)dF^s(w^s) - \int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}\frac{dw^*}{dB}(1 - g'w^*\tau + g(w^s))dF^d(w^d)dF^s(w^s)\right\}$$

$$+\frac{dw^*}{dc}\left\{\int_{\underline{w}^d}^{\kappa(w^*)}(1 - g(w^*))\tau\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s) - \tau^2(w^* - \kappa(\hat{w}))\frac{dw^*}{dB}\kappa'_B f^d(\kappa(\hat{w}))f^s(\hat{w}) + \int_{w^*}^{\hat{w}}\left[\frac{\tau^2}{1-\tau}\frac{dw^*}{dB}(1 - g(w^s))f^d(\kappa(w^s)) - \frac{\tau}{1-\tau}\frac{dw^*}{dB}\kappa'_B f^d(\kappa(w^s))\right.\right.$$

$$\left.\left.\frac{\tau^2}{1-\tau}(w^* - \kappa(w^s))\frac{dw^*}{dB}\kappa'_B f^d(\kappa(w^s))\right]dF^s(w^s) - \int_{w^*}^{\hat{w}}\int_{\underline{w}^d}^{\kappa(w^s)}g'\tau^2\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s) - \int_{\hat{w}}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)}g'\tau^2\frac{dw^*}{dB}dF^d(w^d)dF^s(w^s\right\}$$

To give a better interpretation, the above expression is conveyed, with those respective signs, as follows

$$\frac{dB}{dc}\overbrace{b}^{+} = \frac{d\tau}{dc}\underbrace{\left\{\overbrace{t_1}^{+} + \overbrace{t_2}^{-} - \overbrace{t_3}^{+} - \overbrace{t_4}^{+}\right\}}_{+/-} + \frac{dw^*}{dc}\underbrace{\left\{\overbrace{w_1}^{+} - \overbrace{w_2}^{+} + \overbrace{w_3}^{+} - \overbrace{w_4}^{-} - \overbrace{w_5}^{-}\right\}}_{+/-}$$

K Alternative Derivation of the Optimal Government Policies

In this appendix, the solution for the case where $\overline{n}^d > \overline{n}^s$ is shown. In each case, it is demonstrated that it is possible to extrapolate the conclusion given in the main section. For this explanation, let us define the level of productivity in the dependent sector that is equal to the threshold function evaluated in the upper bound of the self-employed productivity as $\hat{w} = \kappa(\overline{w}^s)$.

K.1 Optimal BLS Rule

The optimal BLS rule is obtained from the government's problem deriving the Lagrangian with respect to the public goods. The Lagrange of the government problem is

$$\begin{aligned}\mathcal{L} = &\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} G\big(U^d(w^d)\big)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} G\big(U^d(w^d)\big)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} G\big(U^d(w^d)\big)dF^d(w^d)dF^s(w^s) \\ &+ \int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} G\big(U^d(w^d)\big)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} G(V^s(w^s,\mathcal{M}))dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} G(V^s(w^s,\mathcal{M}))dF^d(w^d)dF^s(w^s) \\ &- \delta\Bigg\{B + R - \tau\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} w^d dF^d(w^d)dF^s(w^s) - \tau\int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) - \tau\int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} w^d dF^d(w^d)dF^s(w^s) \\ &- \tau\int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) - \tau\int_{\underline{w}^s}^{w^*} w^s F^d\left(\kappa(w^s)\right)dF^s(w^s) - \tau w^*\int_{w^*}^{\overline{w}^s} F^d\left(\kappa(w^s)\right)dF^s(w^s)\Bigg\}\end{aligned}$$

The first-order condition (FOC) with respect to R is

$$\begin{aligned}&\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} G'\big(U^d(w^d)\big)\phi'(R)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} G'\big(U^d(w^d)\big)\phi'(R)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} G'\big(U^d(w^d)\big)\phi'(R)dF^d(w^d)dF^s(w^s) \\ &+ \int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} G'\big(U^d(w^d)\big)\phi'(R)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} G'(V^s(w^s,\mathcal{M}))\phi'(R)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} G'(V^s(w^s,\mathcal{M}))\phi'(R)dF^d(w^d)dF^s(w^s) - \delta = 0\end{aligned}$$

By doing algebra yields

$$\begin{aligned}&\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} \frac{G'\big(U^d(w^d)\big)}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} \frac{G'\big(U^d(w^d)\big)}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} \frac{G'\big(U^d(w^d)\big)}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s) \\ &+ \int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} \frac{G'\big(U^d(w^d)\big)}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} \frac{G'(V^s(w^s,\mathcal{M}))}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} \frac{G'(V^s(w^s,\mathcal{M}))}{\delta}\phi'(R)dF^d(w^d)dF^s(w^s) = 1\end{aligned}$$

The derivative of the Lagrangian regarding consumption in each sector are

$$G'(U^d(w^d))f^d(w^d)f^s(w^s) - \delta f^d(w^d)f^s(w^s) = 0$$
$$G'(V^s(w^s,\mathcal{M}))f^d(w^d)f^s(w^s) - \delta f^d(w^d)f^s(w^s) = 0$$

where the definition of consumption, $C(w) = w - T(w)$, is used to replaces taxes for consumption. By using the above equation to replace the Lagrange multiplier in the FOC of the public good yields

$$\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} \phi'(R)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} \phi'(R)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} \phi'(R)dF^d(w^d)dF^s(w^s)$$
$$+\int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} \phi'(R)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} \phi'(R)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} \phi'(R)dF^d(w^d)dF^s(w^s) = 1$$

Finally, one can use the fact that the marginal utility of consumption is equal to one. Hence, the marginal rate of substitution only $\phi'(R)$. The final expression is

$$\phi\prime(R) = \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} MRS(RC)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} MRS(RC)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} MRS(RC)dF^d(w^d)dF^s(w^s)$$
$$+\int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} MRS(RC)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} MRS(RC)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} MRS(RC)dF^d(w^d)dF^s(w^s) = 1$$

This equation is the same as in the paper, only the limits in the integrals change, but it represents the same result; the solution of the optimal public good provision is the same as in the first-best.

K.2 Optimal Marginal Tax Rate

The optimal marginal tax rate solves the Lagrangian in the government's problem. The Lagrangian of the government problem is

$$\mathcal{L} = \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} G\big(U^d(w^d)\big)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} G\big(U^d(w^d)\big)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} G\big(U^d(w^d)\big)dF^d(w^d)dF^s(w^s)$$
$$+\int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} G\big(U^d(w^d)\big)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} G(V^s(w^s,\mathcal{M}))dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} G(V^s(w^s,\mathcal{M}))dF^d(w^d)dF^s(w^s)$$
$$-\delta\Bigg\{B + R - \tau\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} w^d dF^d(w^d)dF^s(w^s) - \tau\int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) - \tau\int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} w^d dF^d(w^d)dF^s(w^s)$$
$$-\tau\int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) - \tau\int_{\underline{w}^s}^{w^*} w^s F^d(\kappa(w^s))dF^s(w^s) - \tau w^*\int_{w^*}^{\overline{w}^s} F^d(\kappa(w^s))dF^s(w^s)\Bigg\}$$

The first-order condition with respect to τ is

$$
\begin{aligned}
&\int_{\underline{w}^s}^{w^*} (-\kappa'_\tau) G(\kappa(w^s)) f^d(\kappa(w^s)) dF^s(w^s) + \int_{\underline{w}^s}^{w^*} \hat{w}'_\tau G\left(U^d(\hat{w})\right) f^d(\hat{w}) dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} G'_{U^d} U^d_C \frac{dC_d}{d\tau} dF^d(w^d) dF^s(w^s) \\
&+ \int_{\underline{w}^s}^{w^*} (-\hat{w}'_\tau) G\left(U^d(w^d)\right) f^d(\hat{w}) dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} G'_{U^d} U^d_C \frac{dC_d}{d\tau} dF^d(w^s) dF^s(w^s) + \int_{w^*}^{\overline{w}^s} (-\kappa'_\tau) G\left(U^d(\kappa(w^s))\right) f^d(\kappa(w^s)) dF^s(w^s) \\
&\int_{w^*}^{\overline{w}^s} \hat{w}'_\tau G\left(U^d(\hat{w})\right) f^d(\hat{w}) dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} G'_{U^d} U^d_C \frac{dC_d}{d\tau} dF^d(w^d) dF^s(w^s) + \int_{w^*}^{\overline{w}^s} (-\hat{w}'_\tau) G\left(U^d(\hat{w})\right) f^d(\hat{w}) dF^s(w^s) \\
&+ \int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} G'_{U^d} U^d_C \frac{dC_d}{d\tau} dF^d(w^d) dF^s(w^s) + \int_{\underline{w}^s}^{w^*} \kappa'^{\tau} G(V^s(w^s,\mathcal{M})) f^d(\kappa(w^s)) dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} G'_{V^s} V^s_C \frac{dC_s}{d\tau} dF^d(w^d) dF^s(w^s) \\
&+ \int_{w^*}^{\overline{w}^s} \kappa'_\tau G(V^s(w^s,\mathcal{M})) f^d(\kappa(w^s)) dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} G'_{V^s} V^s_C \frac{dC_s}{d\tau} dF^d(w^d) dF^s(w^s) - \delta \Bigg\{ - \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} w^d dF^d(w^d) dF^s(w^s) \\
&- \tau \int_{\underline{w}^s}^{w^*} (-\kappa'_\tau) \kappa(w^s) f^d(\kappa(w^s)) dF^s(w^s) - \tau \int_{\underline{w}^s}^{w^*} \hat{w}'_\tau \hat{w} f^d(\hat{w}) dF^s(w^s) - \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) - \tau \int_{\underline{w}^s}^{w^*} (-\hat{w}'_\tau) \hat{w} f^d(\hat{w}) dF^s(w^s) \\
&- \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} w^d dF^d(w^d) dF^s(w^s) - \tau \int_{w^*}^{\overline{w}^s} (-\kappa'_\tau) \kappa(w^s) f^d(\kappa(w^s)) dF^s(w^s) - \tau \int_{w^*}^{\overline{w}^s} \hat{w}'_\tau \hat{w} f^d(\hat{w}) dF^s(w^s) - \int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d) dF^s(w^s) \\
&- \tau \int_{w^*}^{\overline{w}^s} (-\hat{w}'_\tau) \hat{w} f^d(\hat{w}) dF^s(w^s) - \int_{\underline{w}^s}^{w^*} w^s F^d(\kappa(w^s)) dF^s(w^s) - \tau \int_{\underline{w}^s}^{w^*} w^s \kappa'_\tau f^d(\kappa(w^s)) dF^s(w^s) - \int_{w^*}^{\overline{w}^s} w^* F^d(\kappa(w^s)) dF^s(w^s) \\
&- \tau \int_{w^*}^{\overline{w}^s} w^* \kappa'_\tau f^d(\kappa(w^s)) dF^s(w^s) \Bigg\}
\end{aligned}
$$

To simplify the previous expression, one can use several results and assumptions from the model. First $g = (G_{V^i} \times V^i_C)/\delta$ can be defined as the social valuation of agent's consumption. Second, it needs to be recalled that $U^d(\kappa(w^s)) = V^s(w^s, \mathcal{M})$, implying that some terms are canceling among them. Third, from the IRS's solution, $\kappa'_\tau = 0$ for $w^s \leq w^*$. Finally, the derivative of consumption in regard to marginal tax in each sector are

$$
\frac{dC_d}{d\tau} = -w^d, \qquad \frac{dC_s}{d\tau} = \begin{cases} -w^s & \text{if } w^s < w^* \\ -w^* & \text{if } w^* \leq w^s \end{cases}
$$

By using those facts and doing algebra yields

$$-\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} g(w^d)w^d dF^d(w^d)dF^s(w^s) - \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} g(w^d)w^d dF^d(w^d)dF^s(w^s) - \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} g(w^d)w^d dF^d(w^d)dF^s(w^s)$$

$$-\int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} g(w^d)w^d dF^d(w^d)dF^s(w^s) - \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} g(w^s)w^s dF^d(w^d)dF^s(w^s) - \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} g(w^s)w^* dF^d(w^d)dF^s(w^s)$$

$$= -\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} w^d dF^d(w^d)dF^s(w^s) - \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) - \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} w^d dF^d(w^d)dF^s(w^s) + \tau\int_{w^*}^{\overline{w}^s} \kappa'_\tau \kappa(w^s) f^d(\kappa(w^s))dF^s(w^s)$$

$$-\int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) - \int_{\underline{w}^s}^{w^*} w^s F^d\left(\kappa(w^s)\right)dF^s(w^s) - \int_{w^*}^{\overline{w}^s} w^* F^d\left(\kappa(w^s)\right)dF^s(w^s) - \tau\int_{w^*}^{\overline{w}^s} w^*\kappa'_\tau f^d\left(\kappa(w^s)\right)dF^s(w^s)$$

Note that, $\kappa(w^s) - w^* = \dfrac{w^s - w^*}{1-\tau}$. By using this fact and doing algebra, the optimal marginal tax formula is

$$\frac{\tau}{1-\tau}\int_{w^*}^{\overline{w}^s}\Big[(w^s - w^*)\kappa_\tau(w^s) f^d(\kappa(w^s))\Big]dF^s(w^s) = \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} (1-g(w^d))w^d dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d}(1-g(w^d))w^d dF^d(w^d)dF^s(w^s)$$
$$+\int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} (1-g(w^d))w^d dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d}(1-g(w^d))w^d dF^d(w^d)dF^s(w^s)$$
$$+\int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} (1-g(w^s))w^s dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} (1-g(w^s))w^* dF^d(w^d)dF^s(w^s)$$

This formula is essentially the same as in the main section, only the limits of some integrals change. Moreover, both results represent the same effects and have the same mechanism behind.

K.3 Optimal Budget for the IRS

The optimal IRS's budget comes from the maximization of the Lagrangian of the government's problem

$$\mathcal{L} = \int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} G\big(U^d(w^d)\big)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} G\big(U^d(w^d)\big)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} G\big(U^d(w^d)\big)dF^d(w^d)dF^s(w^s)$$
$$+\int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} G\big(U^d(w^d)\big)dF^d(w^d)dF^s(w^s) + \int_{\underline{w}^s}^{w^*}\int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s,\mathcal{M})\right)dF^d(w^d)dF^s(w^s) + \int_{w^*}^{\overline{w}^s}\int_{\underline{w}^d}^{\kappa(w^s)} G\left(V^s(w^s,\mathcal{M})\right)dF^d(w^d)dF^s(w^s)$$
$$-\delta\Bigg\{B + R - \tau\int_{\underline{w}^s}^{w^*}\int_{\kappa(w^s)}^{\hat{w}} w^d dF^d(w^d)dF^s(w^s) - \tau\int_{\underline{w}^s}^{w^*}\int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) - \tau\int_{w^*}^{\overline{w}^s}\int_{\kappa(w^s)}^{\hat{w}} w^d dF^d(w^d)dF^s(w^s)$$
$$-\tau\int_{w^*}^{\overline{w}^s}\int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d)dF^s(w^s) - \tau\int_{\underline{w}^s}^{w^*} w^s F^d\left(\kappa(w^s)\right)dF^s(w^s) - \tau w^*\int_{w^*}^{\overline{w}^s} F^d\left(\kappa(w^s)\right)dF^s(w^s)\Bigg\}$$

The first-order condition with respect to B is

$$
\begin{aligned}
& w_B^* \int_{\kappa(w^*)}^{\hat{w}} G\big(U^d(w^d)\big) dF^d(w^d) f^s(w^*) + \int_{\underline{w}^s}^{w^*} \hat{w}_B' G\big(U^d(\hat{w})\big) f^d(\hat{w}) dF^s(w^s) + \int_{\underline{w}^s}^{w^*} (-\kappa_B') G\big(U^d(\kappa(w^s))\big) f^d(\kappa(w^s)) dF^s(w^s) \\
& w_B^* \int_{\hat{w}}^{\overline{w}^d} G\big(U^d(w^d)\big) dF^d(w^d) f^s(w^*) \int_{\underline{w}^s}^{w^*} (-\hat{w}_B) G\big(U^d(\hat{w})\big) f^d(\hat{w}) dF^s(w^s) + (-w^*) \int_{\kappa(w^*)}^{\hat{w}} G\big(U^d(w^d)\big) dF^d(w^d) f^s(w^*) \\
& \int_{w^*}^{\overline{w}^s} \hat{w}_B G\big(U^d(\hat{w})\big) f^d(\hat{w}) dF^s(w^s) + \int_{w^*}^{\overline{w}^s} (-\kappa_B') G\big(U^d(\kappa(w^s))\big) f^d(\kappa(w^s)) dF^s(w^s) + (-w_B^*) \int_{\hat{w}}^{\overline{w}^d} G\big(U^d(w^d)\big) dF^d(w^d) f^s(w^*) \\
& + \int_{w^*}^{\overline{w}^s} (-\hat{w}_B') G\big(U^d(\hat{w})\big) f^d(\hat{w}) dF^s(w^s) + w_B^* \int_{\underline{w}^d}^{\kappa(w^s)} G(V^s(w^*, \mathcal{M})) dF^d(w^d) f^s(w^*) + \int_{\underline{w}^s}^{w^*} \kappa_B' G(V^s(w^s, \mathcal{M})) f^d(\kappa(w^s)) dF^s(w^s) \\
& (-w_B^*) \int_{\underline{w}^d}^{\kappa(w^*)} G(V^s(w^*, \mathcal{M})) dF^d(w^d) f^s(w^*) + \int_{w^*}^{\overline{w}^s} \kappa_B' G(V^s(w^s, \mathcal{M})) f^d(\kappa(w^s)) dF^s(w^s) + \int_{w^*}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} G'_{V^s} V_C^s \frac{dC_s}{dB} dF^d(w^d) dF^s(w^s) \\
& - \delta \Bigg\{ 1 - \tau w_B^* \int_{\kappa(w^*)}^{\hat{w}} w^d dF^d(w^d) f^s(w^*) - \tau \int_{\underline{w}^s}^{w^*} \hat{w}_B \hat{w} f^d(\hat{w}) dF^s(w^s) - \tau \int_{\underline{w}^s}^{w^*} (-\kappa_B') \kappa(w^s) f^d \kappa(w^s)) dF^s(w^s) - \tau w_B^* \int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d) f^s(w^*) \\
& \tau \int_{\underline{w}^s}^{w^*} (-\hat{w}_B') \hat{w} f^d(\hat{w}) dF^s(w^s) - \tau(-w_B^*) \int_{\kappa(w^*)}^{\hat{w}} w^d dF^d(w^d) f^s(w^*) - \tau \int_{w^*}^{\overline{w}^s} \hat{w}_B' \hat{w} f^d(\hat{w}) dF^s(w^s) - \tau \int_{w^*}^{\overline{w}^s} \kappa_B' \kappa(w^s) f^d(\kappa(w^s)) dF^s(w^s) \\
& - \tau(-w_B^*) \int_{\hat{w}}^{\overline{w}^d} w^d dF^d(w^d) f^s(w^*) - \tau \int_{w^*}^{\overline{w}^s} (-\hat{w}_B') \hat{w} f^d(\hat{w}) dF^s(w^s) - \tau w_B^* w^* F^d(\kappa(w^*)) f^s(w^*) - \tau \int_{\underline{w}^s}^{w^*} w^s \kappa_B' f^d(\kappa(w^s)) dF^s(w^s) \\
& -\tau(-w_B^*) w^* F^d(\kappa(w^*)) f^s(w^*) - \tau \int_{w^*}^{\overline{w}^s} w^* \kappa_B' f^d(\kappa(w^s)) dF^s(w^s) - \tau \int_{w^*}^{\overline{w}^s} \frac{dw^*}{dB} F^d(\kappa(w^s)) dF^s(w^s) \Bigg\}
\end{aligned}
$$

To simplify the above expression, use some results and assumptions. First, the social valuation of the agent's consumption is defined as $g = G_V' \times V_C / \delta$. Second, by definition $U^d(\kappa(w^s)) = V^s(w^s, \mathcal{M})$, make some terms cancel among them. Finally, the derivative of consumption in the self-employed sector respect to the budget is

$$
\frac{dC_s}{dB} = \begin{cases} 0 & \text{if } w^s < w^* \\ -\tau \dfrac{dw^*}{dB} & \text{if } w^* \leq w^s \end{cases}
$$

At this point, it is important to recall that, in the dependent sector, an increase in the budget for the IRS does not affect taxes. By using those facts and doing algebra yields

$$
\tau \int_{w^*}^{\overline{w}^s} (w^* - \kappa(w^s)) \kappa_B' f^d(\kappa(w^s)) dF^s(w^s) + \tau \int_{w^*}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} w_B^* dF^d(w^d) dF^s(w^s) = 1 + \int_{w^*}^{\overline{w}^s} \int_{\underline{w}^d}^{\kappa(w^s)} g(w^s) \tau w_B^* dF^d(w^d) dF^s(w^s)
$$

This equation represents the same as the one shown in the main section.