Optimal taxes and penalties when the IRS cannot commit to its audit policy*

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Abstract

REVISAR We examine the problem of a utilitarian government that sets taxes and fines for evaders but cannot commit to any enforcement policy. Given the tax law, the government and taxpayers –some of whom are honest– play a report-audit game that, depending on taxes, fines and audit costs, generates either full evasion and no audits, or partial evasion and random auditing. We show that it may be optimal for the government not to fine evaders as a way to commit not to audit. Moreover, social welfare is nonmonotonic in the audit cost.

Keywords: Tax rates - Tax evasion - Enforcement - Audit costs - No commitment - Mixed-strategy equilibrium.

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1 The model

There is a continuum of taxpayers of measure one. Each taxpayer i is either "rich" or "poor": her individual income y_i is a random variable that takes values in the set $\{y_p, y_r\}$, with $y_p < y_r$. The realization of y_i is i's private information. All individual incomes are i.i.d., and the probability that $y_i = y_p$ for any given taxpayer i is $\mu \in (0, 1)$, which is common knowledge.

A key feature of our analysis is that any given taxpayer may be "honest" or "dishonest," and this is her private information as well. Dishonest taxpayers file their tax returns by choosing to report the income level that maximizes their expected utilities. In other words, they may lie provided it is beneficial to them. On the contrary, honest taxpayers always file truthful reports –i.e. they face a cost of lying so large that they will never file a false income report.[1] Any given rich taxpayer is dishonest with probability $\theta \in (0, 1]$, and this is common knowledge –it will be clear later on that whether a poor taxpayer is honest or dishonest is irrelevant, so we ignore this issue.

Taxpayer i's utility function is

$$W_i = u(q_i) + g$$

where q_i is the value of her private-good consumption and g is the government's provision of a public good. The function $u(.)$ is strictly increasing, strictly concave and satisfies the Inada conditions. We normalize $u(0) = 0$, and we assume that $u'(y_r) \in (0, 1)$, so that interior solutions always obtain.

Tax policy is fully described by two elements. First, the "tax law" (t, f), where t is the tax due for rich taxpayers and f is the penalty imposed on those who misreport their incomes and are detected –in that case, the taxpayer owes f in addition to the tax t that corresponds to her true income.[2] Note that only rich taxpayers must pay taxes. As we will later see, this is without loss of generality. We assume ex-post limited liability, so it must be the case that $t + f \leq y_r$. Let $\Omega = \{(t, f) : t, f \geq 0, t + f \leq y_r\}$ denote the set of feasible tax laws.

The second element is audit policy, given by β, the probability with which a low-income report is audited. Each audit costs $c > 0$, and all audits are perfect: if a misreporting taxpayer is audited, she is detected with certainty. The government finances g, the provision of the public good, with all revenues collected from taxpayers: taxes and fines, net of audit costs.

In what follows, we will consider two key dimensions of tax-enforcement policy and examine their interaction with the presence of honest taxpayers. First, we distinguish between two organizational settings that differ according to who designs the tax law (t, f) and who enforces it -i.e. who chooses β. In the "No Delegation" (ND) case, the government undertakes both of these tasks, and does so to maximize a utilitarian social welfare function. On

[1] Alternatively, we may interpret this situation as one where some of the taxpayers do not have a readily available technology for tax evasion.

[2] As most of the literature on tax evasion, we do not allow for rewards for truthful reports. Exceptions to this assumption can be found in Border and Sobel (1987), Mookherjee and Png (1989) and Chander and Wilde (1998) among others.

the other hand, in the "Delegation" (D) case, the same utilitarian government designs the tax law, but it is an independent tax authority (from now on, the IRS) that collects taxes and enforces them. The IRS' objective, given the tax law, is to maximize expected net tax collection.

The second dimension is related to the tax enforcement authority's commitment capability. In case C, the enforcement authority can commit to a specific audit policy –i.e. an audit probability– whereas in case NC she cannot. Combining these two dimensions yields four possible settings, which we will examine below.

All parties involved –the government, taxpayers and, under delegation, the IRS– interact in a two-stage game. In all four possible settings, the first stage is the same: the government selects the tax law, (t, f). In the second stage, tax reports are filed and audits may occur. However, the way this interaction proceeds differs according to the exact setting we wish to study. Under commitment, the tax enforcement authority –i.e. the government in case ND, or the IRS in case $D-$ credibly announces the audit probability β for low-income reports. After this announcement, each taxpayer i reports an income level $\widetilde{y}_i$. Any taxpayer i for whom $\widetilde{y}_i = y_p$ is then audited with probability β. Under no commitment, taxpayers and the tax enforcement authority play a "report-audit game." Taxpayers move first by filing their income reports. Then, the enforcement authority –again, the government or the IRS, depending on the setting– chooses the audit probability for low-income reports.

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The assumptions of the model deserve some comments. As noted by Andreoni et al. (1998), "an unresolved question is whether cheaters should be given the same weight in the social criterion function as honest taxpayers." We follow the literature on tax evasion, which provides a positive answer. Utilities that are linear in the public good simplify equilibrium characterization in the report-audit game (see footnote ACTUALIZAR???). In addition, assuming only two income levels and no taxation for the poor simplifies the presentation of our results. ((In Section 4 we explain why these three assumptions are not as restrictive as they may seem))???. Finally, in this paper the government can commit to taxes and penalties but not to the audit probability. Boadway and Keen (1998) choose a different setup: the government can commit to the audit probability but not to the (capital income) tax rate. They explain this choice as follows: the audit probability is associated with the tax authority's administrative efficiency –and this is more difficult to modify than any tax scheme, as many observers assert. Even though this association seems realistic, we believe it does not necessarily imply commitment to an audit policy. Irrespective of the tax authority's efficiency (which may be irreversible), a government can always renege its announced audit policy and *decrease* the frequency of controls ex post, as is the issue in this paper. In Boadway and Keen (1998), on the contrary, the government would have incentives to raise the audit probability ex post, which may be difficult given the structure of the tax administration.

2 Benchmark: full information

As a benchmark, assume that income levels are publicly known. Audits are unnecessary, and fines irrelevant. Hence, whether tax collection is delegated to the IRS or not is immaterial. The issue of the presence of honest taxpayers is moot in this context.

The government will choose the tax t to maximize

$$\mathbb{E}W = \mu u(y_p) + (1-\mu)u(y_r - t) + g$$

subject to its budget constraint $g = (1-\mu)t$. As the government's objective function is concave, the optimal full-information tax, t^*, is characterized by the first-order condition

$$u'(y_r - t^*) = 1.$$

Given our assumptions, we have $0 < t^* < y_r$. Let $\mathbb{E}W^*$ be the expected welfare level when the government chooses the optimal full-information tax t^*.

In what follows, we move back to the case of interest: income levels are private information.

3 Commitment to the audit policy

We start by assuming that the authority in charge of tax collection (the government or the IRS) can can commit to an audit probability. First, we examine the case (ND, C), where the government itself enforces the tax law. Then, we move to the case where tax collection is delegated to the IRS, i.e. case (D, C).

3.1 No delegation and commitment (ND,C)

In this case, the government can credibly announce the probability β with which it will audit any individual who reports a low income level, y_p. In essence, it is simultaneously choosing the tax law and the audit probability.

As happens in Pestieau et al. (1998), two alternative regimes may emerge. The government may enforce the tax law (a "no-evasion" regime) or, if audits are prohibitively costly, it may choose not not to do so altogether (i.e. select a "full-evasion" regime). Let us examine each possibility in turn.

No evasion In the first regime, the government designs and enforces the tax law so that income is never misreported. Since in this context the revelation principle applies (see Mookherjee and Png, 1990), the optimal fiscal policy can be characterized adopting a mechanism-design approach. The government solves the following program:

$$\mathcal{P}_{NE}^{ND,C}\begin{cases} \underset{t,f,\beta}{Max} \quad \mu u(y_p) + (1-\mu)u(y_r - t) + g \\ subject\ to \\ u(y_r - t) \geq (1-\beta)u(y_r) + \beta u(y_r - t - f) \\ g = (1-\mu)t - \mu\beta c \\ (t,f) \in \Omega \\ \beta \in [0,1] \end{cases}$$

The first constraint imposes, as usual, incentive compatibility, and the second is the government's budget constraint, which incorporates the aggregate audit cost. Let $\widehat{t}_{NE}^{ND,C}$, $\widehat{f}_{NE}^{ND,C}$ and $\widehat{\beta}_{NE}^{ND,C}$ be the values that solve this problem. At the optimum, the government completely deters evasion by setting the maximal penalty

$$\widehat{f}_{NE}^{ND,C} = y_r - \widehat{t}_{NE}^{ND,C}$$

and by auditing low-income reports with probability

$$\widehat{\beta}_{NE}^{ND,C} = 1 - \frac{u(y_r - \widehat{t}_{NE}^{ND,C})}{u(y_r)},$$

i.e. the lowest audit probability that satisfies incentive compatibility. The optimal tax, $\widehat{t}_{NE}^{ND,C}$, is given by the following first-order condition

$$u'(y_r - \widehat{t}_{NE}^{ND,C}) = \frac{1}{1 + \frac{\mu}{1-\mu}\frac{c}{u(y_r)}} < 1. \tag{1}$$

If c goes to zero, $\widehat{t}_{NE}^{ND,C}$ converges to the optimal full-information tax t^*. Moreover, $\widehat{t}_{NE}^{ND,C}$ decreases with c. The government distorts $\widehat{t}_{NE}^{ND,C}$ downwards with respect to t^* in order to reduce the stake for evasion and, thereby, the optimal audit probability. That way, the impact of the rise in c on total audit costs is lowered. In the Appendix we show that, provided $\widehat{\beta}_{NE}^{ND,C} > 0$, expected welfare under no evasion is a decreasing and convex function of c.

However, this is valid as long as c is not too large. As c grows, eventually the optimal way of reaching no evasion is to set $\widehat{t}_{NE}^{ND,C} = 0$ and avoid audits completely –i.e. setting $\widehat{\beta}_{NE}^{ND,C} = 0$. There is no taxation and fines are irrelevant. This corner solution obtains if[3]

$$c \geq c_1 \equiv \frac{(1-\mu)}{\mu}\frac{u(y_r)(1-u'(y_r))}{u'(y_r)}.$$

Expected welfare is given by

$$\mathbb{E}W_{NT} \equiv \mu u(y_p) + (1-\mu)u(y_r),$$

since there is no taxation or public good provision.

[3] The cutoff value c_1 follows from the first-order condition (1), evaluated at $\widehat{t}_{NE}^{ND,C} = 0$.

Full evasion The government may always choose not to enforce the tax law and set $\beta = 0$: all dishonest taxpayers will misreport, and only honest taxpayers will pay their taxes. Naturally, then, how desirable this policy is depends crucially on the existence of honest taxpayers.

If all rich taxpayers are dishonest ($\theta = 1$), full evasion yields the same result as no taxation, independently of the taxes and fines that are actually chosen. When some of the rich are honest ($\theta < 1$), though, the government can collect taxes even when it commits not to audit.[4] Then, it will choose a positive tax level. Fines are irrelevant, and the government chooses taxes by solving

$$\mathcal{P}_{FE}^{ND,C} \begin{cases} \underset{t}{Max} \quad \mu u(y_p) + (1-\mu)\theta u(y_r) + (1-\mu)(1-\theta)u(y_r - t) + g \\ subject\ to \\ g = (1-\mu)(1-\theta)t \\ t \in [0, y_r] \end{cases}$$

Let $\widehat{t}_{FE}^{ND,C}$ be the solution to this problem. The objective function here is –but for a constant factor that does not affect the solution– the same as under full information. Therefore, the government sets $\widehat{t}_{FE}^{ND,C} = t^*$. Expected welfare is clearly higher than $\mathbb{E}W_{NT}$.

Choosing regimes The government will choose the regime that maximizes expected welfare given the audit cost. Figure 1 below shows the resulting expected welfare profiles under no and full evasion, both without and with honest taxpayers. Maximal expected welfare for the government is given by the upper envelope of those welfare profiles.

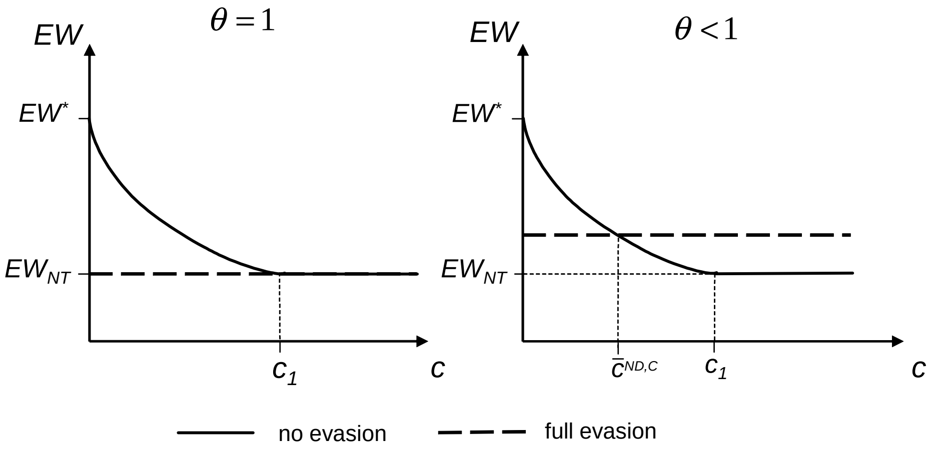

Figure 1: expected welfare in case (ND, C)

[4] In this case, there is no complete revelation of individual types. The government does not have enough instruments to screen honest and dishonest rich taxpayers, so the latter misreport. For similar results, albeit in different contexts, see Kofman and Lawarrée (1996) and Picard (1996).

In both cases, expected welfare takes its full-information level $\mathbb{E}W^*$ when $c = 0$. Then, it is continuous and weakly decreasing in c. When all rich taxpayers are dishonest, the government chooses to enforce the tax law when $c \leq c_1$, If $c > c_1$, it sets taxes at zero and provides no public goods. However, when $\theta < 1$, no enforcement becomes optimal for lower audit costs. The cutoff cost, $\overline{c}^{ND,C}$, where the government is indifferent between full and no enforcement, is lower than c_1. Then, if $c \in (\overline{c}^{ND,C}, c_1)$, the presence of honest taxpayers induces the government to collect positive taxes under no enforcement, when without honest taxpayers it would enforce the tax law fully with relatively lower taxes.

3.2 Delegation and commitment (D,C)

We move now to the context where the government delegates tax enforcement to the IRS. As mentioned above, the latter's objective is to maximize net tax collection. The IRS can commit to an audit probability, just as the government did in the previous case. Now the two-stage game proceeds as follows. The government chooses the tax law (t, f). Then, the IRS selects its credible audit probability β and, knowing the value of β, taxpayers report their incomes. We solve this interaction backwards.

3.2.1 The optimal audit policy for the IRS

Suppose (t, f) is given. The IRS has to select the value of β. Just as happened without delegation, one of two regimes will emerge. The IRS can choose to enforce the tax law or -if auditing is too costly- not to do so.

No evasion In the first regime, the IRS designs its audit strategy to maximize expected net tax collection by avoiding income misreporting: there is no evasion. Once again, the revelation principle applies (see Mookherjee and Png, 1990). The IRS solves the following problem:

$$\begin{cases} \underset{\beta}{Max} \quad (1-\mu)t - \mu\beta c \\ \\ subject\ to \\ \\ u(y_r - t) \geq (1-\beta)u(y_r) + \beta u(y_r - t - f) \\ \\ \beta \in [0,1] \end{cases},$$

where the first constraint ensures incentive compatibility. Let $\widehat{\beta}_{NE}^{D,C}$ be the value that solves this problem. At the optimum, the IRS completely deters evasion by auditing low-income reports with probability

$$\widehat{\beta}_{NE}^{D,C} = \frac{u(y_r) - u(y_r - t)}{u(y_r) - u(y_r - t - f)}.$$

Given this optimal audit strategy, expected net tax collection $(1-\mu)t - \mu\beta c$ decreases linearly with c, until $c = c_2 \equiv \frac{(1-\mu)t}{\mu\beta}$. At that audit cost level, expected net tax collection is zero.

Full evasion Alternatively, the IRS can give up enforcement, i.e. set $\widehat{\beta}^{D,C} = 0$. Under these circumstances, expected tax collection is $(1-\mu)(1-\theta)t$, the amount collected from honest rich taxpayers. Again, the existence of honest taxpayers is key to the desirability of this regime.

The IRS is indifferent between both regimes when

$$(1-\mu)t - \mu\widehat{\beta}_{NE}^{D,C} c = (1-\mu)(1-\theta)t,$$

i.e. when $c = \frac{(1-\mu)\theta t}{\mu\widehat{\beta}_{NE}^{D,C}} \leq c_2$ (the inequality is strict if $\theta < 1$). If c is lower than the cutoff value, the IRS chooses to deter evasion; otherwise full evasion follows. Note again the impact of honest taxpayers on the IRS' regime choice. The cutoff cost grows with θ: naturally, more honest taxpayers make full evasion more attractive.

3.2.2 The optimal tax law

We move back now to the first stage, where the government sets the tax law. The pair (t, f) selected by the government determines the cutoff cost for the IRS' regime choice. Thus, the government should identify the subsets of pairs $(t, f) \in \Omega$ that induce the IRS to select each regime, then find the pairs that maximize expected welfare within each subset, and, finally, compare the maximum attainable expected welfare levels under both regimes to make the final choice. Next, we follow these steps.

Inducing full evasion When the government wishes to induce full evasion, as before, if $\theta = 1$ then t and f are irrelevant, since no taxes will be collected -as long as conditions are such that full evasion is indeed generated by the IRS. However, if $\theta < 1$, honest rich taxpayers become a source of revenue, and specifying the tax they will pay is thus key. The government's problem now is

$$\mathcal{P}_{FE}^{D,C} \begin{cases} \underset{t,f}{Max} \quad \mu u(y_p) + (1-\mu)\theta u(y_r) + (1-\mu)(1-\theta)[u(y_r - t) + t] \\ subject\ to \\ (t, f) \in \Omega \\ c \geq \frac{(1-\mu)\theta t}{\mu\widehat{\beta}_{NE}^{D,C}} \end{cases}$$

The second constraint ensures that, given c, the IRS will generate full evasion. Let $(\widehat{t}_{FE}^{D,C}, \widehat{f}_{FE}^{D,C})$ be the solution to problem $\mathcal{P}_{FE}^{D,C}$. The following proposition characterizes that solution.[5]

Proposition 1 *Let* $c_3 = \frac{(1-\mu)\theta}{\mu} t^*$. *Then,*

- If $0 \leq c < c_3$, $\widehat{t}_{FE}^{D,C} < t^*$ and $\widehat{f}_{FE}^{D,C} = 0$.

[5] All proofs are relegated to the Appendix.

- If $c \geq c_3$, $\widehat{t}_{FE}^{D,C} = t^*$ and $\widehat{f}_{FE}^{D,C} \in [0, \overline{\overline{f}}]$, with $\overline{\overline{f}} < y_r - t^*$.

For intuition, ignore momentarily the second constraint in $\mathcal{P}_{FE}^{D,C}$. As in the no-delegation case above -and in all cases below where full evasion is generated- the government's objective function differs from the one under full information only in a constant term. Then, it would be optimal to set $t = t^*$. If $c \geq c_3$ this is indeed possible. The government optimally chooses a tax t^*, combined with a fine that is low enough. In selecting a low fine, the government provides incentives for the IRS to generate full evasion.

However, if $c < c_3$, it is not possible to set $t = t^*$, $f > 0$ and still generate full evasion. The audit cost is so low that, under those circumstances, the IRS would maximize revenue by generating no evasion. So as to keep inducing full evasion, the government should choose the highest tax[6] that is compatible with the IRS not auditing when $f = 0$: there is *no punishment for detected misreporters*. Intuitively, the government can reduce taxes and/or lower fines to ensure full evasion: any of these actions reduces incentives to audit in the second stage. But lowering fines is always better for the government, since fines will never be collected under full evasion, whereas honest taxpayers always pay their taxes.

Naturally, the expected welfare generated by inducing full evasion varies with c, as Figure 2 shows. When $c = 0$, the IRS will always audit, so that full evasion is only possible under no taxation: $\widehat{t}_{FE}^{D,C} = 0$. As long as $\theta < 1$, for low values of the audit cost expected welfare will be increasing in c. A rise in the audit cost enables the government to select a higher tax (closer to the full-information level) and still induce the IRS to generate full evasion. Once c reaches c_3, and $t = t^*$ is compatible with full evasion, the expected welfare profile becomes flat. In the Appendix we show that expected welfare is an increasing and (weakly) concave function of c, as in Figure 2.

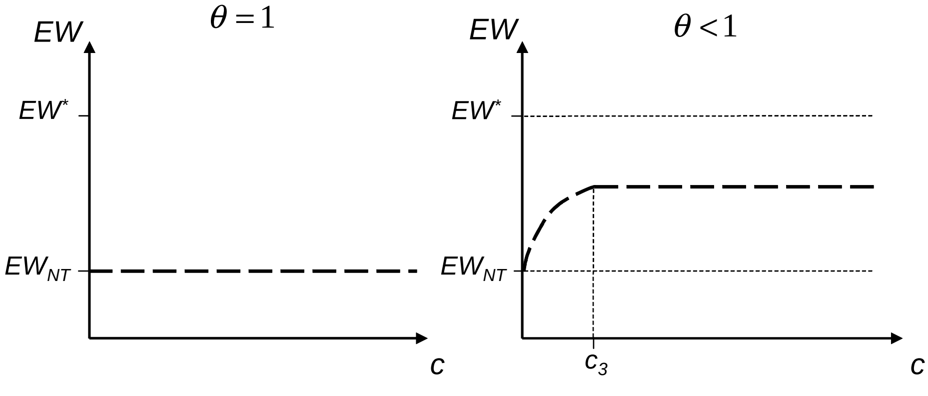

Figure 2: expected welfare in case (D, C) under full evasion

[6]Given that we are considering taxes that are lower than t^*, and since the full-information problem is concave, expected welfare is increasing in t. Hence, the government chooses the highest possible value for t.

Inducing no evasion If the government wants to induce the IRS to generate no evasion, it will choose a pair (t, f) that solves the following problem.

$$\mathcal{P}_{NE}^{D,C} \begin{cases} \underset{t,f}{Max} \quad \mu u(y_p) + (1-\mu)[u(y_r - t) + t] - \mu \widehat{\beta}_{NE}^{D,C} c \\ subject\ to \\ (t, f) \in \Omega \\ \widehat{\beta}_{NE}^{D,C} = \frac{u(y_r) - u(y_r - t)}{u(y_r) - u(y_r - t - f)} \\ c \leq \frac{(1-\mu)\theta t}{\mu \widehat{\beta}_{NE}^{D,C}} \end{cases}$$

By the second constraint, the government anticipates the IRS' second-stage audit policy. The third constraint ensures that no evasion will actually maximize tax revenues for the IRS. As shown in the Appendix, at the optimum the government imposes the maximal penalty $\widehat{f}_{NE}^{D,C} = y_r - t$, to reduce the stake for evasion. The optimal tax $\widehat{t}_{NE}^{D,C}$ is then given by the following first-order condition

$$u'(y_r - \widehat{t}_{NE}^{D,C}) = \frac{1}{1 + \frac{\mu}{1-\mu} \frac{c}{u(y_r)}} < 1,$$

which is equivalent to (1), its analogue under no evasion in case (ND, C). Given that taxes and fines coincide, the lowest audit probability compatible with no evasion maximizes both expected welfare (relevant in case (ND, C)) and expected net tax revenue (relevant here). Then, the IRS' incentives are aligned with the government's, so that delegation is costless in welfare terms. Expected welfare under no evasion, as a function of the audit cost c, coincides with that depicted in Figure 1 above. Finally, as in all cases with no evasion, the presence of honest taxpayers, and the exact value of θ, play no role.

Choosing regimes Naturally, the government will choose a tax law that induces the IRS to generate the regime leading to the highest expected welfare level. This is described in the following proposition.

Proposition 2 *When $c \leq \bar{c}^{D,C} = \bar{c}^{ND,C}$, the government chooses a tax law that induces no evasion. When $c > \bar{c}^{D,C}$, it induces full evasion.*

Figure 3 below shows the expected welfare levels that each possible choice yields.

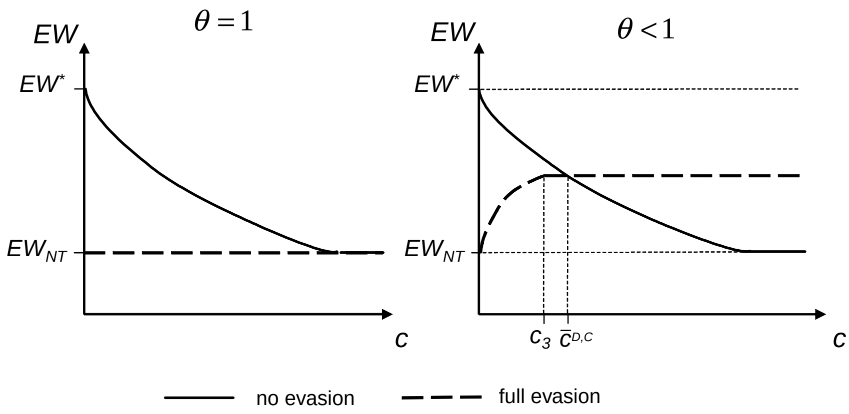

Figure 3: expected welfare in case (D, C)

In the Appendix we show that the expected welfare profile under no evasion intersects the profile with full evasion when the latter is horizontal (i.e., that $\bar{c}^{D,C} > c_3$). When it induces full evasion, the government will do so choosing a tax level t^*. This means that the government induces each regime in case (D, C) in exactly the same situations as in case (ND, C), with the same taxes and penalties –and the expected welfare level reached is naturally the same. In our model, under commitment, whether tax enforcement is delegated or not is inconsequential. Under commitment, then, all expected welfare profiles are (weakly) decreasing in the audit cost c.

The impact of having honest taxpayers is the same as in case (ND, C): honest taxpayers make the full-evasion regime more attractive, and therefore make the government induce full evasion in circumstances where, if $\theta = 1$, it would prefer no evasion.

4 Tax policy under no commitment

We move now to a setting where the tax enforcement authority (the government or the IRS) cannot commit to a specific audit behavior. Hence, once the tax law has been set, that authority and taxpayers interact in a game where the former will choose an equilibrium audit probability. Except in extreme circumstances, then, whenever the authority in charge enforces the tax law, it will tend to do so partially, playing equilibrium mixed strategies in a report-audit game.

As in the previous section, we examine first the case where tax enforcement is carried out by the government itself: no delegation and no commitment, (ND, NC). Then, we move on to the case where the IRS enforces the tax law: delegation and no commitment, (D, NC)

4.1 No delegation to the IRS (ND) and No commitment (NC)

The government now sets the tax law (t, f), but cannot credibly announce an audit probability. Then, given the tax law, it plays a report-audit game with taxpayers. As in both previous cases, the tax law may or may not be enforced, depending on how costly audits are. We solve the two-stage interaction backwards, starting with the report-audit game.

4.1.1 The equilibrium of the report-audit game

Given (t, f), the government's interaction with taxpayers may be described by the following game.

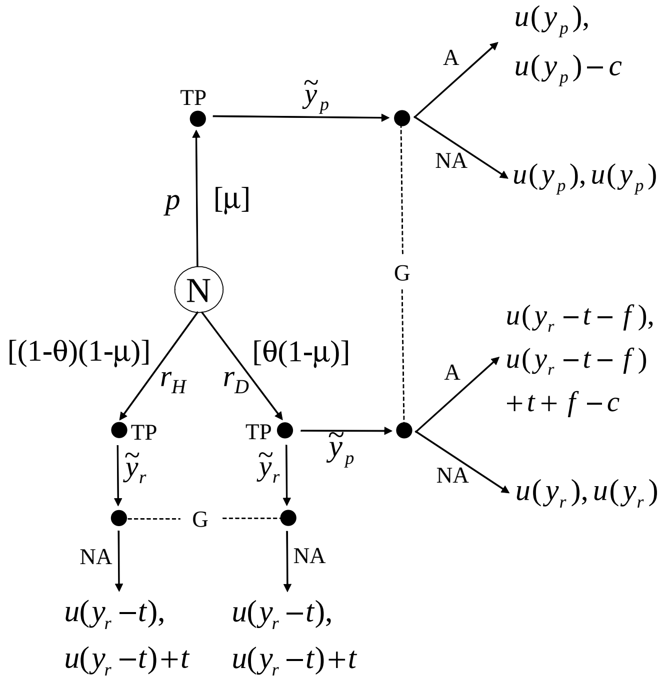

Figure 4: the report-audit game with no delegation

In the game tree, TP (G, N) denotes the taxpayer (respectively, the government, Nature), p denotes a poor individual, r_D (r_H) stands for a dishonest (honest) type for the rich taxpayer, and $\widetilde{y}_i$ is a type-i report. A (NA) indicates that the government audits (does not audit) the taxpayer. The payoffs after each terminal node respectively correspond to the taxpayer and the government. Since the distribution of taxpayers is atomless, none of them takes into account any impact of their choices on public good provision: they just care about their own consumption.[7]

Assume initially that $t > 0$, $f > 0$. The following proposition characterizes the perfect Bayesian equilibria (PBE) of this game.

Proposition 3 *Let* $\widetilde{c} = \frac{(1-\mu)\theta}{(1-\mu)\theta+\mu}(t + f - [u(y_r) - u(y_r - t - f)])$ *denote the value of the audit cost that makes the government indifferent between auditing and not auditing when a dishonest rich taxpayer always misreports.*

[7]Here is where the linearity of taxpayers' utility functions in the public good greatly simplifies the analysis: the effect of each taxpayer's contribution on her own utility through the level of public good provision is independent of other taxpayers' behavior.

- *If* $c > \widetilde{c}$*, there is a unique PBE where the government never audits and a dishonest rich taxpayer always misreports.*
- *If* $c < \widetilde{c}$*, there is a unique PBE in mixed strategies, in which a dishonest rich taxpayer misreports with probability*

$$0 < \widehat{\pi} = \frac{\mu c}{(1-\mu)\theta(t+f-c-[u(y_r)-u(y_r-t-f)])} < 1, \tag{2}$$

and the government audits each announcement $\widetilde{y}_i = y_p$ *with probability*

$$0 < \widehat{\beta} = \frac{u(y_r)-u(y_r-t)}{u(y_r)-u(y_r-t-f)} < 1. \tag{3}$$

- *If* $c = \widetilde{c}$*, there is a continuum of PBEs in mixed strategies, in which the government audits each announcement* $\widetilde{y}_i = y_p$ *with a probability* $\beta \in [0, \widehat{\beta}]$*, and a dishonest rich taxpayer always misreports.*[8]

Leaving aside the borderline case where $c = \widetilde{c}$, Proposition XXX asserts that one of two situations arises, depending on the audit cost. If $c > \widetilde{c}$, we have a new *full evasion* regime: the government never audits and dishonest taxpayers always misreport. If $c < \widetilde{c}$, however, there is *partial evasion* in a mixed-strategy equilibrium of the report-audit game. The government audits any low-income report with probability $\widehat{\beta}$ –which leaves dishonest rich taxpayers indifferent between misreporting and reporting truthfully. Dishonest taxpayers misreport with probability $\widehat{\pi}$, which makes the government –given the updated probabilities it attaches to the taxpayer being rich or poor– indifferent between auditing and not doing so.[9]

4.1.2 The optimal tax law

We move back now to the first stage, where the government sets the tax law anticipating the resulting equilibrium in the report-audit game. Just as in the (D, C) case above, by choosing (t, f) the government decides which regime to induce in the second stage. In this context, this is due to the fact that the tax law determines the equilibrium that follows in the report-audit game. Furthermore, as before, the exact values in (t, f) alter the critical audit cost $\widetilde{c}$ that separates full from partial evasion. Next, we describe the highest expected welfare levels attainable under full and partial evasion, and then compare them to characterize the government's regime choice.

[8] In all these equilibria, the government is indifferent between any audit probabilities, and the behavior of dishonest taxpayers is the same as if $\beta = 0$.

[9] If $f = 0$, misreporting is a weakly dominant strategy for dishonest rich taxpayers. Hence, they will always misreport, except perhaps when the government audits low-income reports with probability one. Then, there is no substantial change in the results stated in Proposition XXX.

We have also assumed that $t > 0$. If $t = 0$, in all equilibria there is no tax collection from dishonest rich taxpayers. Hence, we can assume without loss of generality that there is no misreporting and, therefore, no audits.

Inducing full evasion As before, under full evasion honest rich taxpayers are the only source of revenue. The government's problem is

$$\mathcal{P}_{FE}^{ND,NC}\begin{cases} \underset{t,f}{Max} \quad \mu u(y_p)+(1-\mu)\theta u(y_r)+(1-\mu)(1-\theta)[u(y_r-t)+t] \\ subject\ to \\ (t,f)\in\Omega \\ c\geq \frac{(1-\mu)\theta}{(1-\mu)\theta+\mu}(t+f-[u(y_r)-u(y_r-t-f)]) \end{cases}$$

The second constraint ensures that, given c, the government actually induces full evasion.[10] Let $(\widehat{t}_{FE}^{ND,NC}, \widehat{f}_{FE}^{ND,NC})$ be the solution to problem $\mathcal{P}_{FE}^{ND,NC}$. The following proposition completely characterizes it.

Proposition 4 *Let* $c_4=\frac{(1-\mu)\theta}{(1-\mu)\theta+\mu}\left(t^*-[u(y_r)-u(y_r-t^*)]\right)$. *Then,*

- If $0\leq c<c_4$, $\widehat{t}_{FE}^{ND,NC}<t^*$ and $\widehat{f}_{FE}^{ND,NC}=0$.
- If $c\geq c_4$, $\widehat{t}_{FE}^{ND,NC}=t^*$ and $\widehat{f}_{FE}^{ND,NC}\in[0,\overline{f}]$, with $\overline{f}<y_r-t^*$.

As in previous cases when inducing full evasion , the government would prefer to choose $t=t^*$. Here, doing so is only feasible when $c\geq c_4$. If this inequality holds, t^* is selected together with a fine that ensures second-stage equilibrium full evasion. Lacking commitment capability to its audit behavior, the government chooses a low fine so as not to have incentives to audit when all dishonest rich taxpayers misreport.

If $c<c_4$, t^* is too high for the government not to want to audit low-income reports when it expects full evasion, even if $f=0$. As before, to stay within full evasion, it is optimal to select *no punishment for detected misreporters* and the highest tax[11] compatible with having no incentives to audit.

In addition, we prove in the Appendix that the optimal tax $\widehat{t}_{FE}^{ND,NC}$ satisfies $\lim_{c\to 0}\widehat{t}_{FE}^{ND,NC}=0$, and increases with c. Clearly, a rise in c enables the government to set t at a higher level and still have no incentive to audit in the second stage. When c vanishes, however, audits will follow even from arbitrarily small taxes, so only $\widehat{t}_{FE}^{ND,NC}=0$ remains compatible with full evasion.

Considering the optimal taxes in each possible case, it follows that the highest welfare level attainable under full evasion is

$$\begin{cases} \mu u(y_p)+(1-\mu)\theta u(y_r)+(1-\mu)(1-\theta)\left[u(y_r-\widehat{t}_{FE}^{ND,NC})+\widehat{t}_{FE}^{ND,NC}\right] & \text{if } c<c_4 \\ \mu u(y_p)+(1-\mu)\theta u(y_r)+(1-\mu)(1-\theta)[u(y_r-t^*)+t^*] & \text{if } c\geq c_4 \end{cases}$$

[10] If this constraint holds with equality, as we mentioned above, there is a continuum of PBEs. To avoid technical complications, we assume here that, under such circumstances, the equilibrium where the government never audits is selected.

[11] The reasoning in footnote ??? (EL t MÁS ALTO COMPATIBLE CON FULL EVASION) applies here as well.

If $0 \leq c < c_4$, we show in the Appendix that expected welfare increases linearly with c. This is because a higher audit cost makes higher taxes feasible under full evasion. When $c \geq c_4$, though, expected welfare is constant in c: once the optimal full-information tax can be set within full evasion, further increases in the audit cost have no welfare effects. Figure 5 below shows expected welfare as a function of the audit cost when the government chooses to induce regime full evasion.

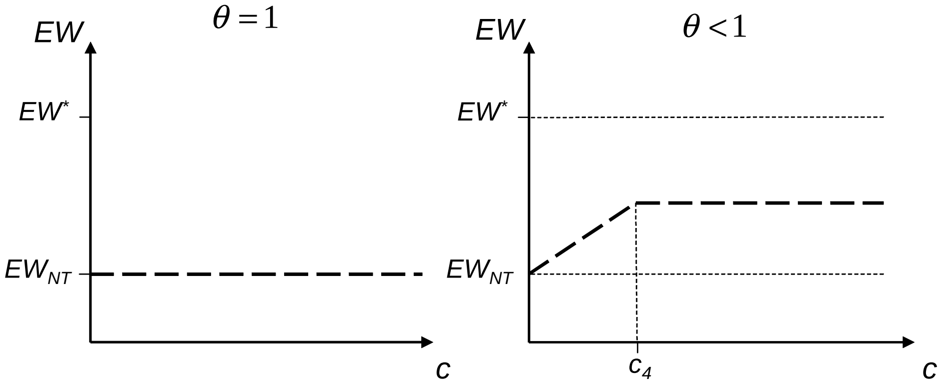

Figure 5: expected welfare in case (ND, NC) under full evasion

Naturally, when all rich taxpayers are dishonest, full evasion is equivalent to no taxation in welfare terms. But as long as $\theta < 1$, expected welfare is strictly higher than $\mathbb{E}W_{NT}$, except when $c = 0$.

Inducing partial evasion If the government wants to induce second-stage partial evasion, it must choose a pair (t, f) such that $c \leq \widetilde{c}$. Expected welfare is then

$$\begin{aligned} \mathbb{E}W = \; & \widehat{\beta}\widehat{\nu}\left[\tfrac{\mu}{\widehat{\nu}}u(y_p) + \tfrac{(1-\mu)\theta\widehat{\pi}}{\widehat{\nu}}u(y_r - t - f)\right] + (1-\widehat{\beta})\widehat{\nu}\left[\tfrac{\mu}{\widehat{\nu}}u(y_p) + \tfrac{(1-\mu)\theta\widehat{\pi}}{\widehat{\nu}}u(y_r)\right] \\ & + (1-\widehat{\nu})u(y_r - t) + g, \end{aligned}$$

where $\widehat{\nu} = \mu + (1-\mu)\theta\widehat{\pi}$ is the probability that any taxpayer i reports $\widetilde{y}_i = y_p$, and

$$g = \widehat{\beta}\widehat{\nu}\left[\frac{(1-\mu)\theta\widehat{\pi}}{\widehat{\nu}}(t + f - c) - \frac{\mu}{\widehat{\nu}}c\right] + (1-\widehat{\nu})t$$

is the government's budget constraint. Replacing $\widehat{\pi}$ and $\widehat{\beta}$ according to (2) and (3) yields,

$$\mathbb{E}W = \; \mu u(y_p) + (1-\mu)u(y_r - t) + (1-\mu)[t + \theta\widehat{\pi}(u(y_r) - u(y_r - t) - t)]. \tag{4}$$

Before moving to the government's problem, let us point out two interesting features in (4). First, the fraction of dishonest rich taxpayers θ has no effect on expected welfare[12]

[12] Despite the differences between the two models, this comparative statics result is the same as in Graetz et al. (1986).

–as long, of course, as it remains positive. Note that expected tax collection –and therefore expected public-good provision– is independent of θ. At the mixed-strategy equilibrium of the report-audit game, the government will be indifferent between auditing a low-income report and not doing so. When auditing, the government pays a cost c and faces, with probability $\mu/\widehat{\nu}$ $(1-\mu/\widehat{\nu})$ a poor (respectively, dishonest rich) taxpayer. When the taxpayer is poor, the government collects nothing. If the taxpayer is rich and dishonest, the government generates a welfare change (net of the audit cost) given by

$$t + f - c - [u(y_r) - u(y_r - t - f)],$$

which does not depend on θ. As not auditing always pays zero, when θ rises the government can only remain indifferent if $\mu/\widehat{\nu}$ stays constant. This, in turn, implies that $\theta\widehat{\pi}$ cannot vary. Therefore, in equilibrium, the increase in θ is exactly offset by the decrease in $\widehat{\pi}$, making aggregate tax evasion –and tax collection– constant.

In addition, the fraction of expected welfare that derives from taxpayers' private consumption is independent of θ as well. Recall that $\widehat{\beta}$ is the audit probability that makes a dishonest rich taxpayer indifferent between reporting truthfully and misreporting. So any such taxpayer's equilibrium expected utility will be $u(y_r - t)$, which is the same as the utility for an honest rich taxpayer. Hence, from the point of view of a utilitarian government, expected utility before observing taxpayers' reports is $\mu u(y_p) + (1-\mu)u(y_r - t)$.

A second interesting feature is that neither the audit cost c nor the fine f appear in (4) except in the very last term. Therefore, as stated by Landsberger et al. (2000), enforcement only contributes to tax collection indirectly, through dishonest rich taxpayers' reporting strategies.

To find the optimal tax law inducing partial evasion, the government solves the following problem

$$\mathcal{P}_{PE}^{ND,NC} \begin{cases} \underset{t,f}{Max} \quad \mu u(y_p) + (1-\mu)[u(y_r - t) + t + \theta\widehat{\pi}(u(y_r) - u(y_r - t) - t)] \\ subject\ to \\ (t, f) \in \Omega \\ \widehat{\pi} = \dfrac{\mu c}{(1-\mu)\theta(t + f - c - [u(y_r) - u(y_r - t - f)])} \\ c \leq \frac{(1-\mu)\theta}{(1-\mu)\theta + \mu}(t + f - [u(y_r) - u(y_r - t - f)]) \end{cases}$$

Again, the last constraint ensures that the government is choosing a tax law that actually induces partial evasion.[13] Let the solution to this problem be $(\widehat{t}_{PE}^{ND,NC}, \widehat{f}_{PE}^{ND,NC})$. Even though the objective function is not globally concave, in the Appendix we prove that the

[13] If the last constraint holds with equality there is a continuum of PBEs in the report-audit game. To avoid technical complications, we assume now that the equilibrium is selected where the government audits with probability $\widehat{\beta}$. As we will point out below, this assumption –just as the previous one under full evasion– could be modified without altering our results.

second-order conditions are satisfied, and thus the first-order, necessary conditions are also sufficient to characterize a maximum. Then, we can apply the Implicit Function Theorem and find how the optimal tax law evolves as the audit cost c changes.

Before specifically describing the optimal tax profile, let us point out a few noteworthy properties of the solution to $\mathcal{P}_{PE}^{ND,NC}$ that are proved in the Appendix. Recall that c_4, defined above, is the minimum cost such that setting a tax t^* and no fine generates full evasion in the second stage of the game. When $c < c_4$, the last constraint in $\mathcal{P}_{PE}^{ND,NC}$ will not bind. In that case, under partial evasion, the government seeks to cut down individual tax evasion by reducing its stake. This is standard in the optimal tax-enforcement literature, which assumes full commitment. The novelty here is that the ex-post limited-liability constraint does not bind either: $\widehat{t}_{PE}^{ND,NC} + \widehat{f}_{PE}^{ND,NC} = t^* < y_r$. This result is independent of θ, the fraction of dishonest taxpayers, which is intuitive: as we mentioned above, this fraction does not affect equilibrium expected welfare. The fact that fines are not set at their maximum legal value is related to government's objective function. It is the government itself −which cares about all taxpayers' welfare, including evaders'− that performs audits ex post, without being able to commit to their frequency.

Why does $\widehat{t}_{PE}^{ND,NC} + \widehat{f}_{PE}^{ND,NC}$ amount to the optimal full-information tax? At the second stage, when the government decides on enforcement, the net benefit from auditing a low-income report is, as we mentioned above, $t + f - c - [u(y_r) - u(y_r - t - f)]$. In the first stage, the government optimally reduces the stake for (individual) tax evasion by maximizing this expression. Note that this net benefit does not depend on the exact values of t and f, but only on $t + f$, and it does so in the exact same way as the government's objective function under full information depends on t. Therefore, the government optimally sets $\widehat{t}_{PE}^{ND,NC} + \widehat{f}_{PE}^{ND,NC} = t^*$.

The following proposition describes the optimal tax profile.

Proposition 5 *Again, let* $c_4 = \frac{(1-\mu)\theta}{(1-\mu)\theta+\mu}\left(t^* - [u(y_r) - u(y_r - t^*)]\right)$. *When* $0 \leq c \leq c_4$, $\widehat{t}_{PE}^{ND,NC} = t^*$ *and* $\widehat{f}_{PE}^{ND,NC} = 0$. *When* $c > c_4$, *no tax law can induce partial evasion.*

The reason why $\widehat{t}_{PE}^{ND,NC}$ equals the full-information tax when $c \leq c_4$ may be seen by examining the government's objective function. The marginal effect of t on the government's welfare while keeping $t + f = t^*$ is $(1 - \mu\ )(1 - \theta\widehat{\pi})[1 - u'(y_r - t)]$, where $\theta\widehat{\pi}$ is constant. The last expression equals zero when $t = t^*$, and thus the optimal fine is zero. Intuitively, dishonest rich taxpayers' reporting behavior only depends on $t + f$. Hence, when $t + f$ is held fixed, the exact value of t only determines the welfare generated by taxing honest rich taxpayers. Then, the solution is the same as in the full-information case.

Taking into account the optimal tax and fine levels, expected welfare under partial evasion is

$$\mu u(y_p) + (1 - \mu)u(y_r - t^*) + (1 - \mu)(1 - \theta\widehat{\pi})t^* \quad \text{if } c \leq c_4$$

As long as $c \leq c_4$, it is straightforward to show that expected welfare is a decreasing and concave function of c, as in Figure 6.

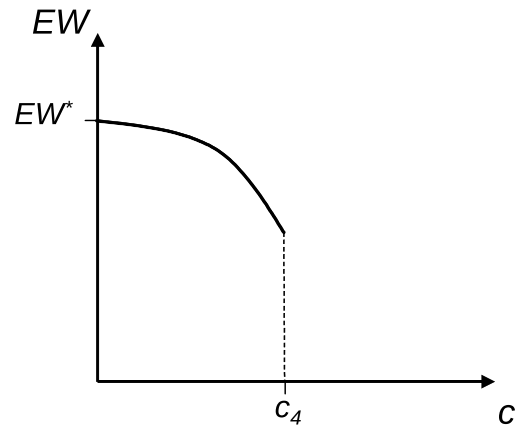

Figure 6: expected welfare in case (ND, NC) under partial evasion

Choosing regimes The government will choose a tax law that induces the regime providing the highest expected welfare.

Proposition 6 *When $c \leq \bar{c}^{ND,NC} = c_4$, the government chooses a tax law that generates partial evasion. When $c > \bar{c}^{ND,NC}$, only the full evasion regime can emerge.*

Figure 7 below combines Figures 5 and 6. Again, the maximum feasible welfare level is represented by the upper envelope of welfare profiles under full and partial evasion.

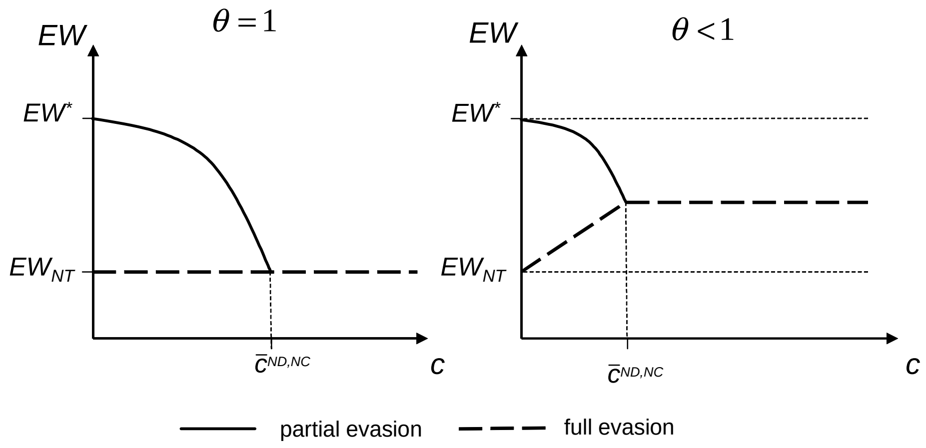

Figure 7: expected welfare in case (ND, NC)

Note first that as c grows to reach c_4, partial evasion becomes impossible (evasion becomes full) and, at the same time, under full evasion expected welfare reaches its highest possible value –since full information taxes are possible in that regime. For that audit cost, in both regimes all rich dishonest taxpayers evade their taxes, and honest rich taxpayers pay t^*. Both regimes yield the same expected welfare. When $c < \bar{c}^{ND,NC} = c_4$, since expected welfare decreases (weakly increases) with c under partial (full) evasion, the government chooses to induce partial evasion. For higher audit costs only full evasion can emerge and the government never audits.

As usual, the presence of honest taxpayers makes full evasion more attractive to the government. As θ falls, expected welfare under full evasion, which is also a lower bound for expected welfare under partial evasion, grows and full evasion becomes more likely.

Maxima expected welfare, given by the upper envelope of the expected welfare profiles in Figure 7, is weakly decreasing in c. So far, this has happened in all three cases we have coverted. However, as we shall see in the following case, this is not always true.

4.2 Delegation to the IRS (D) and No commitment (NC)

We move now to our last case. Tax enforcement is delegated to the IRS, and the latter lacks commitent capability to its audit policy. As in case (ND, NC), the government's tax law choice in the first stage can induce partial or full evasion in the second stage. We proceed, as in all previous cases, solving the problem backwards, starting with the second-stage report-audit game.

4.2.1 The equilibrium of the report-audit game

Under a given tax law (t, f), previously set by the government, taxpayers and the IRS play the following version of the report-audit game. Naturally, the payoffs after each terminal node are respectively those of the taxpayer and the IRS.

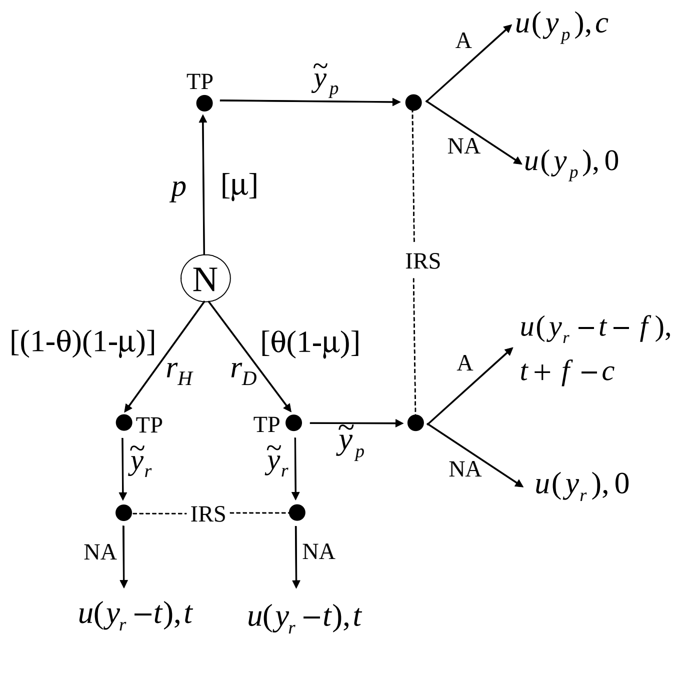

Figure 8: the report-audit game under delegation

Take first the case where $t > 0$, $f > 0$.[14] The PBEs of this game are characterized in the following proposition, proved in Graetz et al. (1986). Just as under no delegation, one of two regimes will arise, depending on the audit cost. For high values of c, *full evasion* obtains: the IRS never audits and dishonest taxpayers always misreport. If the audit cost is low enough, *partial evasion* follows, with a mixed-strategy equilibrium.

Proposition 7 *Let* $c' = \frac{(1-\mu)\theta(t+f)}{(1-\mu)\theta+\mu}$ *denote the value of the audit cost that makes the tax administration indifferent between auditing and not auditing when a dishonest rich taxpayer always misreports.*

- *If* $c > c'$*, there is a unique PBE where the tax administration never audits and a dishonest rich taxpayer always misreports.*
- *If* $c < c'$*, there is a unique PBE in mixed strategies, in which a dishonest rich taxpayer misreports with probability*

$$0 < \widehat{\pi} = \frac{\mu c}{(1-\mu)\theta(t+f-c)} < 1, \tag{5}$$

and the tax administration audits each announcement $\widetilde{y}_i = y_p$ *with probability*

$$0 < \widehat{\beta} = \frac{u(y_r) - u(y_r - t)}{u(y_r) - u(y_r - t - f)} < 1. \tag{6}$$

- *If* $c = c'$*, there is a continuum of PBE in mixed strategies, in which the tax administration audits each announcement* $\widetilde{y}_i = y_p$ *with a probability* $\beta \in [0, \widehat{\beta}]$*, and a dishonest rich taxpayer always misreports.*

4.2.2 The optimal tax law

Moving back to the first stage of the game, we examine now the government's choice of a tax law (t, f). Since, as usual by now, two regimes may emerge, we find the optimal way of inducing each regime and then make the government's final regime comparison.

Inducing full evasion Since only honest rich taxpayers pay their taxes, the government's problem is

[14] Just as under no delegation, if $f = 0$, misreporting is a weakly dominant strategy for dishonest rich taxpayers. Thus, we may assume that they will always misreport. See footnote ??? (ACTUALIZAR EL NÚMERO) for the case where $t = 0$.

$$
\mathcal{P}_{FE}^{D,NC}\begin{cases} \underset{t,f}{Max} \quad \mu u(y_p)+(1-\mu)\theta u(y_r)+(1-\mu)(1-\theta)[u(y_r-t)+t] \\ subject\ to \\ (t,f)\in\Omega \\ c\geq \frac{(1-\mu)\theta(t+f)}{(1-\mu)\theta+\mu} \end{cases}
$$

The only difference with problem $\mathcal{P}_{FE}^{ND,NC}$ is, of course, in the second constraint: the tax law must be such that, now, the IRS and taxpayers play a full-evasion equilibrium in the report-audit game. Let $(\widehat{t}_{FE}^{D,NC}, \widehat{f}_{FE}^{D,NC})$ be the solution to problem $\mathcal{P}_{FE}^{D,NC}$.[15] It is characterized as follows.[16]

Proposition 8 *Let* $c_5 = \frac{(1-\mu)\theta}{(1-\mu)\theta+\mu}t^*$. *Then,*

- If $0 < c < c_5$, $\widehat{t}_{FE}^{D,NC} = \frac{(1-\mu)\theta+\mu}{(1-\mu)\theta}c$ and $\widehat{f}_{FE}^{D,NC} = 0$.
- If $c \geq c_5$, $\widehat{t}_{FE}^{D,NC} = t^*$ and $\widehat{f}_{FE}^{D,NC} \in [0, \overline{\overline{\overline{f}}}]$.

As in the previous case, the government would like to choose $t = t^*$ under full evasion, which is only possible if $c \geq c_5$. In that case, the fine has to be low enough so that equilibrium play in the report-audit game implies full evasion. The maximum chosen fine, $\overline{\overline{\overline{f}}}$, is once again below the highest value compatible with limited liability, $y_r - t^*$. If $c < c_5$, the government then selects the highest tax[17] compatible with second-stage full evasion, and misreporters are not punished. Once more, since taxes are paid by honest taxpayers and fines are not collected, lowering the latter is always preferred by the government.[18]

Maximal welfare under full evasion is now

$$
\begin{cases} \mu u(y_p)+(1-\mu)\theta u(y_r)+(1-\mu)(1-\theta)\left[u(y_r-\widehat{t}_{FE}^{D,NC})+\widehat{t}_{FE}^{D,NC}\right] & \text{if } c<c_5 \\ \mu u(y_p)+(1-\mu)\theta u(y_r)+(1-\mu)(1-\theta)[u(y_r-t^*)+t^*] & \text{if } c\geq c_5 \end{cases}
$$

If $0 < c < c_5$, it is straightforward to show that expected welfare is an increasing and concave function of c. Again, when auditing is more costly, higher taxes become feasible under full evasion, always below the optimal, full-information tax t^*. When $c \geq c_5$, the government can select t^* and expected welfare is constant in c. Figure 9 shows the expected welfare profiles with and without honest taxpayers.

[15] As in Section 4.2.1???? (ACTUALIZAR), we assume here that, if the last constraint holds with equality, the equilibrium where the government never audits is selected.

[16] The proof of this proposition is omitted since it replicates the proof of Proposition 2??? (ACTUALIZAR).

[17] The reasoning in footnote 9???? applies here again. (ACTUALIZAR, AUNQUE CREO QUE LO SACARÍA).

[18] As in **ACTUALIZAR SECCIÓN**, the optimal tax $\widehat{t}_{FE}^{D,NC}$ is increasing in c and satisfies $\lim_{c\to 0} \widehat{t}_{FE}^{D,NC} = 0$.

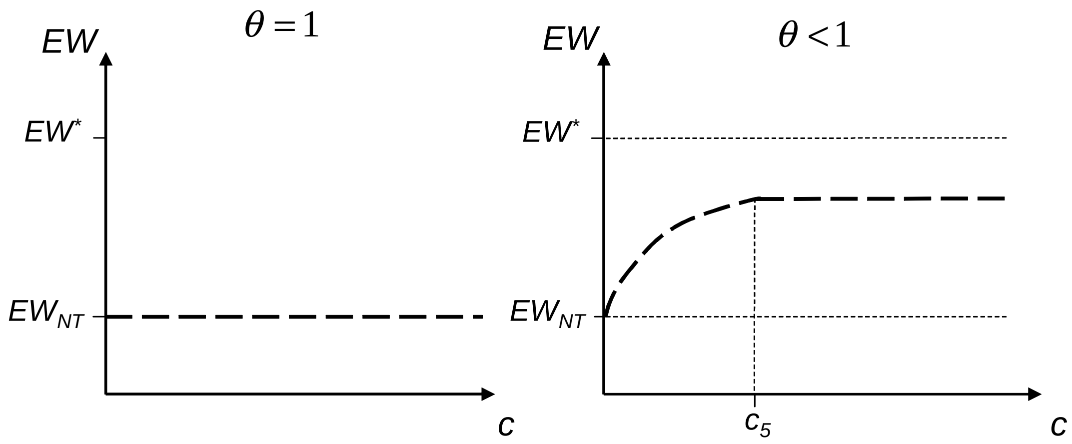

Figure 9: expected welfare in case (D, NC) under full evasion

When $\theta < 1$, the expected welfare profile when inducing full evasion varies with c in the same way as it does in case (D, C) (see Figure 2), although the two profiles are different.

Inducing partial evasion If the government wants to induce second-stage partial evasion, we must then have $c \leq c' = \frac{(1-\mu)\theta(t+f)}{(1-\mu)\theta+\mu}$. A necessary condition for this to be possible is then that $c < c_6 \equiv \frac{(1-\mu)\theta y_r}{(1-\mu)\theta+\mu}$.[19] We now have

$$\begin{aligned}\mathbb{E}W = \; & \widehat{\beta}\widehat{\nu}\left[\tfrac{\mu}{\widehat{\nu}}u(y_p) + \tfrac{(1-\mu)\theta\widehat{\pi}}{\widehat{\nu}}u(y_r - t - f)\right] + (1-\widehat{\beta})\widehat{\nu}\left[\tfrac{\mu}{\widehat{\nu}}u(y_p) + \tfrac{(1-\mu)\theta\widehat{\pi}}{\widehat{\nu}}u(y_r)\right] \\ & + (1-\widehat{\nu})u(y_r - t) + g,\end{aligned}$$

and

$$g = \widehat{\beta}\widehat{\nu}\left[\frac{(1-\mu)\theta\widehat{\pi}}{\widehat{\nu}}(t+f-c) - \frac{\mu}{\widehat{\nu}}c\right] + (1-\widehat{\nu})t$$

is the government's budget constraint. Replacing $\widehat{\pi}$ and $\widehat{\beta}$ according to (5) and (6),

$$\mathbb{E}W = \mu u(y_p) + (1-\mu)u(y_r - t) + (1-\mu)\underbrace{(1 - \frac{\mu c}{(1-\mu)(t+f-c)})}_{1-\theta\widehat{\pi}}t \tag{7}$$

where the last term follows from the government's budget constraint. Public good provision is $(1-\mu)(1-\theta\widehat{\pi})t$, the amount of taxes collected upon those rich who do not misreport.[20]

[19] If this were not the case, all feasible pairs $(t, f) \in \Omega$ would induce full evasion.

[20] $\widehat{\pi}$ is the probability that makes the tax administration indifferent between auditing and not auditing an announcement $\widetilde{y}_i = y_p$. Then, as the government's equilibrium expected tax collection will be, after observing $\widetilde{y}_i = y_p$, equal to 0, expected tax collection *before* observing taxpayers' report is $[\mu + (1-\mu)\theta\widehat{\pi}] .0 + (1-\mu)(1-\theta\widehat{\pi})t$.

Some features that we pointed out in the (ND, NC) case survive once delegation is introduced. Particularly, expected welfare is constant in θ, provided it is smaller than 1. The intuition for this is the exact analogue of the one we described under no delegation. Given that a mixed-strategy equilibrium is played, expected net tax collection has to be zero, since the tax administration will be indifferent between auditing a low-income report and not doing so. When auditing, the administration pays a cost c and faces, with probability $\mu/\widehat{\nu}$ $(1-\mu/\widehat{\nu})$ a poor (respectively, dishonest rich) taxpayer. Correspondingly, dishonest rich taxpayers will be indifferent between reporting the truth or lying, so that their expected utility is also constant in θ. In spite of the fact that expected welfare is independent of θ, this parameter does have an impact on the choice of the optimal tax law under partial evasion, as we will see below.

It is also true that c and f only affect the very last term in (7). Even under delegation, tax-law enforcement only contributes to tax collection via dishonest rich taxpayers' reporting strategies.

The government's problem is now[21]

$$\mathcal{P}_{PE}^{D,NC} \begin{cases} \underset{t,f}{Max} \quad \mu u(y_p) + (1-\mu)u(y_r - t) + (1-\mu)(1-\theta\widehat{\pi})t \\ subject\ to \\ (t, f) \in \Omega \\ \widehat{\pi} = \dfrac{\mu c}{(1-\mu)\theta(t+f-c)} \\ c \leq \frac{(1-\mu)\theta(t+f)}{(1-\mu)\theta+\mu} \end{cases}$$

Let the solution to this problem be $(\widehat{t}_{PE}^{D,NC}, \widehat{f}_{PE}^{D,NC})$. In the Appendix we prove that, as in case (ND, NC), although the objective function of this problem is not globally concave, the second-order conditions hold, and thus the first-order conditions are also sufficient to characterize the maximum. We then carry out the usual comparative statics exercise to determine how the optimal tax law varies with c.

Note first that, unlike in case (ND, NC), it has to be true at the optimum that the ex-post limited liability constraint holds with equality: $\widehat{t}_{PE}^{D,NC} + \widehat{f}_{PE}^{D,NC} = y_r$. The fine is set at the maximal legal limit so as to lower tax evasion by reducing its stake. This is usual in the optimal tax-enforcement literature (which assumes full commitment, under both delegation and no delegation), and has two important consequences. First, the threshold c' that separates full and partial evasion is set at its highest possible level: $c' = c_6$. Therefore, the last constraint in $\mathcal{P}_{PE}^{D,NC}$ is automatically satisfied.

Second, even though the government does not choose the audit strategy, penalizing evaders maximally also (indirectly) sets the level of (individual and aggregate) tax evasion at the second-stage equilibrium. The aggregate level of evasion is $(1-\mu)\theta\widehat{\pi} = \frac{\mu c}{y_r - c}$,

[21] As in case (ND, NC), we assume that, if the last constraint holds with equality, the equilibrium is selected where the tax administration audits with probability $\widehat{\beta}$.

which only depends on the highest possible income level and on the audit cost. In particular, it is increasing in c. The exact value of the optimal tax $\widehat{t}_{PE}^{D,NC}$ does not impact equilibrium aggregate tax evasion.

The next proposition completely characterizes the tax profile.

Proposition 9 *Let* $\widehat{c} = \frac{(1-\mu)[1-u'(y_r)]y_r}{1-(1-\mu)u'(y_r)}$ *and* $\widehat{\theta} = 1 - u'(y_r)$. *There are two possible cases.*

- *Case A:* $\theta \leq \widehat{\theta} \Leftrightarrow c_6 \leq \widehat{c}$.

  $\widehat{t}_{PE}^{D,NC}$ *is such that* $0 < \widehat{t}_{PE}^{D,NC} < t^*$, $\frac{\partial \widehat{t}_{PE}^{D,NC}}{\partial c} < 0$, $\lim_{c \to 0} \widehat{t}_{PE}^{D,NC} = t^*$, *and* $\lim_{c \to c_6} \widehat{t}_{PE}^{D,NC} > 0$.

- *Case B:* $\theta > \widehat{\theta} \Leftrightarrow c_6 > \widehat{c}$.

  When $c < \widehat{c}$, $\widehat{t}_{PE}^{D,NC}$ *satisfies* $0 < \widehat{t}_{PE}^{D,NC} < t^*$, $\frac{\partial \widehat{t}_{PE}^{D,NC}}{\partial c} < 0$, $\lim_{c \to 0} \widehat{t}_{PE}^{D,NC} = t^*$, *and* $\lim_{c \to \widehat{c}} \widehat{t}_{PE}^{D,NC} = 0$. *When* $c \geq \widehat{c}$, *the optimal tax is* $\widehat{t}_{PE}^{D,NC} = 0$.

In both cases, the optimal tax $\widehat{t}_{PE}^{D,NC}$ is lower than the optimal full-information tax t^* and it weakly decreases with c. The intuition is as follows. Clearly, when $c = 0$, there will be no misreporting and the government will select the full-information tax t^*. Assume for convenience that, in this case, $f = 0$. When c becomes positive, though, equilibrium play in the report-audit game shifts to mixed strategies. As c grows, as noted above, so does $\widehat{\pi}$ and thus tax collection falls. This means that the marginal contribution of the tax level t to welfare through public good provision falls. In order to restore first-stage optimality, the marginal cost of the tax level in terms of the rich taxpayer's private consumption has to go down as well. The government's optimal reaction is then to lower t (while raising f, so as to keep their sum equal to y_r). Hence the decrease in welfare due to the rise in the audit cost is dampened with a tax below the full-information one. The same argument explains why the tax distortion grows with c.[22]

At the same time, whenever partial evasion is induced, taxes are lower with than without delegation: $\widehat{t}_{PE}^{D,NC} < \widehat{t}_{PE}^{ND,NC}$. NO ME TERMINA DE CERRAR UNA INTUICIÓN PARA ESTO...VER!

When θ, the probability that a given rich taxpayer is dishonest, is small, case A obtains: $\widehat{t}_{PE}^{D,NC}$ falls with c until $c = c_6$. For higher audit costs, partial evasion is impossible. As c_6 grows with θ, if θ becomes large enough we will eventually have $c_6 > \widehat{c}$.[23] This happens in case B. There, when $c \in [\widehat{c}, c_6]$, the marginal contribution of t to welfare through public good provision is lower than its cost in terms of private consumption even at $t = 0$. Audits are just too costly for the government to even want to induce the tax administration to play an equilibrium in mixed strategies in the report-audit game. Thus, it is optimal for the government to generate a no-taxation regime.

[22] The distortion is measured by the difference $t^* - \widehat{t}_{PE}^{D,NC}$.

[23] In the Appendix, we show that $\widehat{c} = c_6$ iff $\theta = 1 - u'(y_r)$.

Taking into account the optimal tax and fine levels, the expected welfare profile is

$$\mathbb{E}W = \begin{cases} \mu u(y_p) + (1-\mu)u(y_r - \widehat{t}_{PE}^{D,NC}) + (1-\mu)(1-\theta\widehat{\pi})\widehat{t}_{PE}^{D,NC} & \text{if } c \leq \min\{c_6, \widehat{c}\} \\ \mu u(y_p) + (1-\mu)u(y_r) & \text{if } c > \min\{c_6, \widehat{c}\} \end{cases}$$

When $c \leq \min\{c_6, \widehat{c}\}$, it is straightforward to prove, using an envelope argument, that expected welfare decreases with c. When $c > \widehat{c}$, it is constant and equal to the welfare level attained without taxation, $\mathbb{E}W_{NT}$. The welfare profiles for cases A and B in Proposition 9 are depicted in Figure 10 below.

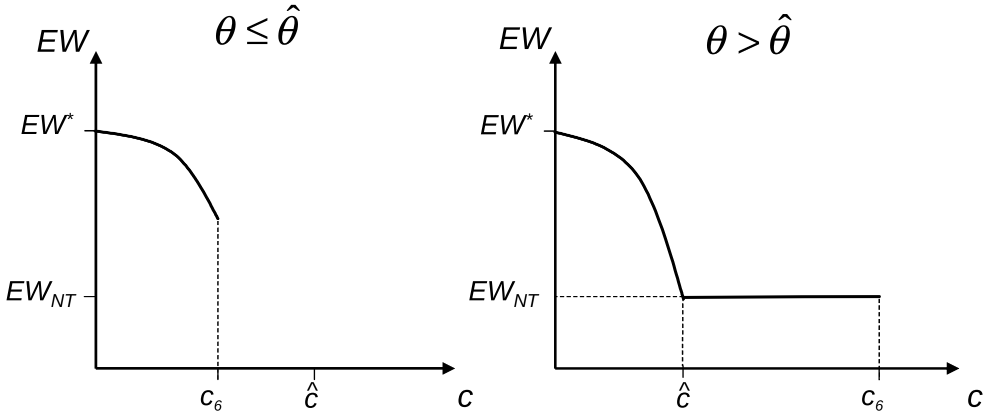

Figure 10: expected welfare in case (D, NC) under partial evasion

Choosing regimes If $\theta = 1$, the government's optimal choice is simple: it will induce partial evasion whenever it is available and compatible with positive taxation. Otherwise, the no-taxation regime will follow. Then, maximum expected welfare will be as in the right-hand panel in Figure 10 –except for the fact that $\mathbb{E}W_{NT}$ is still the expected welfare level reached even if $c > c_6$. As in all previous cases, expected welfare is weakly decreasing in c.

The government's choice when $\theta < 1$ is characterized in the following proposition .

Proposition 10 *Assume $\theta < 1$. Then, there exist an audit cost $\overline{c}^{D,NC}$ such that the government induces partial evasion when $c \leq \overline{c}^{D,NC}$, and it induces full evasion if $c > \overline{c}^{D,NC}$. Furthermore, if $\frac{t^*}{y_r} \leq \frac{u(y_r - t^*) + t^* - u(y_r)}{t^*}$, $c_5 \leq \overline{c}^{D,NC}$, so that maximum expected welfare is weakly decreasing in c. Otherwise, $c_5 > \overline{c}^{D,NC}$ and maximum expected welfare is nonmonotonic in c.*

Figure 11 below shows both cases described in Proposition 10.[24]

[24] In the proof of Proposition 10, we show that $\overline{c}^{D,NC} < c_6$. Then, as c grows, when the government shifts from partial to full evasion the former is still possible.

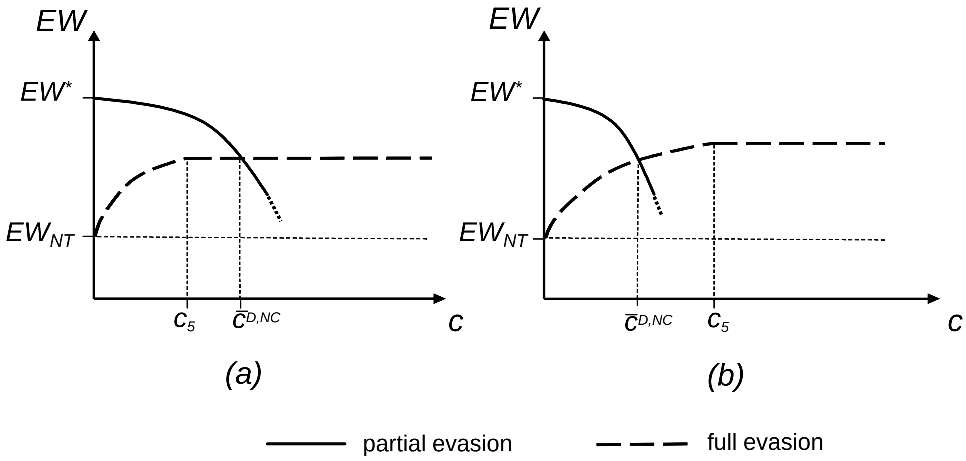

Figure 11: expected welfare in case (D, NC)

When $\theta < 1$ (i.e. when there are some honest rich taxpayers, no matter how few), since expected welfare (weakly) falls with c under partial evasion, while it (weakly) grows with c under full evasion, there exists a positive audit cost $\overline{c}^{D,NC}$ for which both welfare levels coincide. Again, as is intuitive, for low audit costs the government chooses $(\widehat{t}_{PE}^{D,NC}, \widehat{f}_{PE}^{D,NC})$, thereby inducing partial evasion, whereas for high audit costs it chooses a pair $(\widehat{t}_{FE}^{D,NC}, \widehat{f}_{FE}^{D,NC})$ so that regime full evasion obtains. Let us note that it is never optimal for the government to choose the no-taxation regime. Even when $\theta > \widehat{\theta}$ and for any audit cost $c > \widehat{c}$, it can reach higher welfare by setting a positive tax level and low or no fines, so that second-stage full evasion follows. In other words, no taxation is always dominated by full evasion.

Depending on parameter values and the specification of the utility function $u(\cdot)$, two different expected welfare profiles may emerge. As shown in Figure 11(a), expected welfare under partial evasion may intersect expected welfare under full evasion when the latter is flat. This means that maximum expected welfare –the upper envelope of both profiles– monotonically decreases with c, just as in all previous cases. Higher audit costs are also costly in terms of welfare, unless, in a full-evasion regime, audits are not carried out in equilibrium. The transition from one regime to the other when c grows is simple. Once audits become costly enough, the government decides to induce the tax administration not to carry them out by choosing the full-information optimal tax.

However, if $\frac{t^*}{y_r} > \frac{u(y_r - t^*) + t^* - u(y_r)}{t^*}$, a more interesting case appears, which was impossible in all our previous contexts. Expected welfare under partial evasion intersects that under full evasion when the latter is increasing. This implies that maximum expected welfare is nonmonotonic in the audit cost. The government *may strictly gain* when audits become more costly. The intuition for this result is related to the lack of commitment to an audit probability. Under partial evasion, as c grows the government lowers taxes and, in doing so, it also reduces the audit probability chosen by its tax administration. Still, welfare falls. As Figure 11(b) shows, when $c \in (\overline{c}^{D,NC}, c_5)$ it may be the case that the government chooses to eliminate the tax administration's incentives to audit by setting no fine and choosing the maximum tax compatible with full evasion even when, under full evasion, that tax is strictly below the full-information optimal level. In a way, this means that the government chooses to commit not to audit not only by deciding not to fine evaders, but also by setting a low enough tax. An increase in c lowers the cost the government has to pay to commit not to audit, i.e. it makes full evasion compatible with a higher tax, and thereby raises expected welfare.

Needless to say, both cases described in Proposition 10 are possible. Which one arises depends on the curvature of the utility function $u()$ and the value of y_r. According to Chiappori and Paiella (2011), CRRA utility functions with an Arrow-Pratt coefficient of relative risk aversion $\gamma(x) = -xu''(x)/u'(x) \leq 2$ seem to approximate empirically individual behavior under risk. Such an upper bound on relative risk aversion leads to the case depicted in Figure 11 (b), which should then be expected to occur more often.

Why can less risk aversion favor the case described in Figure 11(b)? As we pointed out above in the description of the partial-evasion regime, when $c = 0$ we have a tax t^*. As c grows the tax level is distorted downwards with respect to t^*, since under partial evasion net tax revenue is less significant -dishonest rich taxpayers will misreport more often. Indeed, that distortion grows when taxpayers are less risk averse. The IRS' audit probability keeps dishonest rich taxpayers indifferent between reporting truthfully and misreporting. When the tax is lowered, dishonest taxpayers tend to misreport more, and thus have to be audited more often to keep them indifferent. If taxpayers are less risk averse, the audit probability should rise more, making net tax revenue even less significant in the margin. A larger tax distortion is then first-stage optimal. This, in turn, implies that expected welfare falls more rapidly under partial evasion. (HABRÍA QUE PROBAR ESTO...).

NO SÉ MUY BIEN QUÉ HACER CON LO QUE SIGUE.

Note again that when $\theta = 1$, expected welfare is always weakly decreasing in c, just as in our previous benchmarks. The reason is simple: as all rich are dishonest, regime $R_{FE}^{D,NC}$ yields expected welfare $\mathbb{E}W_{NT}$ for all c. Therefore, $\mathbb{E}W_{FE}^{D,NC}$ is never strictly increasing, and thus the nonmonotonicity result in Figure 10(b) cannot obtain. Clearly, the presence of honest individuals substantially modifies the qualitative results of the model.

VER ESTE PÁRRAFO As we mentioned in the Introduction, many papers explain why penalties for evaders are not optimally set at their maximal legal level. But we are not aware of any other contribution where fines for evaders (or criminals in general) are optimally set at zero in the tax law, as may happen here. In the discussion below, we argue that this result is robust to changes in the model's assumptions. **VER SI HACEMOS EXTENSIONS...** EXTENSIONES PUEDE HABER, PERO ESTE PÁRRAFO QUEDÓ DESCOLOCADO ACÁ... EL PÁRRAFO SIGUIENTE SE PUEDE DEJAR, CREO.

5 Appendix

5.1 Expected welfare in case (ND, C) under no evasion

Let $\mathbb{E}W_{NE}^{ND,C}$ be the expected welfare level achieved under no evasion in this case. Then,

$$\frac{d\mathbb{E}W_{NE}^{ND,C}}{dc} = \frac{d\widehat{t}_{NE}^{ND,C}}{dc} \underbrace{\left[(1-\mu)(1-u'(y_r-\widehat{t}_{NE}^{ND,C})) - \mu \frac{c}{u(y_r)} u'(y_r-\widehat{t}_{NE}^{ND,C})\right]}_{=0} - \mu \widehat{\beta}_{NE}^{ND,C} < 0$$

and

$$\begin{aligned}\frac{d^2\mathbb{E}W_{NE}^{ND,C}}{dc^2} = & \frac{d\widehat{t}_{NE}^{ND,C}}{dc} \left[\frac{d\widehat{t}_{NE}^{ND,C}}{dc} \left((1-\mu)u''(y_r-\widehat{t}_{NE}^{ND,C}) + \mu \frac{c}{u(y_r)} u''(y_r-\widehat{t}_{NE}^{ND,C})\right) - \mu \frac{u'(y_r-\widehat{t}_{NE}^{ND,C})}{u(y_r)}\right] \\ & -\mu \frac{d\widehat{\beta}_{NE}^{ND,C}}{dt} \frac{d\widehat{t}_{NE}^{ND,C}}{dc}.\end{aligned}$$

As the first-order condition holds for any value of the audit cost c, we can differentiate it totally with respect to this parameter, and obtain

$$\frac{d\widehat{t}_{NE}^{ND,C}}{dc} u''(y_r-\widehat{t}_{NE}^{ND,C}) \left(1-\mu+\mu \frac{c}{u(y_r)}\right) = \mu \frac{u'(y_r-\widehat{t}_{NE}^{ND,C})}{u(y_r)}.$$

Hence, as

$$\frac{d\widehat{\beta}_{NE}^{ND,C}}{dt} = \frac{u'(y_r-\widehat{t}_{NE}^{ND,C})}{u(y_r)} > 0$$

and

$$\frac{d\widehat{t}_{NE}^{ND,C}}{dc} = \frac{\mu}{(1-\mu)u(y_r)u''(y_r-\widehat{t}_{NE}^{ND,C}) \left(1+\frac{\mu}{1-\mu}\frac{c}{u(y_r)}\right)^2} < 0,$$

$$\frac{d^2\mathbb{E}W_{NE}^{ND,C}}{dc^2} = -\mu \frac{d\widehat{\beta}_{NE}^{ND,C}}{dt} \frac{d\widehat{t}_{NE}^{ND,C}}{dc} > 0. \blacksquare$$

5.2 Proof of Proposition 1

The optimal tax law inducing full evasion solves

$$\mathcal{P}_{FE}^{D,C} \begin{cases} \underset{t,f}{Max} \quad \mu u(y_p) + (1-\mu)\theta u(y_r) + (1-\mu)(1-\theta)[u(y_r-t)+t] \\ subject\ to \\ (t,f) \in \Omega \\ c \geq \frac{(1-\mu)\theta t}{\mu \widehat{\beta}_{NE}^{D,C}} \end{cases}$$

The Lagrangian of problem $\mathcal{P}_{FE}^{D,C}$ is

$$\begin{aligned}\mathcal{L} = \ & \mu u(y_p) + (1-\mu)\theta u(y_r) + (1-\mu)(1-\theta)[u(y_r - t) + t] \\ & + \phi f - \lambda[t + f - y_r] - \delta[(1-\mu)\theta t - \mu \widehat{\beta}_{NE}^{D,C} c],\end{aligned}$$

where ϕ is the multiplier associated with f's non-negativity constraint, λ is the multiplier associated with the ex-post limited-liability constraint, and δ, the multiplier associated with the constraint that imposes regime full evasion. The first-order conditions of this problem are the following:

$$\begin{cases} \dfrac{\partial \mathcal{L}}{\partial t} = (1-\mu)(1-\theta)[1 - u'(y_r - t)] - \lambda - \delta[(1-\mu)\theta - \mu \frac{d\widehat{\beta}_{NE}^{D,C}}{dt} c] = 0 & (FOC1) \\ \dfrac{\partial \mathcal{L}}{\partial f} = -\lambda + \delta \mu \frac{d\widehat{\beta}_{NE}^{D,C}}{df} c + \phi = 0 & (FOC2) \\ \lambda(t + f - y_r) = 0 \qquad \lambda \geq 0 \qquad t + f - y_r \leq 0 & (CSC1) \\ \delta[(1-\mu)\theta t - \mu \widehat{\beta}_{NE}^{D,C} c] = 0 \qquad \delta \geq 0 \qquad (1-\mu)\theta t - \mu \widehat{\beta}_{NE}^{D,C} c \leq 0 & (CSC2) \\ \phi f = 0 \qquad \phi \geq 0 \qquad f \geq 0 & (CSC3) \end{cases}$$

where

$$\frac{d\widehat{\beta}_{NE}^{D,C}}{df} = -\widehat{\beta}_{NE}^{D,C} \frac{u'(y_r - t - f)}{u(y_r) - u(y_r - t - f)} < 0$$

except when $t = 0$ and $f \in [0, y_r]$, because, in these cases, $\widehat{\beta}_{NE}^{D,C} = 0$. For the moment, we assume that the solution does not adopt these values (and we check later whether this holds). We also compute

$$\frac{d\widehat{\beta}_{NE}^{D,C}}{dt} = \frac{u'(y_r - t)}{u(y_r) - u(y_r - t - f)} + \frac{d\widehat{\beta}_{NE}^{D,C}}{df}.$$

First, assume that $\delta > 0$ at the optimum. From $(FOC2)$, $\phi > 0$. This implies that $f = 0$, $\widehat{\beta}_{NE}^{D,C} = 1$, and thus $\frac{d\widehat{\beta}_{NE}^{D,C}}{dt} = 0$. So $(FOC1)$ becomes

$$(1-\mu)(1-\theta)[1 - u'(y_r - t)] - \delta(1-\mu)\theta = \lambda \geq 0,$$

yielding

$$u'(y_r - t) < 1,$$

which implies that $t < t^*$, and thus $t + f < y_r$, so $\lambda = 0$. The optimal tax is thus $\widehat{t}_{FE}^{D,C} = \frac{\mu c}{(1-\mu)\theta}$ when $c \leq c_3 \equiv \frac{(1-\mu)\theta t^*}{\mu}$.

Now, assume that $\delta = 0$. By $(FOC2)$, either λ and ϕ are both zero or strictly positive. Moreover, $(FOC1)$ becomes

$$(1-\mu)(1-\theta)[1 - u'(y_r - t)] = \lambda \geq 0,$$

which implies that

$$0 < t \leq t^*. \tag{8}$$

Assume that both λ and ϕ are strictly positive. This implies that $t + f = y_r$ and $f = 0$, which contradicts (8). Hence, if at the optimum $\delta = 0$, λ and ϕ are both zero, and thus $\widehat{t}_{FE}^{D,C} = t^*$. In this case, f can adopt any value in $[0, \overline{\overline{f}}]$, where

$$\overline{\overline{f}} = y_r - t^* - u^{-1}\left[u(y_r) - \frac{\mu c}{(1-\mu)\theta t^*}\left(u(y_r) - u(y_r - t^*)\right)\right].$$

This solution applies when $c > c_3$.

As in both solutions $t \neq 0$, we confirm that $\frac{d\widehat{\beta}_{NE}^{D,C}}{df} < 0$, and thus the analysis just shown applies. ■

5.3 Expected welfare in case (D, C) under full evasion

Let $\mathbb{E}W_{FE}^{D,C}$ be the expected welfare level reached when the government optimizes to induce full evasion. Then,

$$\mathbb{E}W_{FE}^{D,C} = \begin{cases} \mu u(y_p) + (1-\mu)\theta u(y_r) + (1-\mu)(1-\theta)\left[u(y_r - \widehat{t}_{FE}^{D,C}) + \widehat{t}_{FE}^{D,C}\right] & \text{if } c \leq c_3 \\ \mu u(y_p) + (1-\mu)\theta u(y_r) + (1-\mu)(1-\theta)[u(y_r - t^*) + t^*] & \text{if } c > c_3 \end{cases}$$

Thus, $\lim_{c \to 0} \mathbb{E}W_{FE}^{D,C} = \mathbb{E}W_{NT}$, and, when $c \leq c_3$,

$$\frac{d\mathbb{E}W_{FE}^{D,C}}{dc} = (1-\theta)\frac{\mu}{\theta}\left[1 - u'(y_r - \widehat{t}_{FE}^{D,C})\right] > 0,$$

and

$$\frac{d^2\mathbb{E}W_{FE}^{D,C}}{dc^2} = (1-\theta)u''(y_r - \widehat{t}_{FE}^{D,C})\left(\frac{\mu}{\theta}\right)^2 < 0.$$

So, when $c \leq c_3$, $\mathbb{E}W_{FE}^{D,C}$ is an increasing and concave function of c. When $c > c_3$, $\mathbb{E}W_{FE}^{D,C}$ is constant. Hence, as a function of c, $\mathbb{E}W_{FE}^{D,C}$ has a profile with a kink at c_3. ■

5.4 The optimal tax law in case (D, C) under no evasion: $(\widehat{t}_{NE}^{D,C}, \widehat{f}_{NE}^{D,C})$

The optimal tax solves

$$\mathcal{P}_{NE}^{D,C} \begin{cases} \underset{t,f}{Max} \quad \mu u(y_p) + (1-\mu)[u(y_r - t) + t] - \mu \widehat{\beta}_{NE}^{D,C} c \\ subject\ to \\ (t, f) \in \Omega \\ \widehat{\beta}_{NE}^{D,C} = \frac{u(y_r) - u(y_r - t)}{u(y_r) - u(y_r - t - f)} \\ c \leq \frac{(1-\mu)\theta t}{\mu \widehat{\beta}_{NE}^{D,C}} \end{cases}$$

The Lagrangian of problem $\mathcal{P}_{NE}^{D,C}$ is

$$\begin{aligned}\mathcal{L} = \ & \mu u(y_p) + (1-\mu)[u(y_r - t) + t] - \mu \widehat{\beta}_{NE}^{D,C} c \\ & -\lambda[t + f - y_r] - \delta[\mu \widehat{\beta}_{NE}^{D,C} c - (1-\mu)\theta t],\end{aligned}$$

where λ is the multiplier associated with the ex-post limited-liability constraint and δ, the multiplier associated with the constraint that imposes regime $R_{NE}^{D,C}$. The first-order conditions of this problem are the following:

$$\begin{cases} \dfrac{\partial \mathcal{L}}{\partial t} = (1-\mu)[1 - u'(y_r - t)] - \mu \frac{d\widehat{\beta}_{NE}^{D,C}}{dt} c - \lambda - \delta[\mu \frac{d\widehat{\beta}_{NE}^{D,C}}{dt} c - (1-\mu)\theta] = 0 & (FOC1) \\ \dfrac{\partial \mathcal{L}}{\partial f} = -\lambda - \mu \frac{d\widehat{\beta}_{NE}^{D,C}}{df} c(1+\delta) = 0 & (FOC2) \\ \lambda(t + f - y_r) = 0 \qquad \lambda \geq 0 \qquad t + f - y_r \leq 0 & (CSC1) \\ \delta[\mu \widehat{\beta}_{NE}^{D,C} c - (1-\mu)\theta t] = 0 \qquad \delta \geq 0 \qquad \mu \widehat{\beta}_{NE}^{D,C} c - (1-\mu)\theta t \leq 0 & (CSC2) \end{cases}$$

where

$$\frac{d\widehat{\beta}_{NE}^{D,C}}{df} = -\widehat{\beta}_{NE}^{D,C} \frac{u'(y_r - t - f)}{u(y_r) - u(y_r - t - f)} < 0$$

except when $t = 0$ and $f \in]0, y_r]$ because, in these cases, $\widehat{\beta}_{NE}^{D,C} = 0$. For the moment, we assume that the solution does not adopt these values (and we check later whether this holds). Also

$$\frac{d\widehat{\beta}_{NE}^{D,C}}{dt} = \frac{u'(y_r - t)}{u(y_r) - u(y_r - t - f)} + \frac{d\widehat{\beta}_{NE}^{D,C}}{df}.$$

From $(FOC2)$ we obtain

$$-\mu \frac{d\widehat{\beta}_{NE}^{D,C}}{df} c(1+\delta) = \lambda > 0.$$

Hence, $t + f = y_r$. Replacing $\frac{d\widehat{\beta}_{NE}^{D,C}}{dt}$ evaluated at the optimum in $(FOC1)$,we obtain

$$(1-\mu)[1 - u'(y_r - t)] - \mu c(1+\delta) \frac{u'(y_r - t)}{u(y_r)} = -\delta(1-\mu)\theta.$$

Next we prove the following lemma.

Lemma 1 At the optimum, $\delta = 0$.

Proof of Lemma 1 Let $\Omega^* = \{(t, f) : t, f \geq 0, t + f = y_r\}$ the set of tax laws that satisfy ex-post limited liability with equality. We know that the optimal tax law must lie on

Ω^*. Consider the profile of expected welfare reached under no evasion, which we denote by $\mathbb{E}W_{NE}^{D,C}$, along Ω^*. As a function of t, its expression is

$$\mathbb{E}W_{NE}^{D,C}\Big|_{\Omega^*} = \mu u(y_p) + (1-\mu)[u(y_r - t) + t] - \mu \left(1 - \frac{u(y_r - t)}{u(y_r)}\right) c.$$

This profile is a concave function of t, with a maximum at a point characterized by the following first-order condition

$$(1-\mu)[1 - u'(y_r - t)] - \mu c \frac{u'(y_r - t)}{u(y_r)} = 0. \tag{9}$$

Hence, if the government were not constrained to chose a tax law generating the no evasion regime, it would choose the tax law characterized by (9). But this tax law is also the solution to the problem $\mathcal{P}_{NE}^{D,C}$ when $\delta = 0$. Hence, we have identified the general solution to this problem.

Finally, as at the optimum $\widehat{t}_{NE}^{ND,C} + \widehat{f}_{NE}^{ND,C} = y_r$ and $\delta = 0, \widehat{t}_{NE}^{D,C}$ is given by the following first-order condition

$$u'(y_r - \widehat{t}_{NE}^{D,C}) = \frac{1}{1 + \frac{\mu}{1-\mu}\frac{c}{u(y_r)}} < 1,$$

which is exactly the same expression that characterizes $\widehat{t}_{NE}^{ND,C}$. Hence, as $\widehat{\beta}_{NE}^{D,C} = \widehat{\beta}_{NE}^{ND,C}$, $\mathbb{E}W_{NE}^{D,C}$ replicates the profile $\mathbb{E}W_{NE}^{ND,C}$. ■

5.5 Proof of Proposition 2

We have shown above that the expected welfare profile $\mathbb{E}W_{FE}^{D,C}$ is an increasing and concave function of c. Let $\mathbb{E}W_{NE}^{D,C}$ be the expected welfare profile under no evasion. As explained in the main text, $\mathbb{E}W_{NE}^{D,C} = \mathbb{E}W_{NE}^{ND,C}$. Then, as proved above, when $c \leq c_1$, expected welfare $\mathbb{E}W_{NE}^{D,C}$ is a decreasing and convex function of c. In order to show that the profile $\mathbb{E}W_{NE}^{D,C}$ crosses the profile $\mathbb{E}W_{FE}^{N,C}$ in its horizontal part, we proceed using the following geometrical argument.

Consider the set $\Phi_\theta = \{c_3(\theta), \mathbb{E}W_{FE}^{D,C}(\theta)\}$. In the plane $(c, \mathbb{E}W)$, this set is the locus of kinks of the profile $\mathbb{E}W_{FE}^{D,C}$, as a function of the parameter θ. Next, we compute the slope of this locus, as follows. Take an arbitrary pair of two values θ_1 and θ_2, with $\theta_1 \neq \theta_2$. These different parameters generate two different profiles of expected welfare, with their corresponding kink. The slope of the segment that joins these two kinks is

$$\frac{\mathbb{E}W_{FE}^{D,C}(\theta_1) - \mathbb{E}W_{FE}^{D,C}(\theta_2)}{c_3(\theta_1) - c_3(\theta_2)} = -\mu \left(\frac{u(y_r - t^*)}{t^*} + 1\right) < 0,$$

which is independent of θ_1 and θ_2. Therefore, in the $(c, \mathbb{E}W)$ plane, the locus Φ_θ is a decreasing straight line.

Now, in the same plane $(c, \mathbb{E}W)$, consider the profile of the expected welfare $\mathbb{E}W_{NE}^{D,C}$. We know that this profile replicates the profile $\mathbb{E}W_{NE}^{ND,C}$. In particular, we know that

$$\frac{d\mathbb{E}W_{NE}^{D,C}}{dc} = -\mu\widehat{\beta}_{NE}^{D,C}(\widehat{t}_{NE}^{D,C}, \widehat{f}_{NE}^{D,C}),$$

and thus

$$\lim_{c\to 0}\frac{d\mathbb{E}W_{NE}^{D,C}}{dc} = -\mu\left(1 - \frac{u(y_r - t^*)}{u(y_r)}\right).$$

As

$$\text{slope of locus } \Phi_\theta = -\mu\left(\frac{u(y_r - t^*)}{t^*} + 1\right) < -\mu < -\mu\left(1 - \frac{u(y_r - t^*)}{u(y_r)}\right) = \lim_{c\to 0}\frac{d\mathbb{E}W_{NE}^{D,C}}{dc},$$

and as $\mathbb{E}W_{NE}^{D,C}$ is convex in c, we conclude that the profile $\mathbb{E}W_{NE}^{D,C}$ is always above the locus Φ_θ. Therefore, the profile $\mathbb{E}W_{NE}^{D,C}$ intersects the profile $\mathbb{E}W_{FE}^{D,C}$ in its horizontal part. ■

5.6 Characterization of the profile $\mathbb{E}W_{FE}^{ND,NC}$

When $c < c_2$, $\lim_{c\to 0}\mathbb{E}W_{FE}^{ND,NC} = \mathbb{E}W_{NT}$,

$$\frac{d\mathbb{E}W_{FE}^{ND,NC}}{dc} = (1-\mu)(1-\theta)\left[1 - u'(y_r - \widehat{t}_{FE}^{ND,NC})\right]\frac{\partial\widehat{t}_{FE}^{ND,NC}}{\partial c} > 0,$$

and

$$\frac{d^2\mathbb{E}W_{FE}^{ND,NC}}{dc^2} = 0.$$

So $\mathbb{E}W_{FE}^{ND,NC}$ increases linearly with c.

Then, when $c \geq c_2$, $\mathbb{E}W_{FE}^{ND,NC}$ is constant. ■

5.7 Proof of Proposition 3

The derivation of the unique, pure-strategy PBE that obtains when $c > \widetilde{c}$ is straightforward.

When $c < \widetilde{c}$, it is not possible that the government audits with probability zero (one) at an equilibrium. Dishonest rich taxpayers would then misreport with probability one (respectively, zero), which in turn would make the government's pure strategy suboptimal. Then the PBE has to be in mixed strategies, so the government has to be indifferent between auditing and not auditing a low-income report. If dishonest rich taxpayers misreport with probability π, when the government receives a low-income report the updated probability it attaches to the taxpayer being poor is μ/ν, where $\nu = \mu + (1-\mu)\theta\pi$. For the government, then, expected welfare when it audits such a report is

$$\frac{(1-\mu)\theta\pi}{\nu}[u(y_r - t - f) + t + f] + \frac{\mu}{\nu}u(y_p) - c$$

whereas when it does not audit expected welfare is

$$\frac{(1-\mu)\theta\pi}{\nu}u(y_r)+\frac{\mu}{\nu}u(y_p).$$

Equating these two expressions and solving yields $\widehat{\pi}$ in (2).

If $c<\widetilde{c}$, $\widehat{\pi}\in(0,1)$. Thus, dishonest rich taxpayers have to be indifferent between reporting truthfully and misreporting. Given that the government audits low-income reports with probability β, if a dishonest rich taxpayer misreports her expected utility is

$$\beta u(y_r-t-f)+(1-\beta)u(y_r).$$

Equating this expression to the utility reached with a truthful report, $u(y_r-t)$, and solving, yields the value of $\widehat{\beta}$ in (3).

When $c=\widetilde{c}$, $\widehat{\pi}=1$. Therefore, any β could be an equilibrium probability of auditing as long as dishonest rich taxpayers find it optimal to misreport. Then, there is a continuum of equilibria, with $\beta\in[0,\widehat{\beta}]$. ■

5.8 Proof of Proposition 4

We present a heuristic proof, based on geometric arguments that will be used again later on. Recall that feasible tax laws $(t,f)\in\Omega$. Let $\Psi=\{(t,f):t,f\geq 0, t+f=t^*\}$. Ψ is a line with slope (-1) in the (t,f) plane. For any given value of the audit cost c, the locus of tax laws that separate full evasion from partial evasion in case (ND,NC) is implicitly given by

$$u(y_r)+\frac{[(1-\mu)\theta+\mu]c}{(1-\mu)\theta}=t+f+u(y_r-t-f). \tag{10}$$

Note that (10) only depends on $t+f$. Then, it also defines a line Γ in the (t,f) plane, with slope (-1). This line shifts farther to the right when c increases. As the government wishes to stay within the full-evasion regime, it has to choose a tax law below Γ.

When $c\geq c_4, \Gamma$ lies above Ψ. Therefore, as the objective function in $\mathcal{P}_{FE}^{ND,NC}$ is –but for a constant factor that does not affect the solution– the same as under full information, $\widehat{t}_{FE}^{ND,NC}=t^*$ and $\widehat{f}_{FE}^{ND,NC}\in[0,\overline{f}]$, where $\overline{f}$ is implicitly given by

$$u(y_r)+\frac{[(1-\mu)\theta+\mu]c}{(1-\mu)\theta}=t^*+\overline{f}+u(y_r-t^*-\overline{f}).$$

In fact, $\overline{f}$ is the highest fine compatible with full evasion when $t=t^*$. By construction, if $\Gamma\subset\Omega$, $\overline{f}\leq y_r-t^*$. Therefore, when $c\geq c_4$, expected welfare is constant in c.

On the other hand, when $c<c_4$, Γ lies below Ψ and thus t^* is no longer attainable. When this is the case, expected welfare increases with t. Therefore, $\widehat{t}_{FE}^{ND,NC}<t^*$, and the optimal tax is implicitly given by

$$u(y_r)+\frac{[(1-\mu)\theta+\mu]c}{(1-\mu)\theta}=\widehat{t}_{FE}^{ND,NC}+u(y_r-\widehat{t}_{FE}^{ND,NC}), \tag{11}$$

where $\widehat{t}_{FE}^{ND,NC}$ is the highest tax compatible with full evasion when $\widehat{f}_{FE}^{ND,NC}=0$.

Properties of $\widehat{t}_{FE}^{ND,NC}$ Clearly, $\lim_{c\to 0}\widehat{t}_{FE}^{ND,NC}=0$. Applying the Implicit Function Theorem to (11) yields

$$\frac{d\widehat{t}_{FE}^{ND,NC}}{dc}=\frac{[(1-\mu)\theta+\mu]}{(1-\mu)\theta}\frac{1}{1-u'(y_r-\widehat{t}_{FE}^{ND,NC})}>0,$$

because $u(.)$ is concave and $\widehat{t}_{FE}^{ND,NC}<t^*$. Moreover,

$$\frac{d^2\widehat{t}_{FE}^{ND,NC}}{dc^2}=\frac{-u''(y_r-\widehat{t}_{FE}^{ND,NC})}{1-u'(y_r-\widehat{t}_{FE}^{ND,NC})}\left(\frac{\partial\widehat{t}_{FE}^{ND,NC}}{\partial c}\right)^2>0. \blacksquare$$

5.9 Proof of Proposition 5

The optimal tax law solves

$$\mathcal{P}_{PE}^{ND,NC}\begin{cases}\underset{t,f}{Max}\quad \mu u(y_p)+(1-\mu)[u(y_r-t)+t+\theta\widehat{\pi}(u(y_r)-u(y_r-t)-t)]\\ subject\ to\\ (t,f)\in\Omega\\ \widehat{\pi}=\dfrac{\mu c}{(1-\mu)\theta(t+f-c-[u(y_r)-u(y_r-t-f)])}\\ c\leq\frac{(1-\mu)\theta}{(1-\mu)\theta+\mu}(t+f-[u(y_r)-u(y_r-t-f)])\end{cases}.$$

If we neglect temporarily the last constraint, the Lagrangian of this problem is

$$\mathcal{L}=\mu u(y_p)+(1-\mu)[u(y_r-t)+t+\theta\widehat{\pi}(u(y_r)-u(y_r-t)-t)]-\lambda(t+f-y_r),$$

where λ is the multiplier associated with the ex-post limited-liability constraint. The first-order conditions of the relaxed problem are as follows:

$$\begin{cases}\dfrac{\partial\mathcal{L}}{\partial t}=(1-\mu)(1-\theta\widehat{\pi})[1-u'(y_r-t)]+(1-\mu)\theta[u(y_r)-u(y_r-t)-t]\frac{\partial\widehat{\pi}}{\partial t}-\lambda=0 & (FOC1)\\ \dfrac{\partial\mathcal{L}}{\partial f}=(1-\mu)\theta[u(y_r)-u(y_r-t)-t]\frac{\partial\widehat{\pi}}{\partial f}-\lambda=0 & (FOC2)\\ \lambda(t+f-y_r)=0\qquad \lambda\geq 0\qquad t+f-y_r\leq 0 & (CSC1)\end{cases}$$

From $(FOC2)$ we obtain

$$\lambda=(1-\mu)\theta[u(y_r)-u(y_r-t)-t]\frac{\partial\widehat{\pi}}{\partial f},\qquad (FOC2')$$

where

$$\frac{\partial\widehat{\pi}}{\partial f}=-\frac{\mu c[1-u'(y_r-t-f)]}{(1-\mu)\theta(t+f-c-[u(y_r)-u(y_r-t-f)])^2}.$$

As $\frac{\partial \widehat{\pi}}{\partial t} = \frac{\partial \widehat{\pi}}{\partial f}$, we can plug $(FOC2')$ in $(FOC1)$ to obtain

$$(1-\mu)(1-\theta\widehat{\pi})[1-u'(y_r-t)] = 0,$$

which implies that, at the optimum, $\widehat{t}_{PE}^{ND,NC} = t^*$.

Next we show that $\widehat{f}_{PE}^{ND,NC} = 0$. Assume that $\widehat{\lambda} > 0$. Then, by $(CSC1)$, $\widehat{t}_{PE}^{ND,NC} + \widehat{f}_{PE}^{ND,NC} = y_r$. As the utility function satisfies the Inada conditions, $\frac{\partial \widehat{\pi}}{\partial f} > 0$. From $(FOC2)$ evaluated at the optimum, we obtain

$$(1-\mu)\theta[u(y_r) - u(y_r - t^*) - t^*]\tfrac{\partial \widehat{\pi}}{\partial f} = \widehat{\lambda} > 0.$$

But, as the utility function is concave and $u'(y_r - t^*) = 1$, the term in brackets is negative, which yields a contradiction. So, at the optimum, $\widehat{\lambda} = 0$, which implies that $\frac{\partial \widehat{\pi}}{\partial f} = 0$. Hence $u'(y_r - \widehat{t}_{PE}^{ND,NC} - \widehat{f}_{PE}^{ND,NC}) = 1$. But as $\widehat{t}_{PE}^{ND,NC} = t^*$, $\widehat{f}_{PE}^{ND,NC} = 0$.

So far we have ignored the last constraint in problem $\mathcal{P}_{PE}^{ND,NC}$. Note that if $c \le c_4 = \frac{(1-\mu)\theta}{(1-\mu)\theta+\mu}\left(t^* - [u(y_r) - u(y_r - t^*)]\right)$, this inequality holds in the solution to the relaxed problem. Hence, the constraint is not binding and we have characterized the solution to $\mathcal{P}_{PE}^{ND,NC}$. When $c > c_4$, partial evasion is impossible. As $\widetilde{c}$ reaches the maximum c_4 when $\widehat{t}_{PE}^{ND,NC} + \widehat{f}_{PE}^{ND,NC} = t^*$, there is no pair (t, f) that can make $\widetilde{c} > c_4$. Hence, for all $c > c_4$, $c > \widetilde{c}$ and thus full evasion obtains.

5.9.1 Second-order conditions of problem $\mathcal{P}_{PE}^{ND,NC}$

Although the objective function (4) is not globally concave, we can show that the second-order conditions for a maximum hold, and thus the first-order conditions of the relaxed problem are sufficient to characterize the optimal schedule $(\widehat{t}_{PE}^{ND,NC}, \widehat{f}_{PE}^{ND,NC})$. Let's denote by $\mathbf{s} = (t, f)$, compute the following partial derivatives, and evaluate them at the optimum $\widehat{\mathbf{s}} = (t^*, 0)$

$$\begin{aligned}
\frac{\partial^2 \mathcal{L}(\widehat{\mathbf{s}})}{\partial t^2} &= (1-\mu)(1-\theta\widehat{\pi})u''(y_r - t^*) + (1-\mu)\theta[u(y_r) - u(y_r - t^*) - t^*]\tfrac{\partial^2 \widehat{\pi}(\widehat{\mathbf{s}})}{\partial t^2} \\
\frac{\partial^2 \mathcal{L}(\widehat{\mathbf{s}})}{\partial f \partial t} &= (1-\mu)\theta[u(y_r) - u(y_r - t^*) - t^*]\tfrac{\partial^2 \widehat{\pi}(\widehat{\mathbf{s}})}{\partial f \partial t} \\
\frac{\partial^2 \mathcal{L}(\widehat{\mathbf{s}})}{\partial f^2} &= (1-\mu)\theta[u(y_r) - u(y_r - t^*) - t^*]\tfrac{\partial^2 \widehat{\pi}(\widehat{\mathbf{s}})}{\partial f^2}
\end{aligned}$$

where

$$\tfrac{\partial^2 \widehat{\pi}(\widehat{\mathbf{s}})}{\partial t^2} = \tfrac{\partial^2 \widehat{\pi}(\widehat{\mathbf{s}})}{\partial f^2} = \tfrac{\partial^2 \widehat{\pi}(\widehat{\mathbf{s}})}{\partial f \partial t} = -\frac{\mu c}{(1-\mu)\theta}\frac{u''(y_r - t^*)}{(1-\mu)\theta[t^* - c - u(y_r) + u(y_r - t^*)]^3}.$$

As $c \le c_4$, $t^* - c - u(y_r) + u(y_r - t^*) \ge 0$. Hence these three last derivatives are positive.

Next, we verify whether the Hessian matrix

$$\bigtriangledown^2_{ss}\mathcal{L}(\widehat{\mathbf{s}}) = \begin{bmatrix} \dfrac{\partial^2\mathcal{L}(\widehat{\mathbf{s}})}{\partial f^2} & \dfrac{\partial^2\mathcal{L}(\widehat{\mathbf{s}})}{\partial t\partial f} \\ \dfrac{\partial^2\mathcal{L}(\widehat{\mathbf{s}})}{\partial f\partial t} & \dfrac{\partial^2\mathcal{L}(\widehat{\mathbf{s}})}{\partial t^2} \end{bmatrix}$$

is semi-definite negative.

- By concavity of the utility function, $\dfrac{\partial^2\mathcal{L}(\widehat{\mathbf{s}})}{\partial f^2} < 0$.
- We compute

  $$\frac{\partial^2\mathcal{L}(\widehat{\mathbf{s}})}{\partial f^2}\cdot\frac{\partial^2\mathcal{L}(\widehat{\mathbf{s}})}{\partial t^2}-\frac{\partial^2\mathcal{L}(\widehat{\mathbf{s}})}{\partial f\partial t}\cdot\frac{\partial^2\mathcal{L}(\widehat{\mathbf{s}})}{\partial t\partial f} = (1-\mu)^2(1-\theta\widehat{\pi})\theta u''(y_r-t^*)[u(y_r)-u(y_r-t^*)-t^*]\tfrac{\partial^2\widehat{\pi}(\widehat{\mathbf{s}})}{\partial f^2} > 0.$$

  Hence, the abovementioned Hessian matrix is semi definite, ensuring that the second-order conditions hold at the optimum. ■

5.10 Expected welfare in case (ND, NC) under partial evasion

Let $\mathbb{E}W_{PE}^{ND,NC}$ be the expected welfare profile under partial evasion, i.e.

$$\mathbb{E}W_{PE}^{ND,NC} = \mu u(y_p) + (1-\mu)u(y_r - t^*) + (1-\mu)(1-\theta\widehat{\pi})t^*$$

Differentiating the expected welfare yields

$$\frac{d\mathbb{E}W_{PE}^{ND,NC}}{dc} = -\frac{\mu[u(y_r)-u(y_r-t^*)-t^*]^2}{[c+u(y_r)-u(y_r-t^*)-t^*]^2} < 0$$

and

$$\frac{d^2\mathbb{E}W_{PE}^{ND,NC}}{dc^2} = \frac{2\mu[u(y_r)-u(y_r-t^*)-t^*]^2}{[c+u(y_r)-u(y_r-t^*)-t^*]^3}.$$

As $c \leq c_4$, the term in brackets in both denominators is negative, and thus $\mathbb{E}W_{PE}^{ND,NC}$ is concave in c. ■

5.11 Proof of Proposition 6

Assume that $c \leq c_4$. Let's study the behavior of the function

$$\Delta(c) \equiv \mathbb{E}W_{PE}^{ND,NC}(c) - \mathbb{E}W_{FE}^{ND,NC}(c),$$

where $\mathbb{E}W_{FE}^{ND,NC}$ is the expected welfare profile under full evasion. We know that

$$\lim_{c\to 0}\Delta(c) = \mathbb{E}W^* - \mathbb{E}W_{NT} > 0,$$

$$\frac{d\Delta(c)}{dc} = \frac{d\mathbb{E}W_{PE}^{ND,NC}(c)}{dc} - \frac{d\mathbb{E}W_{FE}^{ND,NC}(c)}{dc} < 0$$

and

$$\lim_{c\to c_4} \Delta(c) \quad = \lim_{c\to c_4} \mathbb{E}W_{PE}^{ND,NC}(c) - \lim_{c\to c_4} \mathbb{E}W_{FE}^{ND,NC}(c)$$

where

$$\begin{aligned}
\lim_{c\to c_4} \mathbb{E}W_{PE}^{ND,NC}(c) &= \mu u(y_p) + (1-\mu)u(y_r - t^*) + (1-\mu)[t^* + \theta \lim_{c\to c_4} \widehat{\pi}[u(y_r) - u(y_r - t^*) - t^*]] \\
&= \mu u(y_p) + (1-\mu)u(y_r - t^*) + (1-\mu)t^* + \frac{\mu c_2[u(y_r) - u(y_r - t^*) - t^*]}{[t^* + u(y_r - t^*) - u(y_r) - c_2]} \\
&= \lim_{c\to c_4} \mathbb{E}W_{FE}^{ND,NC}(c)
\end{aligned}$$

Thus, $\lim_{c\to c_2} \Delta(c) = 0$. Hence, as the function $\Delta(c)$ is continuous and decreasing on $[0, c_4]$ and $\lim_{c\to 0} \Delta(c) > 0 = \lim_{c\to c_4} \Delta(c)$, the government chooses the optimal tax law that generates the partial evasion regime when $c \le \overline{c}^{ND,NC} \equiv c_4$. ■

5.12 Proof of Proposition 9

When $c < c_6 \equiv \frac{(1-\mu)\theta y_r}{(1-\mu)\theta+\mu}$, the optimal tax law under partial evasion solves the following problem

$$\mathcal{P}_{PE}^{D,NC} \quad \begin{cases} \underset{t,f}{Max} \quad \mu u(y_p) + (1-\mu)u(y_r - t) + (1-\mu)(1-\theta\widehat{\pi})t \\ subject\ to \\ t + f \le y_r \\ \widehat{\pi} = \frac{\mu c}{(1-\mu)\theta(t+f-c)} \\ c < \frac{(1-\mu)\theta(t+f)}{(1-\mu)\theta+\mu}. \end{cases}$$

If we neglect the last constraint, the Lagrangean of this problem is

$$\mathcal{L} = \mu u(y_p) + (1-\mu)u(y_r - t) + (1-\mu)(1-\theta\widehat{\pi})t - \lambda(t + f - y_r),$$

where λ is the multiplier associated with the ex-post limited liability constraint.

The first-order conditions of the relaxed problem are the following

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial t} = -(1-\mu)u'(y_r - \widehat{t}) + (1-\mu)(1-\theta\widehat{\pi}) - (1-\mu)\theta\widehat{t}\frac{\partial\widehat{\pi}}{\partial t} - \widehat{\lambda} = 0 & (FOC1) \\ \frac{\partial \mathcal{L}}{\partial f} = -(1-\mu)\theta\widehat{t}\frac{\partial\widehat{\pi}}{\partial f} - \widehat{\lambda} = 0 & (FOC2) \\ \widehat{\lambda}(\widehat{t} + \widehat{f} - y_r) = 0 \qquad \widehat{\lambda} \ge 0 & (CSC1) \end{cases}$$

From $(FOC2)$ we obtain

$$\widehat{\lambda} = -(1-\mu)\theta \frac{\partial \widehat{\pi}}{\partial f}\widehat{t}.$$

As

$$\frac{\partial \widehat{\pi}}{\partial f} = -\frac{\mu c}{(1-\mu)\theta (t+f-c)^2} < 0,$$

$\widehat{\lambda} > 0$. Hence the ex-post limited liability constraint binds and thus the last constraint is also satisfied. Then, plugging the value of $\widehat{\lambda}$ in $(FOC1)$ and knowing that $\frac{\partial \widehat{\pi}}{\partial t} = \frac{\partial \widehat{\pi}}{\partial f}$, we obtain

$$u'(y_r - \widehat{t}_{PE}^{D,NC}) = (1-\theta\widehat{\pi}) \tag{12}$$

which is the reduced form of the first-order condition that defines the optimal tax $\widehat{t}_{PE}^{D,NC}$. So, by concavity of the utility function $u(\ )$, $\widehat{t}_{PE}^{D,NC} < t^*$.

5.12.1 Second-order conditions of problem $\mathcal{P}_{PE}^{D,NC}$

Let's denote $\mathbf{s} = (t,f)$. As the maximand of $\mathcal{P}_{PE}^{D,NC}$ is not globally concave, we have to verify if the Hessian matrix of the function

$$\mathcal{L}(\mathbf{s},\lambda) = \mu u(y_p) + (1-\mu)u(y_r - t) + (1-\mu)(1-\theta\widehat{\pi})t - \lambda(t+f-y_r)$$

evaluated at $(\widehat{\mathbf{s}}, \widehat{\lambda})$ is negative semidefinite in the tangent sub-space

$$T(c,\widehat{\mathbf{s}}) = \{\mathbf{z} = (z_1, z_2) \in \mathbb{R}^2 \mid \langle \nabla h(\widehat{\mathbf{s}}), \mathbf{z}\rangle = 0\}$$

where $\nabla h(\widehat{\mathbf{s}})$ is the gradient of the binding ex-post limited liability constraint, evaluated at $\widehat{\mathbf{s}}$. The Hessian matrix of $\mathcal{L}(\mathbf{s},\lambda)$, evaluated at $(\widehat{\mathbf{s}}, \widehat{\lambda})$, is equal to

$$\nabla^2_{ss}\mathcal{L}(\widehat{\mathbf{s}}, \widehat{\lambda}) = \begin{bmatrix} \frac{\partial^2 \mathcal{L}(\widehat{\mathbf{s}},\widehat{\lambda})}{\partial t^2} & \frac{\partial^2 \mathcal{L}(\widehat{\mathbf{s}},\widehat{\lambda})}{\partial t \partial f} \\ \frac{\partial^2 \mathcal{L}(\widehat{\mathbf{s}},\widehat{\lambda})}{\partial f \partial t} & \frac{\partial^2 \mathcal{L}(\widehat{\mathbf{s}},\widehat{\lambda})}{\partial f^2} \end{bmatrix}$$

where

$$\begin{aligned} \frac{\partial^2 \mathcal{L}(\widehat{\mathbf{s}},\widehat{\lambda})}{\partial t^2} &= -(1-\mu)\theta \left(2\frac{\partial \widehat{\pi}(\widehat{\mathbf{s}},\widehat{\lambda})}{\partial t} + t\frac{\partial^2 \widehat{\pi}(\widehat{\mathbf{s}},\widehat{\lambda})}{\partial t^2}\right) + (1-\mu)u''(y_r - t) \\ \frac{\partial^2 \mathcal{L}(\widehat{\mathbf{s}},\widehat{\lambda})}{\partial f \partial t} &= -(1-\mu)\theta \left(\frac{\partial \widehat{\pi}(\widehat{\mathbf{s}},\widehat{\lambda})}{\partial f} + t\frac{\partial^2 \widehat{\pi}(\widehat{\mathbf{s}},\widehat{\lambda})}{\partial f \partial t}\right) \\ \frac{\partial^2 \mathcal{L}(\widehat{\mathbf{s}},\widehat{\lambda})}{\partial f^2} &= -(1-\mu)\theta t\frac{\partial^2 \widehat{\pi}(\widehat{\mathbf{s}},\widehat{\lambda})}{\partial f^2} \end{aligned}$$

The set $T(c,\widehat{\mathbf{s}})$ is characterized by all points $\mathbf{z} = (z_1, z_2) \in \mathbb{R}^2$ whose coordinates are given by

$$\begin{cases} z_1 \\ z_2 = -z_1 \end{cases}$$

Hence

$$\begin{aligned}\mathbf{z}.\nabla^2_{ss}\mathcal{L}(\widehat{\mathbf{s}},\widehat{\lambda}).\mathbf{z}^T &= (z_1)^2\left(\tfrac{\partial^2\mathcal{L}(\widehat{\mathbf{s}},\widehat{\lambda})}{\partial t^2} - 2\tfrac{\partial^2\mathcal{L}(\widehat{\mathbf{s}},\widehat{\lambda})}{\partial f\partial t} + \tfrac{\partial^2\mathcal{L}(\widehat{\mathbf{s}},\widehat{\lambda})}{\partial f^2}\right)\\ &= (z_1)^2(1-\mu)u''(y_r - \widehat{t}^{D,NC}_{PE}) < 0\end{aligned}$$

which effectively verifies second-order conditions for a maximum. As the Slater condition holds (i.e. there is only one biding constraint at the optimum), the first-order conditions are necessary and sufficient to characterize the optimal schedule $(\widehat{t}^{D,NC}_{PE}, \widehat{f}^{D,NC}_{PE})$. Hence we can do some comparative statics, as follows.

5.12.2 Comparative statics

In order to completely characterize the optimal schedule $(\widehat{t}^{D,NC}_{PE}, \widehat{f}^{D,NC}_{PE})$, let's compute $\frac{d\widehat{t}^{D,NC}_{PE}}{dc}$ when $c < c_2$. Applying the Implicit Funtion Theorem, we can show that

$$\frac{d\widehat{t}^{D,NC}_{PE}}{dc} = \frac{\theta}{u''(y_r - \widehat{t}^{D,NC}_{PE})}\frac{d\widehat{\pi}}{dc} < 0$$

because

$$\frac{d\widehat{\pi}}{dc} = \frac{\mu y_r}{(1-\mu)\theta(y_r - c)^2} > 0.$$

5.12.3 Domain of positive taxation

As we have assumed that the audit-reporting game is played only when $t > 0$, let's find the parametric region (with respect to the audit cost c) where this inequality holds. From (12), we can find the value $\widehat{c}$ that generates the limit case $\widehat{t}^{D,NC}_{PE} \to 0$. This value is implicitly characterized by the following expression

$$u'(y_r) = (1 - \frac{\mu\widehat{c}}{(1-\mu)(y_r - \widehat{c})})$$

After some manipulations, we obtain

$$\widehat{c} = \frac{(1-u'(y_r))(1-\mu)y_r}{1-u'(y_r)(1-\mu)}.$$

To compare $\widehat{c}$ to c_6, let's find the sign of

$$\widehat{c} - c_6 = \frac{\mu(1-\mu)y_r[1-u'(y_r)-\theta]}{[1-u'(y_r)(1-\mu)][(1-\mu)\theta+\mu]}.$$

So $\widehat{c} \gtreqless c_6$ if and only if $\theta \gtreqless 1 - u'(y_r) \equiv \widehat{\theta}$.

Finally, with all these results, we can easily derive that $\lim_{c\to\widehat{c}} \widehat{t}^{D,NC}_{PE} = 0$ and $\lim_{c\to 0} \widehat{t}^{D,NC}_{PE} = t^*$.

■

5.13 Proof of Proposition 10

As before, let $\mathbb{E}W_{FE}^{D,NC}$ ($\mathbb{E}W_{PE}^{D,NC}$) be the expected welfare levels optimally reached in case (D, NC) under full evasion (partial evasion).

Lemma 2 For any $\theta < 1$, $\mathbb{E}W_{FE}^{D,NC}(c_6) > \mathbb{E}W_{PE}^{D,NC}(c_6)$.

Proof of Lemma 2 When $c = c_6 \equiv \frac{(1-\mu)\theta y_r}{(1-\mu)\theta+\mu}$,

$$\mathbb{E}W_{FE}^{D,NC}(c_6) = \mu u(y_p) + (1-\mu)\theta u(y_r) + (1-\mu)(1-\theta)[u(y_r - t^*) + t^*]$$

and

$$\mathbb{E}W_{PE}^{D,NC}(c_6) = \mu u(y_p) + (1-\mu)u(y_r - \widehat{t}_{PE}^{D,NC}(c_6)) + (1-\mu)(1-\theta)\widehat{t}_{PE}^{D,NC}(c_6)$$

where $\widehat{t}_{PE}^{D,NC}(c_6)$ is characterized by

$$u'(y_r - \widehat{t}_{PE}^{D,NC}(c_6)) = 1 - \theta,$$

because

$$\widehat{\pi}(c_6) = \frac{\mu c_6}{(1-\mu)\theta(y_r - c_6)} = 1.$$

When $\theta \to 0$, $c_6 \to 0$ and so $\widehat{t}_{PE}^{D,NC}(c_6) \to t^*$. Hence $\mathbb{E}W_{PE}^{D,NC}, \mathbb{E}W_{FE}^{D,NC} \to \mathbb{E}W^*$. Then, we compute

$$\frac{d\mathbb{E}W_{FE}^{D,NC}(c_6)}{d\theta} = (1-\mu)\left[u(y_r) - u(y_r - t^*)\right] - (1-\mu)t^* < 0$$

and, using an envelope argument,

$$\frac{d\mathbb{E}W_{PE}^{D,NC}(c_6)}{d\theta} = -(1-\mu)\widehat{t}_{PE}^{D,NC}(c_6) < 0.$$

Taking limits

$$\lim_{\theta\to 0}\frac{d\mathbb{E}W_{FE}^{D,NC}(c_6)}{d\theta} > \lim_{\theta\to 0}\frac{d\mathbb{E}W_{PE}^{D,NC}(c_6)}{d\theta} = -(1-\mu)t^*.$$

Hence, $\mathbb{E}W_{PE}^{D,NC}(c_6)$ decreases more than $\mathbb{E}W_{FE}^{D,NC}(c_6)$ when $\theta \to 0$. As

$$\frac{d^2\mathbb{E}W_{PE}^{D,NC}(c_6)}{d\theta^2} = -(1-\mu)\frac{d\widehat{t}_{PE}^{D,NC}(c_6)}{d\theta} = -\frac{1-\mu}{u''(y_r - \widehat{t}_{PE}^{D,NC}(c_6))} > 0$$

and

$$\lim_{\theta\to\theta_1}\mathbb{E}W_{PE}^{D,NC}(c_6) = \mathbb{E}W_{NT} < \lim_{\theta\to\theta_1}\mathbb{E}W_{FE}^{D,NC}(c_6)$$

(because $\mathbb{E}W_{FE}^{D,NC}(c_6)$ is decreasing in θ and equal to $\mathbb{E}W_{NT}$, but only when $\theta = 1$), then, for all $\theta < 1, \mathbb{E}W_{PE}^{D,NC}(c_6) < \mathbb{E}W_{FE}^{D,NC}(c_6)$, with equality when $\theta = 1$. ■

As $\mathbb{E}W_{PE}^{D,NC}$ decreases with c, and $\mathbb{E}W_{FE}^{D,NC}$ weakly increases, this lemma ensures that there exists $\overline{c}^{D,NC}$ such that both expected welfare levels coincide.

Now, we compare $\mathbb{E}W_{FE}^{D,NC}(c_5)$ with $\mathbb{E}W_{PE}^{D,NC}(c_5)$. When $c = c_5 \equiv \frac{(1-\mu)\theta t^*}{(1-\mu)\theta+\mu}$,

$$\mathbb{E}W_{FE}^{D,NC}(c_5) = \mu u(y_p) + (1-\mu)\theta u(y_r) + (1-\mu)(1-\theta)[u(y_r - t^*) + t^*]$$

and

$$\mathbb{E}W_{PE}^{D,NC}(c_5) = \mu u(y_p) + (1-\mu)u(y_r - \widehat{t}_{PE}^{D,NC}(c_5)) + (1-\mu)(1-\theta\widehat{\pi}(c_5))\widehat{t}_{PE}^{D,NC}(c_5),$$

where

$$\widehat{\pi}(c_5) = \frac{\mu c_5}{(1-\mu)\theta(y_r - c_5)} = \frac{\mu t^*}{(1-\mu)\theta(y_r - t^*) + \mu y_r}$$

and $\widehat{t}_{PE}^{D,NC}(c_5)$ is characterized by

$$u'(y_r - \widehat{t}_{PE}^{D,NC}(c_5)) = 1 - \theta\widehat{\pi}(c_5).$$

When $\theta \to 0$, $c_5 \to 0$ and so $\widehat{t}_{PE}^{D,NC}(c_5) \to t^*$. Hence $\mathbb{E}W_{PE}^{D,NC}, \mathbb{E}W_{FE}^{D,NC} \to \mathbb{E}W^*$.

Then, we compute

$$\frac{d\mathbb{E}W_{FE}^{D,NC}(c_5)}{d\theta} = (1-\mu)\left[u(y_r) - u(y_r - t^*)\right] - (1-\mu)t^* < 0$$

and, using an envelope argument,

$$\begin{aligned}\frac{d\mathbb{E}W_{PE}^{D,NC}(c_5)}{d\theta} &= -(1-\mu)\widehat{t}_{PE}^{D,NC}(c_5)\left(\widehat{\pi}(c_5) + \theta\frac{d\widehat{\pi}(c_5)}{d\theta}\right) \\ &= -(1-\mu)\widehat{t}_{PE}^{D,NC}(c_5)\widehat{\pi}(c_5)\frac{\mu y_r}{(1-\mu)\theta(y_r-t^*)+\mu y_r} < 0.\end{aligned}$$

Taking limits

$$\lim_{\theta\to 0}\frac{d\mathbb{E}W_{PE}^{D,NC}(c_5)}{d\theta} = -(1-\mu)\frac{(t^*)^2}{y_r}.$$

Moreover, as

$$\frac{d\widehat{t}_{PE}^{D,NC}(c_5)}{d\theta} = \frac{\mu^2 t^* y_r}{\left[(1-\mu)\theta(y_r - t^*) + \mu y_r\right]^2 u''(y_r - \widehat{t}_{PE}^{D,NC}(c_5))} < 0,$$

$$\begin{aligned}\frac{d^2\mathbb{E}W_{PE}^{D,NC}(c_5)}{d\theta^2} &= -\frac{1-\mu}{(1-\mu)\theta(y_r-t^*)+\mu y_r}\cdot \\ &\left[\mu y_r\widehat{\pi}(c_5)\frac{d\widehat{t}_{PE}^{D,NC}(c_5)}{d\theta} - 2\frac{\mu(1-\mu)\widehat{\pi}(c_5)\widehat{t}_{PE}^{D,NC}(c_5)(y_r-t^*)y_r}{(1-\mu)\theta(y_r-t^*)+\mu y_r}\right] > 0.\end{aligned}$$

From the previous analysis, two different cases emerge, as follows.

1. If $\frac{t^*}{y_r} < \frac{(1-\mu)[u(y_r)-u(y_r-t^*)-t^*]}{t^*}$, $\lim_{\theta\to 0}\frac{d\mathbb{E}W_{PE}^{D,NC}(c_5)}{d\theta} > \frac{d\mathbb{E}W_{FE}^{D,NC}(c_5)}{d\theta}$. As $\mathbb{E}W_{FE}^{D,NC}(c_5)$ is linear in θ but $\mathbb{E}W_{PE}^{D,NC}(c_5)$ is decreasing and convex, $\mathbb{E}W_{PE}^{D,NC}(c_5) > \mathbb{E}W_{FE}^{D,NC}(c_5)$ for all $\theta < 1$. As $c_5 < c_6$ and, by Lemma 2, $\mathbb{E}W_{PE}^{D,NC}(c_6) < \mathbb{E}W_{FE}^{D,NC}(c_6)$, we must have $c_5 < \overline{c}^{D,NC}$: $\mathbb{E}W_{PE}^{D,NC}$ must cross $\mathbb{E}W_{FE}^{D,NC}$ from above, when the latter is flat.

2. If $\frac{t^*}{y_r} > \frac{(1-\mu)[u(y_r)-u(y_r-t^*)-t^*]}{t^*}$, $\lim\limits_{\theta\to 0}\frac{d\mathbb{E}W_{PE}^{D,NC}(c_5)}{d\theta} < \frac{d\mathbb{E}W_{FE}^{D,NC}(c_5)}{d\theta}$. Let $\overline{\theta}$ denote the value of θ such that $c_5 = \widehat{c}$. As $\mathbb{E}W_{PE}^{D,NC}(c_5)$ is a decreasing and convex function of θ that verifies $\lim\limits_{\theta\to\overline{\theta}} \mathbb{E}W_{PE}^{D,NC}(c_5) = \mathbb{E}W_{NT}$, $\mathbb{E}W_{PE}^{D,NC}(c_5) < \mathbb{E}W_{FE}^{D,NC}(c_5)$ for all $\theta < 1$. As $\mathbb{E}W_{PE}^{D,NC}(c_6) < \mathbb{E}W_{FE}^{D,NC}(c_6)$, we must have $c_5 > \overline{c}^{D,NC}$: $\mathbb{E}W_{PE}^{D,NC}$ must cross $\mathbb{E}W_{FE}^{D,NC}$ from above when the latter is increasing.