

Reducing the Heat Island Effect: A Mathematical Model of Green Roof Design

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Abstract

Green roofs ease the heat island effect. Optimizing green roof design helps achieve this goal more efficiently. This paper proposes two energy models of green roofs to estimate surface temperature, and validates them with experimental evidence.

Architects and engineers optimizing their green roof design can use the proposed energy balance models in any equation-solving software. The mathematical models were validated with measured soil temperatures taken from the green roof at the University of Wisconsin – Milwaukee’s Golda Meir Library.

The energy balance model explains how surface colour, soil depth and plant types affect the surface temperature of a green roof. The green roof surface temperature can be reduced by lighter surface colour, shallower soil depth, and plants with lower internal leaf resistance and larger leaf size.

Key Innovations

- Allow a green roof to be simulated independently without developing the whole building performance.
- Stand-alone equation that can be scripted in any equation-solving platform.

Practical Implications

The proposed energy models can be independently implemented by architects and engineers to optimize their green roof design in any equation-solving software or component-based algorithmic modelling software. For example, MATLAB, Grasshopper+Python/MATLAB, TRNSYS+MATLAB, and so on.

Introduction

Green roofs have many potential benefits. Reducing the urban heat island effect by lowering the roof surface temperature is one of the major reasons that developers widely apply green roofs to their buildings. The heat island effect is caused by urban sprawl. Green roofs can help to reduce it (Susca, Gaffin & Dell’Osso, 2011).

For the well-insulated buildings, the impact of green roofs on building energy performance is minimal (Jaffal & Belarbi, 2012). Therefore, a green roof performance is not necessarily simulated with the whole building energy performance. The most widely used green roof model is

developed and integrated in EnergyPlus by D.J. Sailor (2008). However, the green roof component built in EnergyPlus cannot be separately run on other platforms. The goal of this research is to create an open-source and simplified heat transfer model of a green roof to estimate the roof’s surface temperature. The model has green roof materials and assemblies as parameters and climate data as inputs. However, weather conditions and building characteristics are viewed as exogenous, as they are not elements of green roof design. Only variable elements of green roofs are studied here.

In the proposed energy model, the calculation of radiation, convection and evaporation effects on surface temperature are simplified to use fewer unknown variables. For that reason, this model only requires basic weather data and an initial soil temperature reading to estimate the green roof surface temperature in any equation-solving program.

Nomenclature

Q	Energy flux within the time step, J/m ²
\bar{I}_{solar}	Avg. solar radiation within the time step, W/m ²
α_{solar}	Solar absorptance, %
ϵ_{surf}	Surface emissivity, %
σ	Stefan–Boltzmann constant, 5.67 x 10 ⁻⁸ W/m ² · K ⁴
k_e	Effective conductivity, W/m · K
T_{surf}	Surface Temperature, K
h_c	Convection coefficient, W/m ² · K
d	Soil depth at soil temperature measurement, 0.05m
λ	Latent heat of vaporization, approx. 2.43 x 10 ⁶ J/kg
T_{sky}	Effective sky temperature, K
T_{air}	Dry bulb temperature, K
T_{dp}	Dew point temperature, °C
V	Wind speed, m/s
k_f	Thermal of conductivity of fluid in soil, W/m · K
k_s	Thermal of conductivity of dry soil solid, W/m · K
x_s	Volume fraction of solids in unit soil volume
x_f	Volume fraction of fluid in unit soil volume
E	Evaporation rate, kg/(m ² s)
ρ_{air}	Air density, kg/m ³
C	Dalton number, assume 1.5 × 10 ⁻³

$\Delta\bar{q}$	The difference of humidity between the surface and the air, kg/kg
e	Water vapor pressure Pa
P	Total atmospheric pressure Pa
ΔG	Soil heat storage, J/m ²
U_{bldg}	Existing roof U-value, W/m ² K
T_{bldg}	Building surface temperature, K
$h_{c,g}$	Thermal resistance of air space between canopy and ground, 4 ~ 5.67 w/m ² ·°C
d_l	The saturation density of water vapor in the leaf intercellular air spaces as a function of leaf temperature, kg/m ³
d_a	The saturation density of water vapor in the air as a function of air temperature, kg/m ³
h	Relative humidity of the air, %
r_l	Internal leaf resistance, 100~2000 s/m
r_a	Surface boundary-layer resistance
ω_a	Humidity ratio, kg/kg
ρ_a	Dry air density, kg/m ³
B	0.622 for air, kg/kg
P_w	Saturated water vapor pressure, mbar
$P_{w,l}$	Water vapor pressure on leaf, mbar
$P_{w,a}$	Water vapor pressure of air, mbar
P_{total}	Ambient total pressure, mbar
A	6.116441 for water between -20°C ~ 50°C
Δz	Distance between two points
T_{soil}^{i+1}	The soil temp. at the end of the time step, K
T_{soil}^i	The soil temp. at the beginning of the time step, K
ρ_d	Dry bulk density, kg/m ³
c_d	Specific heat capacity of dry soil, J/kg·K
c_w	Specific heat capacity of water, J/kg·K
w	Water content, kg/kg
m	7.591386 for water between -20°C ~ 50°C
T_n	240.7263 for water between -20°C ~ 50°C
T	Ambient temperature, °C
RH	Relative humidity
W	The dimension at right angles to the width of a leaf, m
D	Width of a leaf in the direction of the air flow, m
k_2	200 s ^{1/2} /m

Green roof mathematical model

Mathematical models can help build simulation programs and explore the performance of green roofs in energy saving and water retention. A mathematical model of a green roof consists of two parts: The energy balance model and the mass balance model. Based on the Law of Conservation of Energy and Mass, the heat and mass flow in green roofs will always follow:

$$Input - Output = Energy \text{ or } mass \text{ storage} \quad (1)$$

In the energy balance model, as shown in Figure 1, the heat transfer approaches include solar radiation, sky radiation, heat convection, heat conduction and heat storage in the soil.

In the mass balance model, as shown in Figure 1, which is also the water balance, the water flows include precipitation, evapotranspiration, condensation, drainage and retention.

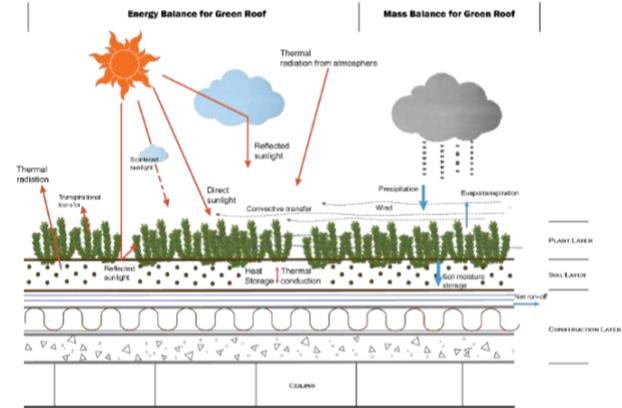


Figure 1. Energy and water balance in green roofs

Figure 1. Energy balance and water balance in green roofs

The energy flow on a green roof surface is influenced by solar radiation, sky radiation, heat conduction, heat convection and evaporation (Sailor, 2008). The evaporation is also the latent heat released or absorbed during the water-vapor conversion.

$$Q_{solar} + Q_{rad} + Q_{cond} + Q_{conv} - Q_{evap} = 0 \quad (2)$$

Then we have,

$$Q_{solar} = \bar{I}_{solar} \Delta t \alpha_{solar} \quad (3)$$

$$Q_{rad} = \varepsilon_{surf} \sigma \Delta t (T_{sky}^4 - T_{surf}^4) \quad (4)$$

$$Q_{cond} = \frac{k_e}{d} \Delta t (T_{soil} - T_{surf}) \quad (5)$$

$$Q_{conv} = h_c \Delta t (T_{air} - T_{surf}) \quad (6)$$

$$Q_{evap} = \lambda E \Delta t \quad (7)$$

Through onsite measurement, it is easily found that the surface temperatures of a bare soil and a vegetation-covered surface are different. Therefore, the energy balance model of a green roof should be separately developed for a bare soil and a vegetation-covered green roof.

Bare Soil Energy Balance: Surface Temperature

The energy transfer flow of the bare soil is illustrated in Figure 2. The surface of the bare soil absorbs the solar radiation, absorbs or releases radiation from or to the sky, convects heat with the air on the surface, and then conducts the heat down to the soil and building. There is

also some heat restored in the soil during heat conduction.

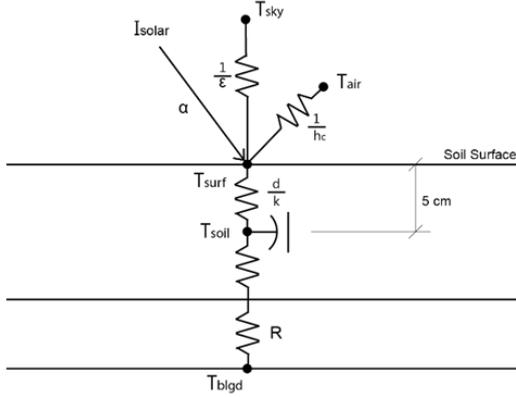


Figure 2. Energy flow for the bare soil.

The energy balance of the heat flux through the bare soil surface is as follows:

$$\bar{I}_{solar} \alpha_{solar} + \epsilon_{soil,surf} \sigma (T_{sky}^4 - T_{surf}^4) + h_c (T_{air} - T_{surf}) + \frac{k_e}{d} (T_{soil} - T_{surf}) - \lambda E = 0 \quad (8)$$

The average solar radiation can be measured or provided by the weather data, for example, TMY-3. Solar absorption α_{solar} is determined by the surface color.

The radiation between sky and surface is estimated by the emissivity function, the Stefan–Boltzmann constant, and the difference between the fourth powers of the sky and surface temperatures.

The emissivity of the moist soil can be estimated by the emissivity of the dry soil and the saturated soil:

$$\epsilon_{soil,surf} = \epsilon_{soil,dry} + (\epsilon_{soil,sat} - \epsilon_{soil,dry}) x_s \quad (9)$$

And $(T_{sky}^4 - T_{surf}^4)$ in Equation 8 can be expressed as:

$$(T_{sky}^4 - T_{surf}^4) = (T_{sky}^2 + T_{surf}^2)(T_{sky}^2 - T_{surf}^2) = (T_{sky}^2 + T_{surf}^2)(T_{sky} + T_{surf})(T_{sky} - T_{surf}) \quad (10)$$

For two arbitrary surfaces that have close temperatures, an estimated average temperature \bar{T} can be introduced to simplify the sky radiation heat transfer (Duffie & Beckman, 1980).

$$4\bar{T}^3 = (T_2^2 + T_1^2)(T_2 + T_1) \quad (11)$$

Since the surface temperature is close to the air temperature, we can get an approximate \bar{T} using the average of sky temperature and ambient air temperature.

$$\bar{T} = \overline{T_{sky}, T_{air}} \quad (12)$$

Then $(T_{surf}^4 - T_{soil}^4)$ can be expressed as:

$$(T_{surf}^4 - T_{soil}^4) = 4\bar{T}^3 (T_{surf} - T_{soil}) \quad (13)$$

Therefore, the energy balance for a node in the soil can be expressed as:

$$\bar{I}_{solar} \alpha_{solar} + 4\epsilon_{soil,surf} \sigma \bar{T}^3 (T_{sky} - T_{surf}) + h_c (T_{air} - T_{surf}) + \frac{k_e}{d} (T_{soil} - T_{surf}) - \lambda E = 0 \quad (14)$$

The method of estimating sky temperature T_{sky} is a function of the dew point temperature, dry bulb

temperature, and number of hours since midnight t (Berdahl & Martin, 1984).

$$T_{sky} = T_{air} [0.711 + 0.0056T_{dp} + 0.000073T_{dp}^2 + 0.013 \cos(15t)]^{0.25} \quad (15)$$

The convection heat transfer happens on the surface boundary, and it can be estimated as a function of the convection coefficient and the difference between the air temperature and surface temperature. The convection coefficient, h_c , can be estimated with a given wind speed (Watmuff et al, 1977). And this wind speed, V , is monitored by the weather station.

$$h_c = 2.8 + 3.0V \quad (16)$$

The effective thermal conductivity k_e can be estimated by the idealized models of heat flow through a unit cube of moist soil (Farouki, 1982).

$$\frac{1}{k_e} = \frac{x_s}{k_s} + \frac{x_f}{k_f} \quad (17)$$

x_f and x_s are monitored by the weather station. k_s and k_f can be estimated by tests or found in the engineering property charts.

For the water retained in the topsoil, the evaporation can be estimated by Equation 18 (Oke, 2002):

$$E = \rho_{air} CV \Delta \bar{q} \quad (18)$$

The air density is calculated using the built-in function of the Energy Equation Solver (EES) with a known dry bulb temperature, dew point temperature and air pressure, as provided by the weather stations.

The humidity can also be calculated with a known temperature and pressure (Gates, 2012):

$$q = \frac{0.622e}{P - 0.379e} \cong \frac{0.622e}{P} \quad (19)$$

Tetens' formula for temperatures above 0°C define the water vapor pressure as indicated below (Monteith & Unsworth, 2007):

$$e = 0.61078 \exp\left(\frac{17.27T}{T + 237.3}\right) \quad (20)$$

Therefore, the evaporation rate can be estimated by the soil temperature and ambient air temperature:

$$E = \rho_{air} CV \frac{0.622}{P} 0.61078 \left(\exp\left(\frac{17.27T_{soil}}{T_{soil} + 237.3}\right) - \exp\left(\frac{17.27T_{air}}{T_{air} + 237.3}\right) \right) \quad (21)$$

To solve the equation above, the surface temperature T_{surf} can be estimated as follows:

$$T_{surf,soil} = \frac{\bar{I}_{solar} \alpha_{solar} + 4\epsilon_{soil,surf} \sigma (\bar{T})^3 T_{sky} + h_c T_{air} + \frac{k_e}{d} T_{soil} - \lambda E}{h_c + 4\epsilon_{soil,surf} \sigma (\bar{T})^3 + \frac{k_e}{d}} \quad (22)$$

Bare Soil Energy Balance: Soil Temperature

The surface temperature, $T_{surf,soil}$, can be calculated by the equation 22 when the soil temperature is known. However, in most cases, soil temperature thermometers are not installed for green roofs. If the soil temperature is unknown, an energy balance in the soil can be derived as a function of the heat conduction between the soil surface and building surface, and the heat storage.

$$\Delta G = \frac{k_e}{d} (T_{surf,soil}^i - T_{soil}^i) + \left(\frac{1}{\frac{d}{k_e} + U_{bldg}} \right) (T_{bldg} - T_{soil}^i) \quad (23)$$

The heat capacity of the soil will store heat. The heat storage between any two points of the soil is

$$\Delta G = \int_z^0 C_v \cdot \frac{\partial T}{\partial T} dz = \Delta z \cdot C_v \cdot \frac{T_{soil}^{i+1} - T_{soil}^i}{\Delta t} \Big|_{0 \sim z} \quad (24)$$

$$C_v = \rho_d (c_d + w c_w) \quad (25)$$

Combining the equations above, the energy balance of the soil temperature is as below:

$$\frac{2dC_v(T_{soil}^{i+1} - T_{soil}^i)}{\Delta t} = \frac{k_e}{d} (T_{surf,soil}^i - T_{soil}^i) + \left(\frac{1}{\frac{d}{k_e} + U_{bldg}} \right) (T_{bldg} - T_{soil}^i) \quad (26)$$

The soil temperature T_{soil}^{i+1} can be expressed as:

$$T_{soil}^{i+1} = T_{soil}^i + \frac{\Delta t}{2C_v d} \left[\frac{k_e}{d} (T_{surf,soil}^i - T_{soil}^i) + \left(\frac{1}{\frac{d}{k_e} + U_{bldg}} \right) (T_{bldg} - T_{soil}^i) \right] \quad (27)$$

The $T_{surf,soil}^i$ in Equation 27 was replaced with the surface temperature Equation 22, achieving the following soil temperature equation:

$$T_{soil}^{i+1} = \frac{\frac{\Delta t k_e}{2C_v d^2} (I_{solar} \alpha_{solar} + 4\epsilon_{veg, surf} \sigma \bar{T}^3 T_{sky} + h_c T_{air} - \lambda E)}{h_c + 4\epsilon_{soil, surf} \sigma \bar{T}^3 + \frac{k_e}{d}} + \frac{\frac{\Delta t}{2C_v d} T_{bldg}}{\frac{d}{k_e} + U_{bldg}} - \left[\frac{\Delta t k_e}{2C_v d^2} + \frac{\frac{\Delta t}{2C_v d}}{\frac{d}{k_e} + U_{bldg}} \right] T_{soil}^i \quad (28)$$

$$\lambda E = \lambda \rho_{air} C V \frac{0.622e}{P} 0.61078 \left(\exp \left(\frac{17.27 T_{soil}^i}{T_{soil}^i + 237.3} \right) - \exp \left(\frac{17.27 T_{air}}{T_{air} + 237.3} \right) \right) \quad (29)$$

Note: The T_{soil}^i in Equation 28 is in Kelvin units, but the T_{soil}^i in Equation 29 is in Celsius units.

Vegetation-Covered Soil Energy Balance: Surface Temperature

Compared to the uncovered bare soil, the vegetation-covered green roof includes the plant transpiration and extra heat transfer between the vegetation and soil surface. The energy transfer flow of the vegetation-covered surface is illustrated in Figure 3. The surface of the vegetation absorbs the solar radiation, absorbs or releases radiation from or to the sky, convects heat with the air on the surface, and then conducts the heat down to the air between the vegetation and soil surface, the soil, and the building. There is some heat stored in the soil during heat conduction.

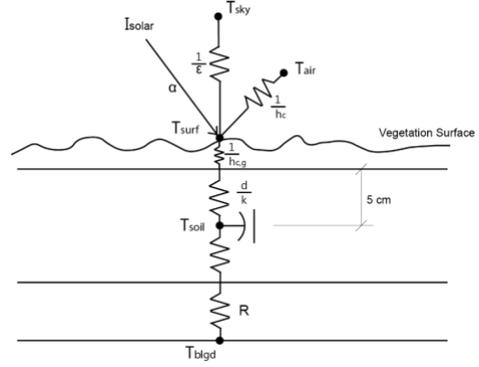


Figure 3. Energy flow for the vegetation-covered soil.

For vegetation-covered surfaces, the energy balance includes the transpiration of the plants. The heat flux through the vegetation surface is:

$$\bar{I}_{solar} \propto I_{solar} + 4\epsilon_{veg, surf} \sigma \bar{T}^3 (T_{sky} - T_{surf,veg}) + h_c (T_{air} - T_{surf,veg}) + \frac{1}{\frac{d}{k_e} + \frac{1}{h_{c,g}}} (T_{soil} - T_{surf,veg}) - \lambda E = 0 \quad (30)$$

Thermal resistance of air space between canopy and ground, $h_{c,g}$, is from 4 to 5.67 w/m² C (ASHRAE, 2013). E is the rate of transpiration, kg/(m²s), which is also the rate at which water vapor escapes from a leaf is as below (Oke, 2002):

$$E = \frac{d_l - h d_a}{r_l - r_a} \quad (31)$$

The internal leaf resistance, r_l , is between 100 and 200 s/m depending on the plant type (Oke, 2002). To simplify the calculation, we assume the air density on the leaf and atmosphere are approximately the same. Therefore,

$$d_l \approx d_a = \omega_a \rho_a \quad (32)$$

The humidity ratio can be estimated as below (Oyj, 2013):

$$\omega_a = \frac{BP_w}{(P_{total} - P_w)} \quad (33)$$

If low accuracy is allowed, a simpler formula of P_{ws} can be estimated as follows:

$$P_w = P_{ws} \cdot \frac{RH}{100} = A \cdot 10^{\frac{mT}{T+T_n}} \cdot \frac{RH}{100} \quad (34)$$

A 6.116441 for water between -20°C ~ 50°C

m 7.591386 for water between -20°C ~ 50°C

T_n 240.7263 for water between -20°C ~ 50°C

T Ambient temperature, °C

RH Relative humidity

P_w Saturated water vapor pressure, mbar

$$r_a = k_2 \frac{w^{0.2} D^{0.3}}{v^{0.5}} \quad (35)$$

k_2 is approximately 200 s^{1/2}/m.

The dimension of a leaf of the sedum plant is about 0.01m x 0.01 m, so

$$r_a = 200 \times \frac{0.01^{0.2} \times 0.01^{0.3}}{v^{0.5}} = \frac{20}{v^{0.5}} \quad (36)$$

Solving Equation 30 with Equation 31-36, the surface temperature of vegetation is

$$T_{surf,veg} = \frac{\bar{I}_{solar} \alpha_{solar} + 4\epsilon_{veg,surf} \sigma (\bar{T})^3 T_{sky} + h_c T_{air} + \frac{1}{\frac{d}{k_e} + \frac{1}{h_{c,g}}} T_{soil} - \lambda \left[\frac{(1-h)d_a}{r_l - v^{0.5}} \right]}{h_c + 4\epsilon_{canopy} \sigma (\bar{T})^3 + \frac{1}{\frac{d}{k_e} + \frac{1}{h_{c,g}}}} \quad (37)$$

Vegetation-Covered Soil Energy Balance: Soil Temperature

Equation 37 can be used to predict the surface temperature on the vegetation of a green roof with the known soil temperature. Following the same derivation method of the bare soil temperature, we can achieve an energy balance with the function of the heat conduction between the vegetation surface and building surface, and heat storage in the soil:

$$\frac{2C_v d (T_{soil}^{i+1} - T_{soil}^i)}{\Delta t} = \frac{1}{\frac{d}{k_e} + h_{c,g}} (T_{surf,veg}^i - T_{soil}^i) + \left(\frac{1}{\frac{d}{k_e} + U_{bldg}} \right) (T_{bldg} - T_{soil}^i) \quad (38)$$

The $T_{surf,veg}^i$ in Equation 38 was replaced with the surface temperature Equation 37, giving a soil temperature equation:

$$T_{soil,veg}^{i+1} = \frac{\frac{\Delta t}{2C_v d} \left(\frac{1}{\frac{d}{k_e} + h_c} \right) \left\{ \bar{I}_{solar} \alpha_{solar} + 4\epsilon_{veg,surf} \sigma (\bar{T})^3 T_{sky} + h_c T_{air} - \lambda \left[\frac{(1-h)d_a}{r_l - v^{0.5}} \right] \right\}}{h_c + 4\epsilon_{veg,surf} \sigma (\bar{T})^3 + \frac{k_e}{d}} + \left\{ \frac{\frac{\Delta t}{2C_v d} T_{bldg}}{\frac{d}{k_e} + U_{bldg}} - \left[\frac{\Delta t}{2C_v d} \left(\frac{1}{\frac{d}{k_e} + h_{c,g}} \right) + \frac{\frac{\Delta t}{2C_v d}}{\frac{d}{k_e} + U_{bldg}} - \left[\frac{\Delta t \left(\frac{1}{\frac{d}{k_e} + h_{c,g}} \right)^2}{2C_v d (h_c + 4\epsilon_{veg,surf} \sigma (\bar{T})^3 + \frac{k_e}{d})} \right] \right] \right\} T_{soil,veg}^i \quad (39)$$

Validation

The mathematical models of green roofs' heat transfer are simulated using the numerical computing software, MatLab. The coefficients in the models were based on experiments, estimation and known measured results.

The validation data were measured from the Golda Meir Library's green roof at University of Wisconsin – Milwaukee in 2014. To validate the energy balance model, the modelled soil temperature was compared with the measured one. At each point in time, Equations 28 and 39 were simulated in MatLab to calculate the soil temperature, and then compared to the measured data.

The root-mean-square deviation (RMSD) and the standard error (SE) of the differences were used as the

criteria to investigate the errors. The modelled soil temperatures were validated with the measured ones from April to October 2014. For the bare soil surface, the sample mean of the difference is 0.5°C, RMSD=3.15°C and the SE is 0.0235. For the soil temperature beneath the canopy, the average difference of the simulation is 1.31°C, RMSD=2.34°C and the SE is 0.015.

To better illustrate the difference between measured and the simulated data, Figures 4 and 5 compare the calculated and measured soil temperature of bare soil and vegetation-covered soil in August 2014.

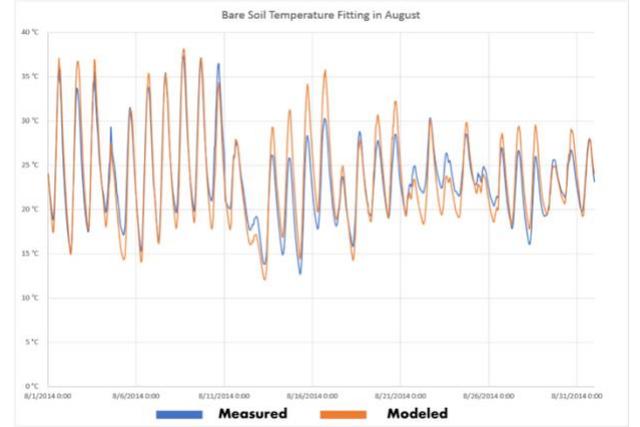


Figure 4. Measured and calculated bare soil temperature comparison in August.

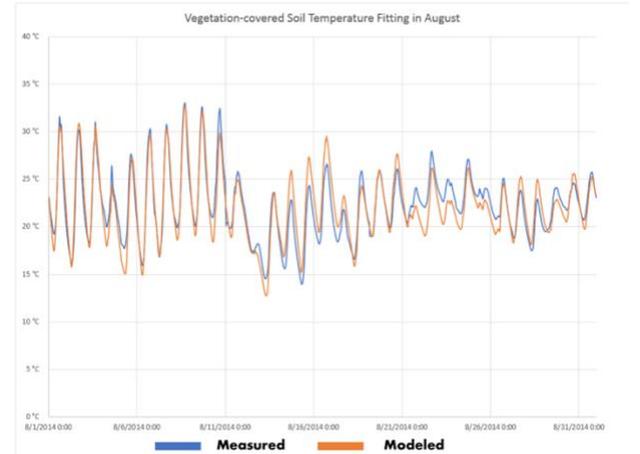


Figure 5. Measured and calculated vegetation-covered soil temperature comparison in August.

Application of the Energy Model

Energy Flux Density

With the surface temperature computed in MatLab, Q_{solar} , Q_{rad} , $Q_{conduction}$ and $Q_{evaporation}$ can be calculated using Equation 2-7. Figures 6 and 7 show the average hourly energy flux density in August 2014 for bare soil and vegetation-covered surfaces. When Q is positive, the surface is absorbing heat, and otherwise is losing heat.

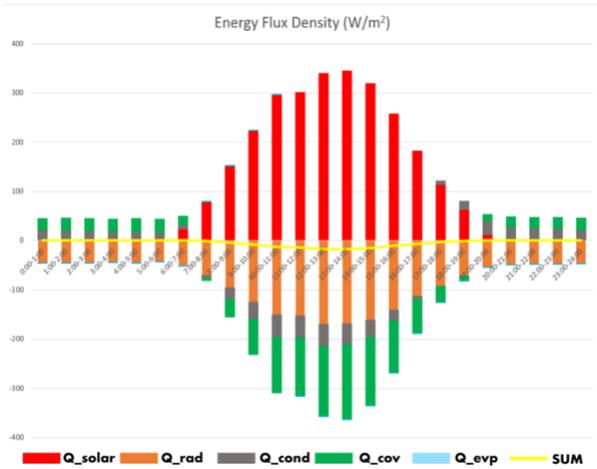


Figure 6. The average energy flux density for bare soil surface in August, 2014.

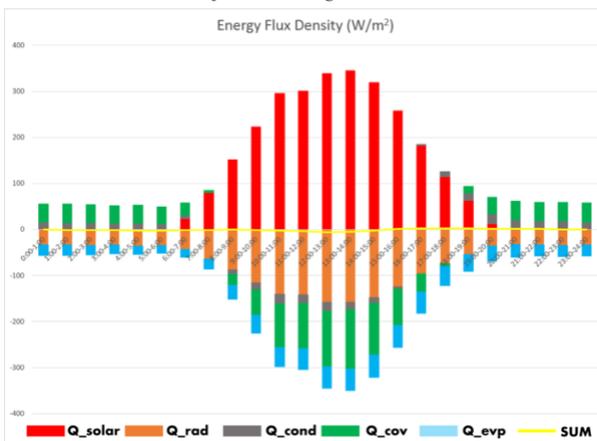


Figure 7. The average energy flux density for vegetation-covered soil surface in August, 2014.

The solar radiation, Q_{solar} , is the major heat source for both bare soil and vegetation-covered surface. In August, solar radiation appears from 6 a.m., increases to a peak in the afternoon from 13:00 to 14:00 (1 p.m. to 2 p.m.), and then decreases gradually toward sunset at around 18:00 to 19:00 (6 p.m. to 7 p.m.). In the simulation, I assumed the same absorption for both bare soil and vegetation-covered surfaces.

The diffusive radiation from the atmosphere, Q_{rad} is negative throughout the day. That means the surface temperature is always higher than the sky temperature, which makes sense during summer.

The heat conduction, Q_{cond} , is negative during the day and positive during the night. That means, whether there is vegetation coverage, the soil is absorbing heat during the day and losing heat during the night. It also demonstrates that the heat is conducted down to the building through the soil during the day and released from the building through the soil at night.

Temperature

As shown in Figure 8 of the surface temperature comparison, the surface temperature changes reflect the change in solar radiation immediately. In the morning, the surface temperature increases more rapidly. Based on the rate of increase of temperature and solar radiation in the

morning, we can see that the temperature increase is sharper than the solar radiation increase. This means that when the surface receives solar radiation in the morning, it heats up more quickly. After solar radiation peaks, the temperature starts to go down, but it does not reduce the surface temperature as much as it does in the morning. The thermal inertia, which is also the heat capacity of the surface, is the main reason for this phenomenon.

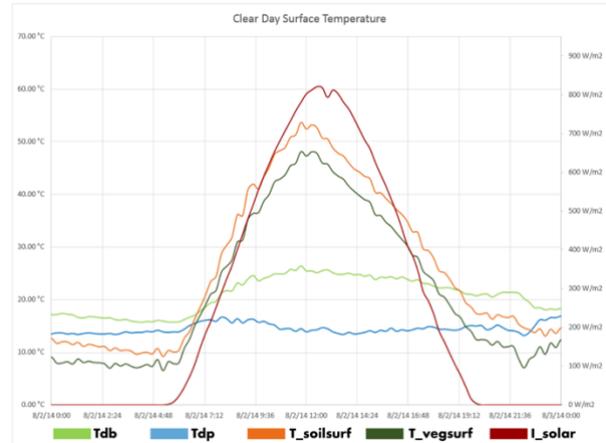


Figure 8. The comparison of the surface temperature on a clear day, August 2nd, 2014.

Figure 9 shows the bare and vegetation-covered surface temperature comparison on an overcast day. Just as on a clear day, the daytime surface temperature still varies closely in step with the change of solar radiation. As on a clear day, the soil surface temperature is still higher than the vegetation-covered surface temperature. The temperature difference between these two surfaces on a clear day is much higher than that on an overcast day.

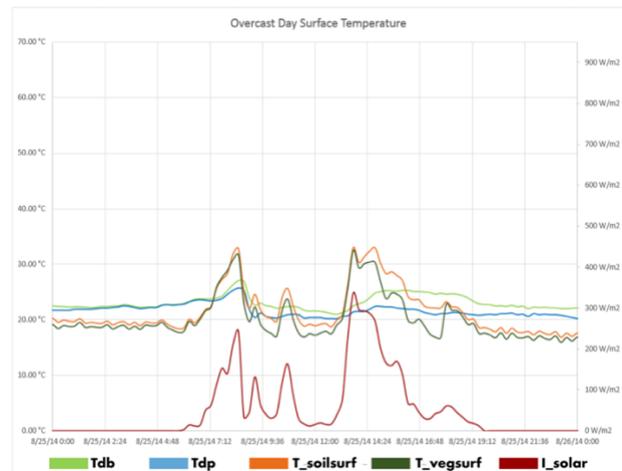


Figure 9. The comparison of the surface temperature on an overcast day, August 25th, 2014.

The Effects of Colour, Soil Depth and Plant Type on Surface Temperature

One of the goals that green roofs can achieve is to mitigate the heat island effect. The key to bringing down the heat island effect is reducing the surface temperature.

As stated in the bare soil surface temperature mathematical model, shown in Equations 21 and 22, I_{solar} , T_{sky} , T_{air} , h_c , V are variables that cannot be

controlled. But α_{solar} , ϵ_{surf} , d are variables that can be controlled by green roof designers. α_{solar} and ϵ_{surf} are determined by surface color and texture. Soils have similar textures, so the color of the soil is something the designers can more easily control. Soil depth d is also controlled by green roof designers.

Because ϵ_{surf} is proportional to α_{solar} , and they are both determined by the color of the surface, I used the solar absorption level, α_{solar} , as the indicator to see how the color of the surface affects the surface temperature.

Figures 10 and 11 show the effects of the surface colour and the soil depth on the surface temperature on a clear day and on an overcast day. On a clear day in summer, between sunrise and sunset, the surface temperature of the bare soil is higher than the air temperature. When α_{solar} increases by 33%, the soil surface temperature rises about 10 °C during peak solar radiation at noon. However, when d increases by 50%, the surface temperature only rises about 2°C during peak solar radiation at noon. In addition, Figure 10 shows that, the higher the solar radiation, the more influential α_{solar} and d . During the night, the surface temperature is lower than the air temperature and the α_{solar} has no impact on surface temperature. Also, the thicker the soil the lower the surface temperature, which is the opposite of the effect during the daytime.

On an overcast day in summer, as on a clear day, Figure 11 shows that α_{solar} is still the primary influence on the surface temperature. When α_{solar} increases by 33%, the soil surface temperature rises about 5°C at peak solar radiation. On an overcast day, the peak solar radiation does not necessarily happen at noon, because it is also influenced by cloud conditions. When soil depth d increases by 50%, the surface temperature rises less than 1°C. During the night, the effect of α_{solar} and d on the bare soil surface under overcast conditions is similar to that on a clear day.

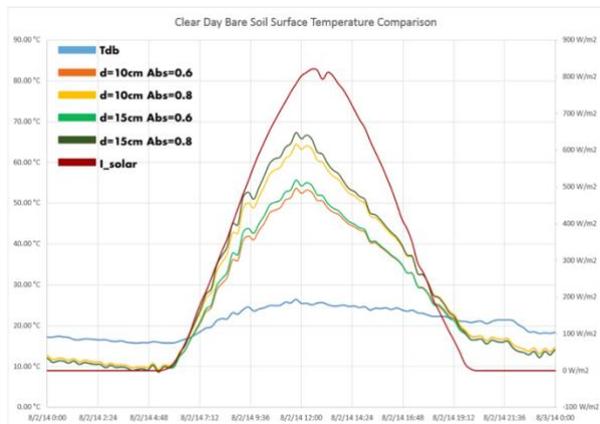


Figure 10. Bare soil surface temperature with soil depth and solar absorption variations on a clear day, August 2nd, 2014.

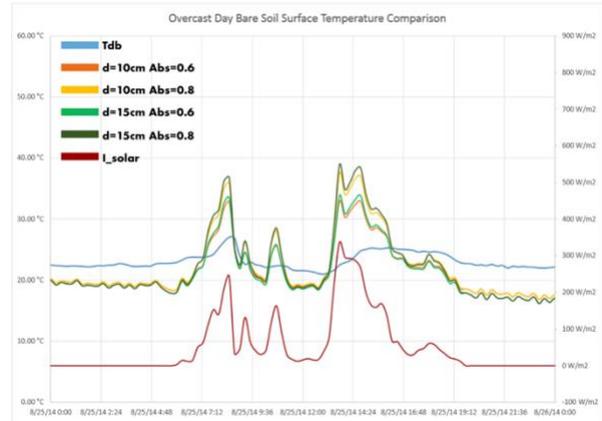


Figure 11. Bare soil surface temperature with soil depth and solar absorption variations on an overcast day, August 25th, 2014.

For vegetation-covered surfaces, as shown in Equation 37, I_{solar} , T_{sky} , T_{air} , h_c , V , h are fixed by natural conditions. But α_{solar} , ϵ_{canopy} , d , r_l , W , and D are controlled by green roof designers. The vegetation color and texture determine α_{solar} and ϵ_{canopy} . The leaves have similar textures, so the color of the leaves is what the designers are concerned with. The green roof designers decide the soil depth d with the needs of the plants in mind. Internal leaf resistance r_l is in a range of 100-2000 s/m. The level of r_l is determined by the type of plant. 200 s/m is the most common internal leaf resistance, but plants that thrive in relatively arid conditions have a higher internal leaf resistance. The dimension of a leaf is W and D , which represents the size of the leaves, have influence on surface temperature too.

The surface temperatures of three plants with different internal leaf resistances and sizes on a clear day are shown in Figure 12. The sedum, which requires the least amount of maintenance and survives in harsh conditions, has the highest temperature. A native plant with the same internal leaf resistance as the grass is cooler than the grass because of its larger leaf size. However, on a cloudy day, as shown in Figure 13, the difference was minimal during the day, and there was no difference at night. At the time with low solar radiation, the surface temperature is even lower than the air temperature.

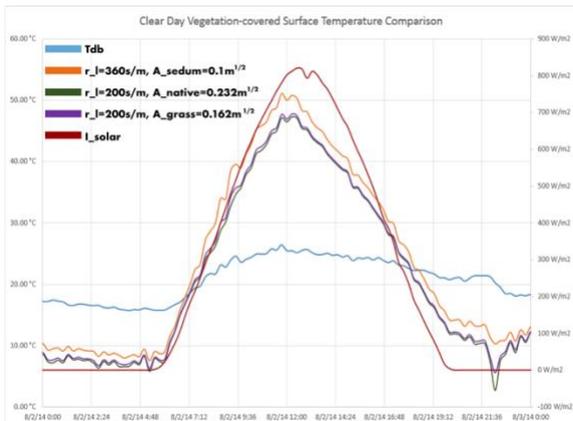


Figure 12. Vegetation-covered surface temperature with internal leaf resistance and leaf size variations on a clear day, August 2nd, 2014.

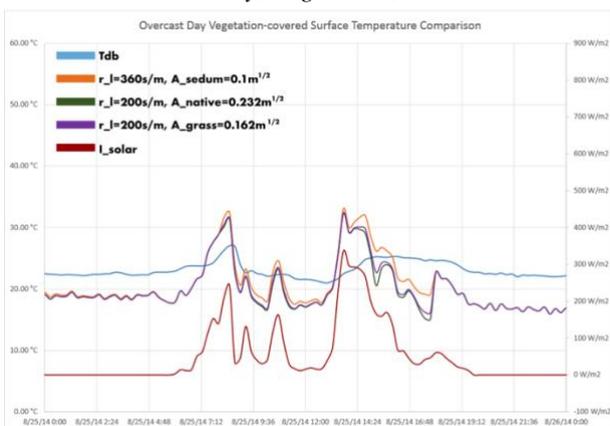


Figure 13. Vegetation-covered surface temperature with internal leaf resistance and leaf size variations on an overcast day, August 25th, 2014.

Conclusion

With the energy model proposed in this paper, three green roof elements - surface colour, soil depth and plant type - can be used to reduce surface temperature.

Reducing the solar absorption of a roof surface is the key to lowering the surface temperature during the day. Especially in a high-radiation environment, the colour of the surface affects solar absorption. The lighter the colour, the lower the surface temperature.

Minimizing the soil depth also reduces the surface temperature in warm weather. In a high-radiation environment, the surface temperature is higher than the soil temperature. A shallower soil stores less heat and therefore keeps the surface temperature lower.

The latent heat transferred by evapotranspiration reduces the surface temperature of the vegetation. Therefore, increasing the evaporation rate cools down the green roof surface. A plant with lower internal leaf resistance and

larger leaf size has a higher evaporation rate. The native plant performs best in reducing surface temperature; the grass performs second-best; and the worst-performing is the sedum.

For the tested climate, a lighter colour, shallower soil layer, and plants with lower internal leaf resistance and larger leaf will work best to reduce the heat island effect.

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