Study on the relationship between amplitude and wavelength of internal wave based on Euler numerical model

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Introduction

It is very important to estimate the amplitude and wavelength of internal waves from radar images. Different amplitudes usually correspond to different wavelengths. In order to further study, Euler models of KdV and eKdV in multilayer flow system were established based on Gerris software, and the internal waves in stable state under different stratification conditions were simulated. The results of euler model were analyzed with SAR data and field measurement data.

Methods

Setting of initial field of Euler numerical model

KdV theory：

$$\eta(x,z)=\eta_0\varphi(z)sech^2\left(\frac{x}{L}\right)$$

$$w(x,z)=-\frac{\partial c_f\eta(x,z)}{\partial x}$$

$$u(x,z)=\frac{\partial c_f\eta(x,z)}{\partial z}$$

eKdV theory：

$$\eta(x,z)=\frac{\eta_0\psi(z)}{b+(1-b)cosh^2(\lambda x)}$$

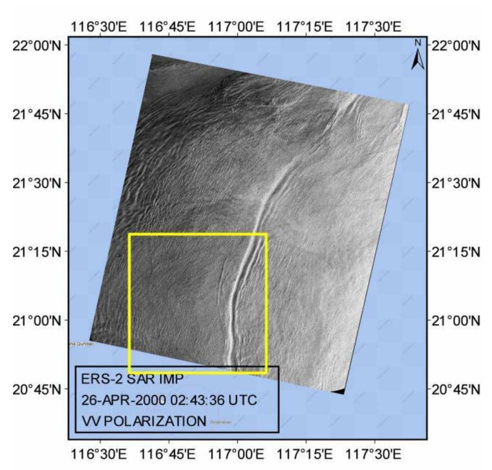
$$\psi(z)=\varphi(z)-\eta_0T(z)$$

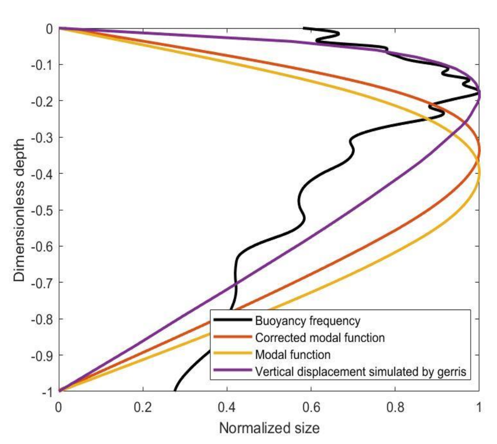
$$w(x,z)=-\frac{\partial c_f\eta(x,z)}{\partial x}$$

$$w(x,z)=-\frac{\partial c_f\eta(x,z)}{\partial x}$$

η:Vertical displacement of each layer of the internal wave
w:Vertical velocity of each layer of internal wave
u:Horizontal velocity of each layer of internal wave
φ(z): Modal function
ψ(z)：Corrected modal function
T(z): corrective function
c_f ：internal wave phase velocity
L: KdV half width
$1/\lambda$：eKdV half width

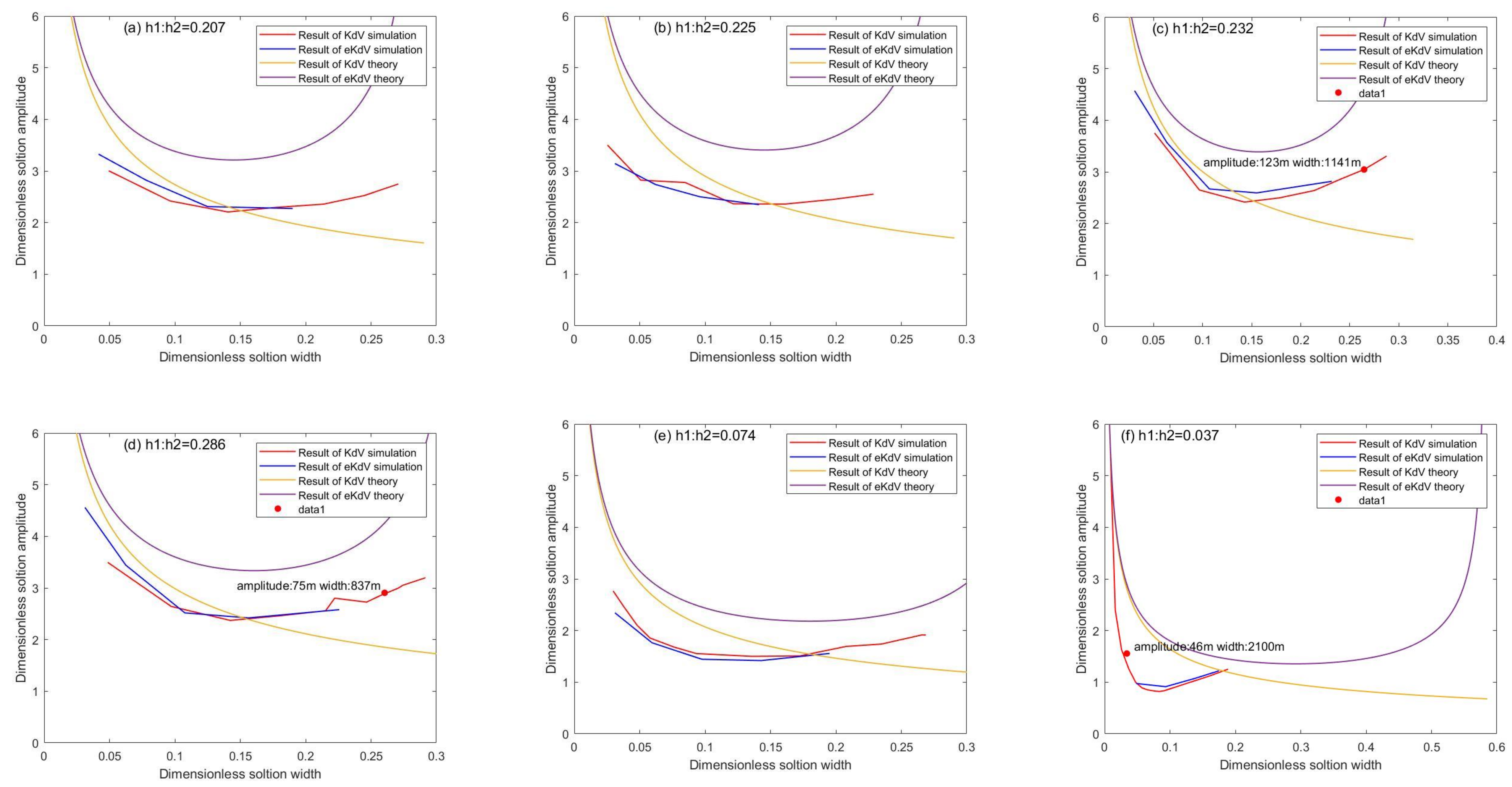
Results





The picture above is the satellite image of the internal wave and the vertical plane curve of different parameters at the site, including Buoyancy frequency, Corrected modal function, Modal function and Vertical displacement simulated by gerris.

Results



The results of gerris simulation under different layering conditions and the relationship between different amplitude wavelengths and amplitudes of KdV theory and eKdV theory under multi-layer flow system. The points marked are measured values.

experiment number	Small amplitude (m)	Large amplitude (m)	Measured amplitude (m)	Depth (m)	H1:H2	KdV theoretical width (m)	eKdV theoretical width (m)	SAR image width (m)	gerris simulation width (m)
c	27	123	>100	375	0.23	759.081	1579.38	1141	1127
d	27	75	73	288	0.28	536.89	1200.99	837	821
f	46	390	50	1350	0.04	4344.37	4444.07	2100	2203

The simulation results in experiment c,d,f were compared with SAR data and measured data, The data marked in red are the final results.

Conclusion

- The curve of the maximum offset position of each layer in the final stable state of the numerical simulation is close to the Buoyancy frequency, and the offset position of each layer in the eKdV theory is closer to the final stable result than that in the KdV theory.
- In the numerical simulation results, the wavelength of eKdV and KdV decreases first and then increases with the increase of amplitude. The simulated results are closer to the measured ones.
- When H1 and H2 are close, in the case of small amplitudes, the results simulated by gerris are close to both KdV and eKdV. In the case of large amplitude, the results are closer to the eKdV theory. When H1/H2 is sufficient small, the numerical simulation results have a big difference with KdV and eKdV at large amplitude.
- Compared with SAR data and measured data, the simulated amplitude results were determined, when H1/H2> 0.2, the result is a large amplitude, and the wavelength of the numerical simulation is between the wavelength of eKdV and KdV theory. When H1/H2<0.1, the result is small amplitude, and the wavelength of numerical simulation is smaller than that of KdV and eKdV.

Reference

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